REPORT ON THE IN-TANK MIXING STUDY

EE-100
REPORT ON THE IN-TANK MIXING STUDY

by

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Abstract

Scale model experiments of mixing in rectangular tanks by line bubble plumes are described with the object being to predict mixing of dispersant into crude oil in full sized oil tanker compartments. The experimental tank length was 1/10th of the length of anticipated full sized tanks. The gas bubble plume was created by air exiting from a line source at the bottom of tanks filled with water. Fluoroscein dye was used in the tanks to trace the mixing. From samples of tank fluid collected during the experiments, the average dye concentration variation was calculated and used to describe the amount of mixing left to be completed.

Through dimensional analysis it was determined that the Froude number was the most important dimensionless group and the independent variables of the system were defined. The resulting change in mixing during the experiments as the independent variables were varied was noted. The data was nondimensionalized and plotted in log-log form so that a straight line could be fitted through the points. To fit the data, the slope of the line increases when the fluid reaches a partially mixed state. An exponential relationship between average concentration variation and mixing time was derived. Dispersant mixing times in full sized tanks containing crude oil are predicted.

Experiments with oil and dispersant were conducted. The degree of mixing was measured using a radioactive tracer in the dispersant. No significant differences were noticed between oil mixing and water mixing for similar high Reynolds numbers. The dispersant concentration variations with time in light crude, agree with the mathematical model predictions. The model could not be used for a viscous oil such as cold Bunker.
Résumé

Cette étude a pour objet l'élaboration d'un modèle mathématique servant à prédire l'efficacité de mélange d'un dispersant ajouté à du pétrole brut dans les réservoirs de pétrolier. Un modèle réduit a été utilisé pour vérifier l'efficacité d'un rideau de bulles de gaz comme système de mélange dans un réservoir rectangulaire. Le réservoir utilisé avait un dixième de la longueur et de la largeur réelle. Le rideau de bulles d'air était créé avec l'aide d'un tuyau perforé, placé au fond d'un réservoir d'eau, attaché à une ligne d'air. Un colorant, fluorescine, a été utilisé pour évaluer l'effet de mélange. La variation de la concentration moyenne de colorant était établie suite à un échantillonnage vertical et horizontal. Cette concentration moyenne de colorant servait d'indicateur de "mélange non-complété", mesure utilisée pour l'analyse des résultats.

Une analyse dimensionnelle a révélé que le nombre de Froude était le plus important groupe adimensionnel à considérer pour le modèle réduit. Les variables indépendantes du modèle mathématique ont été établies. La variation de ces variables indépendantes affectait l'efficacité du mélange. À partir des résultats obtenus, transformés sous une forme sans dimension et tracés sur un graphe log-log, une corrélation linéaire a été établie entre le temps de mélange et la variation de la concentration moyenne. Une courbe exponentielle décrit donc la relation entre ces deux variables. À l'aide de cette courbe, le temps de mélange d'un dispersant dans un réservoir de pétrolier, grandeur réelle, a été estimé.

Une série d'expériences avec un pétrole brut léger et un dispersant a été faite. Le degré de mélange a été mesuré avec l'aide d'un produit radioactif soluble dans le dispersant. Les mêmes tendances qu'avec le mélange eau/colorant ont été remarquées pour un nombre de Reynolds semblable, c'est-à-dire élevé. L'évolution dans le temps des valeurs de mélange non-complété de pétrole léger/dispersant concorde avec les prédictions du modèle mathématique développé en utilisant l'eau comme liquide. Il en serait autrement pour un pétrole visqueux tel que le Bunker à basse température.
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1. Introduction

Oil tankers operating in the Arctic could suffer long delays between damage of the tanker and the arrival of help. Installation of gas bubble plume systems in the compartments of oil tankers has been suggested as a method which the tanker crew could use to mix treating agents into the oil in the damaged compartment(s). Dispersant is the most likely treating agent although sinking agents and gelling agents should also be considered. It is proposed that the gas and dispersant could enter the tanker compartments at the bottom through perforated pipes. The turbulent action of the plume would serve to mix the dispersant into the oil through the mixing in the plume and the induced convection which carries oil into the plume. The dispersant would have to be mixed into the oil before significant amounts of oil have escaped. This method would also be a more effective method of adding dispersant to the oil. The conventional approach is to drop the dispersant by plane onto the oil slick. Large quantities of dispersant are required in order for it to reach all the oil in the slick in appropriate amounts. Dispersant that is pre-mixed in the compartment can be added to the oil in amounts close to the theoretical ratio required and all of the oil can be treated.

The purpose of this report is to investigate the feasibility of using gas bubble plumes for the in-tank mixing of dispersant in such a situation. The report also compares three arrangements of the plume: a plume along one wall of the tank, along two coincident walls, and a "T" arrangement.
To study the mixing, scale model tests were carried out in a cubical tank 1.22 m along each wall and in a rectangular tank with a width and length of 1.22 m and a height of 2.44 m. Water is used as the test fluid to be mixed. Fluorescein dye in water is added to the tank fluid in place of dispersant to trace the mixing. The model is approximately 1/10th the anticipated size of a prototype, or full scale, compartment. By coincidence, this choice of length ratio and model fluid gives Reynolds number similarity between the model and prototype.

The dimensionless group scaled is the Froude number since it is the group which describes the buoyant momentum of the plume.

In the model tests the gas bubble plume is created by air exiting from a perforated 1.27 cm copper pipe in the bottom of the tank. Dye is introduced into the tank for a short time only once the plume has started since the research concentrates on determining the mixing of the tank from the gas bubble plume instead of the combined action of the dye injection and gas bubble plume. When the dye solution is pumped into the tank before the plume is started it can contribute significantly to the mixing. While this is an advantage in the prototype minimizing this mixing during the model tests allows a better determination of the gas bubble plume mixing. In an actual full size tank the pumping of the dispersant would begin at the same time as the mixing.

Tank fluid is sampled by twelve small gear pumps which pump fluid continuously during the experiments. Samples are collected at the pump outlets and measured in a fluorometer to determine the dye concentration.
Twelve points from one horizontal cross sectional area of the tank are sampled at a time and four to six different cross sectional areas are sampled, depending on the fluid depth. The average concentration variation of the dye in the tank, $U$, is calculated and plotted against the mixing time.

During the experiments gas flow rates, fluid depth, bubble plume arrangements, and surface tension were all varied. Flow rates from 0.32 l/s to 2.0 l/s and fluid depths from 0.66 m to 2.44 m were tested. During one experiment the surface tension was halved with the addition of a surfactant to the tank water.

As expected tank mixing increases ($U$ decreases) as the gas flow rate increases but the increase levels off as the gas flow rate increases and appears to approach an asymptotic value. Varying the fluid depth does not have any effect on the mixing except at small depths where the increase in mixing is attributed to a disproportionate increase in the ratio of the mixing zone to the fluid volume. This small depth effect decreases as the gas flow rate increases. The "T" plume arrangement is the most effective mixer with the two-arm plume (plumes along two coincident walls) next and the one-arm plume (a plume along one tank wall) last.

Using dimensional analysis, a characteristic system time is defined. The optimum form of the characteristic time is found by comparing the scatter of the data as the characteristic time is varied. Plotted nondimensionally the data is convex to the origin suggesting an exponential relationship between average concentration variation and dimensionless mixing time.
When the data is plotted nondimensionally in log-log form it has a knee at \( U = 0.3 \) so that a straight line drawn through the data increases its downwards slope after this point. The correlation of the data to such a curve, fitted using least squares, is within the limits of experimental error and a final form of the relationship between average concentration variation, \( U \), and dimensionless mixing time is derived from the curve. In the final form, \( U \) is a function of mixing time, gas flow rate, tank wall length, and bubble plume length.

The result can be used to predict the mixing of dispersant into crude oil once the independent variables of the system are known. It was found that fine bubble plumes were efficient mixers and should be able to mix full size tanks within the time limits required.
WATER AND DYE EXPERIMENTS

2. Test Materials and Apparatus

Most of the experiments were done in a cubical tank made of 6 mm (1/4 in.) thick molded polyethylene, measuring 1.22 m on a side and open at the top. This is a convenient size for the laboratory while still being large compared to bubble diameter so it was expected to yield well-modeled mixing. For example a compartment in a oil tanker is typically 12 m deep and roughly cubical, so the model tank gives a scale of 1:10 for model:prototype length dimensions. The bottom of the tank is fitted with a drain and valve.

Ordinary tap water is used as the tank fluid. It is the fluid in which the bubble plume acts and into which the tracer, fluorescein dye, is mixed. The water has a surface tension of 0.065 N/m, a viscosity of approximately $1 \times 10^{-6} \text{ m}^2/\text{s}$ and it is at or slightly below room temperature, 20°C. This choice of model tank size and fluid has a Reynolds number of the same order as the prototype tank and fluid.

The perforated pipe used to create the bubble plume is made from 12.7 mm (1/2 in.) diameter copper tubing. Figure 2.1 shows a diagram of the bubble plume pipe. This size corresponds to a 0.127 m (5 in.) diameter pipe in the prototype or full scale tank although the full scaled gas delivery pipe need not be this diameter. The bubble plume pipe has two "arms", each 1.17 m long. The gas and fluid is released into the tank through 1.59 mm (1/16 in.) diameter holes along the top of the pipe arms. Each arm has eighteen of these holes, spaced 60.3 mm apart. The sum of the area of the holes in both the arms is equal to half of the inside radial cross sectional area of the copper tubing. The bubble plume pipe arms are joined at right angles to each
Note: Not drawn to scale
All dimensions are in inches
All tubing and fittings are 1.27 cm (1/2 in.) copper

1.27 (1/2 in.) copper tubing
0.159 dia holes on 6.03 centers
0.159 dia holes on 5.72 centers
inlet for air, dye and water
1.27 (1/2 in.) ball valve

Figure 2.1: two-arm bubble plume pipe
other and placed in the bottom of the tank so that the arms run along two coincident walls. A length of copper tubing rises vertically from the intersection of the bubble plume pipe arms to the top of the tank, where it is connected to the gas and fluid supply lines. Fluid refers to the two fluids which enter the tank during the experiments: a fluorescein and water solution, which is referred to as the dye, and water. A valve is placed in one of the arms, near their intersection, so that it can be closed off when desired. In this manner it is possible to determine if the mixing is improved when two arms, rather than one, are used to create the bubble plume. These two configurations are called the one-arm and two-arm bubble plumes.

Another pipe configuration that was tested is called the 'T' bubble plume pipe. It is made of the same material as the other pipe with the same size and total number of holes as shown in Figure 2.2. It is in the shape of a 'T': one length of pipe is along a wall of the tank and the stem runs along the middle of the tank bottom. Copper tubing rises vertically from the intersection of the stem and cap until it meets the gas and fluid supply lines at the top of the tank.

In some experiments the dye and water is pumped into the tank through the bubble plume pipe arms, but in other experiments the dye and water is pumped into the tank through a separate dye arm, 1.17 m long, made from 12.7 mm (1/2 in.) diameter copper tubing. The dye arm has 16 holes each 3.18 mm (1/8 in.) in diameter to allow the dye into the tank and are spaced as shown in Figure 2.3. The dye arm is attached to the pump outlet by copper tubing that runs vertically from the center of the dye arm to the top of the tank. From the top of the tank, the tubing is connected to the pump
Figure 2.2: T bubble plume

Note: Not drawn to scale
All dimensions are in centimeters
All tubing and fittings are 1.27 cm (1/2 in.) copper
Note: Not drawn to scale
All dimensions are in centimeters
All tubing and fittings are 1.27 cm (1/2 in.) copper

0.318 dia holes on 6.98 centers
inlet for fluid
copper T fitting
0.318 dia holes on 6.98 centers
1.27 cm (1/2 in.) copper tubing
cap

cap

Figure 2.3: Dye arm
outlet by flexible hose. The dye arm is placed horizontally in the tank so that it is suspended halfway between the surface of the tank fluid and the tank bottom, directly above one of the bubble plume arms. It is placed to insure that the dye will enter the tank in the midheight of the bubble plume. It is only used with the one-arm bubble plume.

A 2.44 m high, rectangular tank with sides 1.22 m and 1.16 m, constructed from plywood and coated with epoxy resin, is used for experiments requiring a fluid depth of 2.44 m. The same two-arm bubble plume pipe is used to create the bubble plume, the only difference being that a 2.5 m length of copper tubing is needed to reach from the bubble plume pipe at the bottom of the tank to the connections for the air and fluid at the top of the tank. When the dye arm is used instead of the bubble plume arms to deliver the dye and water, it is placed 1.22 m from the bottom of the tank, in the same manner as before.

The gas used is air from a central air supply which is at a pressure of 830 kPa gage (120 psi). Meters, installed in the air supply lines, measure the mass flow rate which was controlled by metering valves. The pressure in the supply pipes at the bottom of the tank is less than one quarter of the atmospheric pressure. Because of the large range of gas flows required in the experiments, two gas flow meters are used: a Fisher Scientific variable area flow meter in the range of 0.05 to 0.60 l/s, and a square-edged orifice meter in the range of 2.0 to 5.0 l/s. The orifice meter consists of a square edged orifice of 7.936 mm (5/16 in.) diameter placed in the gas flow line according to ASME specifications, a pressure gage at the inlet side of the orifice, and a manometer which measures pressure loss across the orifice as
shown in Figures 2.4 and 2.5. Mass flow rate is calculated from the pressure and manometer readings. A pressure gage at the inlet side of the variable area flow meter is also used to calculate the mass flow rate from the meter readings. Pressure regulators in the air supply lines lower the pressure and hold it constant for each gas flow rate.

The fluorescent dye, fluorescein, is used as the tracer to determine the mixing. It is diluted with tap water to concentrations in the range of 0.5 to 50 parts per million and stored in a 120 l barrel. This solution is called the dye. From the barrel it is pumped into the tank by a vane pump at a flow rate of 0.16 to 0.20 l/s. A turbine type flow meter, placed in the fluid line, measures the flow rate of the dye. Water, in another 120 l barrel, is also pumped into the tank, at the same flow rates, by the vane pump as shown in Figures 2.4 and 2.5.

The air and fluid (dye and water) lines all meet at a "T" at the top of the tank lid. In each line, just before the T, a ball valve is installed to start and stop the flows and to prevent the air supply from evacuating the fluid from the vane pump and fluid lines. Schematics of the air and fluid supply lines to the tank are shown in Figure 2.4 and 2.5.

To sample the tank fluid as the mixing progresses, twelve small gear pumps pump fluid continuously from sample points at different locations in the tank. The flow rate of the pumps is small, 16 to 20 ml/s, and the total volume of tank fluid moved by the sampling pumps is at most only three percent of the total fluid volume being mixed during an experiment. The sampling pumps are housed on a tank lid which is constructed from plywood.
Figure 2.4: Schematic of air and fluid lines with air and fluid pumped into the tank through the bubble plume pipe

N.B. The orifice meter was used for gas flows of 1.1 and 2.0 L/s and the variable area flow meter was used for gas flows of 0.32 and 0.48 L/s.
Figure 2.5: Schematic of air and fluid lines: dye and water pumped into the tank through the dye arm

N.B. The orifice meter was used for gas flows of 1.1 and 2.0 L/s and the variable area flow meter was used for gas flows of 0.32 and 0.48 L/s.
The tank lid is not close fitting—the tank fluid at the top is at atmospheric pressure. The lid only serves as a place to attach the sampling apparatus.

3.18 mm (1/8 in.) I.D. Vin-Con flexible tubing is used for the inlet and outlet lines of the sampling pumps. The pumps are not self-priming. To keep them primed, a 6.35 mm (1/4 in.) toggle valve is closed when the pumps are not running. The total length of inlet and outlet lines is the same for each sampling point: 3.30 m for the 1.22 m tank, and 4.52 m for the 2.44 m tank.

To position the inlet lines at the sampling points in the tank, the inlet end of each line is fixed to a vertical rod. The tubing extends 13 mm past the end of the rod. Twelve rods, 6.35 x 12.7 mm (1/4 x 1/2 in.) rectangular rods for the 1.22 m tank and 12.7 x 12.7 x 6.35 mm (1/2 x 1/2 x 1/4 in.) angles for the 2.44 m tank, are held vertically by angle brackets on the lid of the tank and pass through holes in the tank lid to reach the tank fluid. Horizontally there are twelve sampling points, one for each pump. The rods and brackets are set out on a grid: four rows of sampling points spaced 0.356 m apart from each other, each row containing three sampling points spaced 0.533 m apart. There is a 0.076 m border between the outside sampling points and the sides of the tank. Diagrams of the sampling points are given in Appendix B. Each rod is fixed to its bracket with two screws. To change the height of the sampling point, each rod has holes for the screws all along its length so it can be moved up and down as shown in Figure 2.6. The tank fluid is sampled at four heights, 0.05, 0.20, 0.48, and 0.61 m, from the tank bottom when the fluid depth is 0.66 m; five heights, 0.15, 0.33, 0.61, 0.89, and 1.07 m, when the fluid depth is 1.22 m; and six
heights, 0.15, 0.51, 0.86, 1.56, 1.93, and 2.29 m, when the fluid depth is 2.44 m.

Tank fluid is collected in fluorometer cuvettes, 10 mm O.D. (8 mm I.D.), 75 mm long, at the outlet end of the sampling pump tubing. The cuvettes are held in test tube racks and the pump outlets are situated above the racks. Tank fluid not collected in the cuvettes falls back into the tank. To sample the tank fluid at one height, fifty four cuvettes are needed. Afterwards they are cleaned and reused in future experiments.

A diagram of the sampling system, showing only one of the twelve rod-tubing-pump assemblies, is drawn in Figure 2.6

The amount of fluorescein dye in each cuvette is measured by a model A-4 Farand fluorometer and compared to the amount of fluorescein in a fully mixed sample of tank fluid. The fluorometer is adjusted to compensate for background signals from fluorescein present in the tank fluid from previous experiments.

For experiments at reduced surface tension, the surface active agent "Zonyl" A, manufactured by E. I. du Pont De Nemours & Co., is added to the tank fluid. Surface tensions are measured by a "Fisher Scientific" surface tensiomet.
Figure 2.6: Sampling System

Note: Only one of twelve assemblies shown
Not drawn to scale

- Screw hole
- Rod
- Screw
- Bracket
- Toggle valve
- Guide for outlet
- Outlet
- Cuvette
- Test tube rack
- Slider
- Tank lid
- Gear pump
- 0.476 cm (3/16 in.) I.D. tubing
- 1.27 cm
- Inlet
- Tank
3. Procedure

Two slightly different procedures, which shall be called procedure \#1 and procedure \#2, were used during the course of the experiments. Procedure \#2 was developed after preliminary analysis of the data from experiments using procedure \#1 indicated that injecting the dye into the fluid through the one-arm bubble plume pipe was contributing to the overall mixing. Although such a contribution to the mixing may be beneficial in actual process operations; minimizing it allows a more accurate determination of the principal bubble plume mixing. The main differences between procedures is that a different pipe, with larger holes, is used for the dye injection in procedure \#2, and the dye is pumped into the tank over a shorter period of time.

To begin procedure \#2, once the tank has been filled with tap water to the desired fluid depth (0.66, 1.22, or 2.44 m), the gas is set at the chosen flow rate and pressure by adjustments of the flow meter and pressure regulator in the case of the variable area flow meter or by opening a valve in the air line to the orifice meter. Once the gas flow has established the bubble plume and associated turbulent flow in the tank fluid (this takes at most thirty seconds), two samples (i.e., two cuvettes) of the tank fluid are collected for use as background samples and then the dye injection is started.

To start the dye flow, the valve between the fluorescein dye solution and the vane pump is opened. Then the vane pump is turned on and immediately
begins to pump the fluorescein dye solution into the tank at a flow rate of 0.165 to 0.205 l/s. The dye is injected into the tank via the dye arm. Immediately following, without turning off the pump, the dye valve is closed and the valve between the water supply and the pump is opened and water is pumped into the tank for fifteen seconds, at the same flow rate, to push all of the dye out of the dye arm into the tank. After this the vane pump is turned off. During the time that the pump is on, or immediately after it is turned off, the sampling pumps are turned on.

Once the sampling pumps are on and all the dye has been injected, the fluid sampling can begin. For each experiment a sampling time interval is chosen and samples are collected, usually in four sets, each one time interval apart; the first sample set being collected at the end of the first time interval. The sample collection time starts 12 seconds after the vane pump is turned on to account for the start of the mixing time and the travel time of the tank fluid between tank and sampling pump outlet. The mixing time is thought of as starting midway though the dye injection. Since it takes five seconds from the starting of the vane pump for the dye to reach the first hole in the dye or bubble plume arm pipe and seven seconds for the dye to reach the hole at the end of the pipe, on average it takes six seconds for the dye to reach the tank from the start of pumping. The dye is pumped into the tank for five seconds so the midpoint of pumping is two and one-half seconds, adding these together, the midpoint of the dye injection is eight and one-half seconds. It takes three and one-half seconds for the fluid to travel from a sampling point in the tank to the outlet of a sampling pump (four to five seconds when the tank depth is 2.44 m). Altogether this means
that the sample collection time starts twelve seconds after the vane pump is turned on.

If samples are to be collected every 30 seconds the first samples would be taken 42 seconds after the vane pump was started, the next at 72 seconds, and so on.

Samples are collected by pushing test tube racks, containing twelve cuvettes, underneath the outlets of the sampling pumps and then pulling the racks away when the cuvettes have been filled. Normally four sets of samples are collected. The accuracy of the sampling time is plus or minus two seconds: one second for the accuracy of the starting time and one second for sample collection, either filling cuvettes too early or leaving them at the pump outlets too long.

Once the sets of samples are collected, the tank is allowed to continue mixing for another ten to fifteen minutes and then four samples are collected for use as fully mixed samples.

To prepare for the measurement of fluorescein in the samples, the fluorometer is calibrated so that the background samples give zero relative fluorescence and the fully mixed samples give 50%–60% of the full scale of the instrument. All the samples are measured (including the fully mixed samples) and the readings, the percent needle deflection of full scale, are recorded on a standard data sheet. The accuracy of the fluorometer measurements, unless special care is taken, is 2%. One data sheet is filled
out for each rod height of the experiment. All the pertinent information for each experiment is also recorded on the data sheet: rod height, gas flow rate, fluid flow rate, pumping time for the dye and water, fluid depth, pipe configuration, and sampling time interval.

After all the samples have been measured the cuvettes are emptied and each one is rinsed twice with tap water, twice with distilled water, and then dried. Since the cuvettes are reused, cleaning insures that fluoroscien is not left in the cuvettes to contaminate new samples. Every week or two all of the cuvettes are soaked in a solution of 20% hydrochloric acid and 80% water for a thorough cleaning, and then rinsed and dried as before.

To continue the experiment the rods are adjusted to a new height in the tank—so the sampling inlets are all at a new location—and the procedure outlined above is repeated from the starting of the bubble plume to the cleaning of the test tubes. An experiment is complete when samples have been collected at all the rod heights (four heights for the 0.66 m fluid depth, five for 1.22 m, and six for 2.44 m).

To start a new experiment, the gas flow rate or the fluid depth is changed. The 0.66 m fluid depth and 1.22 m fluid depth experiments are carried out in the 1.22 m plastic tank while the 2.44 m fluid depth experiments are done in the 2.44 m wooden tank. When the 0.66 m depth is tested a 5 cm aluminum angle is suspended in the tank, above the fluid surface, underneath the sampling pumps outlets, to catch the fluid from the sampling pumps and
prevent it from falling directly into the tank. The angle dumps the fluid back into the tank about 4 cm above the fluid surface, over the bubble plume. This precaution is only taken at the 0.66 m depth since the fluid surface is 0.65 m below the pump outlets (compared to 0.15 m when the depth is 1.22 m) and the falling liquid could have enough energy to contribute to the mixing, especially since the total fluid volume is not large.

Using procedure 2, data for each height was collected at four flow rates: 0.32, 0.46, 1.1, and 2.0 l/s.

The original procedure, procedure 1, differed from 2 in the manner of dye injection and the start of the mixing time. For procedure 1, the tank fluid is initially still (the bubble plume is off) and the dye is injected into the tank via the bubble plume pipe arm(s) for 20 seconds followed by water for 15 seconds to flush the dye out of the pipe. After this is done the vane pump is turned off, the gas supply valve is opened, and the bubble plume starts. At this point the mixing time starts and the sample collection time starts three seconds later. If samples are to be taken every 30 seconds then the first sample is collected 30 seconds after the gas valve is opened, and the rest at 60, 90 and 120 seconds from the start.

Using procedure 1 data was collected at flow rates of 0.16, 0.32, 0.46 l/s when the depth was 0.66m; 0.16, 0.32, 0.46, 1.1, and 2.0 l/s at 1.22 m; and 0.32, 0.48, and 1.1 l/s at 2.44 m.
During one experiment (flow rate of 0.32 l/s, fluid depth of 1.22 m) the surface tension of the tank water was lowered by adding 0.020 l (0.0011 % of the total fluid volume) of "Zonyl"-A, a surface active agent. The "Zonyl" A reduced the surface tension of the tank fluid by 54 %. Tank fluid containing 0.0011 % "Zonyl" A by volume has a surface tension of 0.35 N/m, compared to 0.65 N/m without "Zonyl" A. Procedure #1 was used during this experiment.

The "T" and two-arm bubble plume were tested at a fluid depth of 1.22 m and a flow rate of 0.46 l/s. For these configuration procedure #3 is used:

in order for the dye to be injected along the total horizontal length of the plume, the dye and water were pumped into the tank via the bubble plume pipe. While the dye and water are being pumped into the tank, the bubble plume is kept running. The rest of the experimental procedure, including the timing of the dye injection and sample collecting, is identical to procedure #2.

At each depth one or more experiments were carried out without a gas bubble plume to try to determine the mixing in the tank from the different methods of dye injection. During these "no air" experiments, either procedure is used but the gas supply valve is never opened.
4. Data Analysis

The purpose of the data analysis is to reduce the data collected from each experiment into a meaningful description of the amount of mixing left to be completed in the tank at each sample collection time. From fluorometer measurements the concentration of dye in each sample and the concentration of dye in the fully mixed tank is known. The tank is fully mixed when the dye is evenly distributed throughout the tank fluid. During the experiments, if six random samples did not differ by more than 2% the tank was considered fully mixed.

The average of the sample concentration variations from the fully mixed concentration is used to describe the mixing. This quantity will be called $U$. Ideally $U$ is the square root of the integral over the entire tank of the square of the normalized variation of the dye concentration at each point in the tank.

$$U^2 = \frac{1}{V} \int_{V} \left( \frac{c_{dV} - c_f}{c_f} \right)^2 dV$$  \hspace{1cm} (4.1)$$

where $c_{dV}$ is the dye concentration in the incremental volume $dV$, $c_f$ is the fully mixed concentration, and $V$ is the tank volume. The variation, $c_{dV}-c_f$, is divided by $c_f$ in order to normalize it.
Since it is impossible to measure the concentration at each point in the tank, the tank concentration is measured at a finite number of points, enough so that all the areas of the tank are represented but not so many as to interfere with the mixing. The horizontal cross sectional area of the tank is sampled at 12 points and 4 to 6 cross sectional areas are used in each experiment, depending on the fluid depth. Each horizontal cross sectional area is identified by its height above the tank bottom. The total number of points sampled at each depth is the number of heights sampled multiplied by 12: 48 points for the depth of 0.66 m, 60 for 1.22 m, and 72 for 2.44 m. Instead of an integral over the entire tank the summation of concentration variations at all the sampling points is calculated.

\[ U^2 = \sum_{i=1}^{N} \left( \frac{c_i - c_f}{c_f} \right)^2 w_i \]  

(4.2)

where \( c_i \) is the dye concentration at the \( i \)th sampling point, \( w_i \) is the volume of tank fluid associated with each sample point divided by the total volume of the tank fluid, and \( N \) is the number of sampling points. The boundaries of the sampling volumes were the tank walls, the fluid surface, and the midpoint of the distances between the sampling points and neighboring sampling points above, below and to the side. For the calculations and values of \( w_i \) for each fluid depth see Figures A.1-4.

When \( U \) equals zero all of the \( c_i \)'s are equal to \( c_f \) and the dye is completely mixed into the tank fluid. If \( U \) is equal to 0.4 then the dye concentration at any point in the tank is on the average either 40% more or 40% less than the fully mixed concentration. Ideally \( U \) should approach infinity when the dye
has just been released. In this case most of the $c_i$'s will be zero but a few will have very high values since the dye will be highly concentrated in a small region.

To form a description of how the mixing proceeds with time, samples are taken at four time intervals. The complete data set for one air flow and one fluid depth consists of 192, 240 or 288 measurements depending on fluid depth. A computer program is used to reduce the data and find the average concentration variation, $U$, at each time sampled. The algorithm reads the data file of the experiment, computes the values of $U$ and outputs a table of the sample collection times and the corresponding values of $U$.

For each experiment, a curve of concentration variation with mixing time can be plotted. Curves from different experiments can be compared to see which mix faster and how changes in the parameters have affected the mixing.
5. Results

Examining the results from experiments in which the bubble plume is not used and the only mixing is by the injection of dye and water into the tank via the one-arm bubble plume pipe (procedure *1), it can be seen in Figure 5.1 that the injection is premixing the tank at the 1.22 m and 0.66 m depth. Injecting the dye into the tank via the separate dye arm (procedure *2) halves the premixing at the 1.22 m fluid depth. This gives reason to believe that the data collected from experiments using procedure *1 may not accurately measure the mixing of the tank fluid by the bubble plume but rather the data measures an overall mixing that has contributions from the dye injection and from the bubble plume mixing. Many experiments were redone using the procedure *2, injecting the dye and water into the tank through a separate dye arm after the bubble plume is started, so that the bubble plume mixing could be isolated and measured. A comparison of the two procedures for the same air flow rate and fluid depth reveals that the tank fluid is considerably premixed when procedure *1 is used but that the effect of the premixing decreases as fluid depth, gas flow rate, and mixing time increase. The different procedures have no noticeable effect at the 2.44 m fluid depth, enough effect at 1.22 m to make the value of $U$ at $t = 30$ sec in the curves of $Q = 0.32 \text{l/s}$ disagree by 12%, and an effect at 0.66 m that is so great that all of the data from that fluid depth using procedure *1 had to be discarded; see Figure 5.2 for the curves of procedure *1 and *2 at $h=0.66$ m and $Q = 0.48 \text{l/s}$.
FIGURE 5.1 Some Comparative Measurements for Procedures 1 and 2
Figure 5.2: Tank mixing for procedures #1 and #2. h = 0.66 m, Q = 0.48 l/s
The data from procedure 
1 which has been used for the analysis is that
from h = 1.22 m, Q = 0.48, 1.1, and 2.0 l/s; h = 2.44 m, Q = 0.32, 0.48, and 1.1
l/s; and the last three data points from the curve h = 1.22 m, Q = 0.32 l/s. All
of the data collected using procedure 12 has been used in the data analysis.

Figure 5.3 is a graph of U versus t for different gas flow rates at h = 1.22 m
with the bubble plume along only one wall of the tank. At any value of U the
mixing time, t, decreases as the gas flow rate is increased.

Figure 5.4 is the result obtained when the surface tension of the tank fluid
was halved by adding the surfactant "Zonyl" A. Although this experiment
used procedure 1 (the value of the first data point may be affected by
premixing) it can be seen that the lowered surface tension appears to have
no effect on the mixing when this curve is compared to another at the same
flow rate and depth also using procedure 1 and a curve at the same flow
rate and depth using procedure 2. No conclusion can be drawn from this
experiment because it is possible that the change in surface tension was
not great enough to change the bubble size or that "Zonyl" A only changes
static surface tension significantly and has less effect on the dynamic
changes that take place when bubbles are formed.

Continuing with the same basic comparison of U versus mixing time, it can
be seen in Figure 5.5 that for gas flow rates from 0.32 to 2.0 l/s the mixing
of the 1.22 m tank and the 2.44 m tank are the same, given that the
experimental error in the U measurements is 2% and the error in mixing
time is ± two seconds. The obvious conclusion is the mixing is independent
of fluid depth. However the mixing curves of the 0.66 m tank do not coincide
Figure 5.3: Tank mixing as the gas flow rate increases. h = 1.22 m
Figure 5.4: Tank mixing as surface tension is reduced.

$\Delta \sigma = 0.037 \text{ N/m, procedure #1}$

$\sigma = 0.065 \text{ N/m, procedure #1}$

$\Sigma \sigma = 0.065 \text{ N/m, procedure #2}$

$\text{mixing time, } t (\text{seconds})$

$h = 1.22 \text{ m}, Q = 0.48 \ell/\text{s}$
Figure 5.5: Tank mixing as height and gas flow rate increase
with the mixing of the 1.22 m and 2.44 m tank of the same flow rates. The three fluid depths are compared in Figure 5.6 and 5.7. For concentration variations greater than 0.20, the 0.66 m tank mixes in 65% to 75% of the time of the larger tanks when the flow rate is 0.32 l/s and 80% to 90% of the time when the flow rate is 0.46 l/s. It seems that as the gas flow rate increases the mixing of the 0.66 m fluid depth approaches that of the 1.22 m and 2.44 m depths.

At low fluid depths the faster mixing is most likely caused by a disproportionate increase in the ratio of the mixing zone to fluid volume. At the origin of the bubble plume $k$ is not constant and the plume does not have a linear shape. Also a typical bubble size for air in water is 13 mm which is greater than the bubble plume width over a significant portion of the bubble plume at a fluid depth of 0.66 m. Doubling the fluid depth from 1.22 m to 2.44 m had no noticeable effect on the mixing so increasing the fluid depth even further should not bring about any changes except those related to pressure. The midheight pressure change from 0.66 m to 1.22 m is 3% and from 1.22 m to 2.44 m is 5% so the difference in mixing cannot be attributed to a change in the pressure ratio. Since the emphasis of the analysis is to describe the mixing of large tanks, the effect of low fluid depth has not been investigated and data from this depth is not used. It should be kept in mind that the analysis does not apply to low fluid depths (less than 1.0 m).

At the same gas flow rate, the mixing of the two-arm bubble plume and the "T" bubble plume is faster than the one-arm bubble plume--see Figure 5.8. The mixing of these two plumes was measured for a fluid depth of 1.22 m.
Figure 5.6: Tank mixing as fluid depth decreases. $Q = 0.32 \, \ell/s$
Figure 5.7: Tank mixing as fluid depth decreases. $Q = 0.48 \text{ l/s}$

- $h = 0.66 \text{ m}$
- $h = 1.22 \text{ m}$
- $h = 2.44 \text{ m}$
Figure 5.8: Two-arm and T bubble plumes compared to the one-arm bubble plume. \( h = 1.22 \text{ m} \)
and a flow rate of 0.48 l/s. The two-arm plume at 0.48 l/s mixes faster than the one-arm plume at 0.48 l/s but not as fast as the one-arm plume at 1.1 l/s. The "T" plume at 0.48 l/s mixes as fast as the one-arm plume at 2.0 l/s.

Theoretically the two-arm plume is twice as long as the one-arm plume and the "T" plume is three times longer although the actual pipe used to create the "T" plume is only two times longer. This is because a bubble plume in the middle of the tank is twice as wide as a plume along a tank wall so it has double the effective length. During the experiments the plume pipes did not work as planned. The pressure drop across the 1.59 mm (1/16 in) holes in the perforated pipe was not large enough to force the gas to flow to the ends of the pipe and out of all the holes so the plume was very small or nonexistent along approximately one quarter of the total bubble plume length for both the "T" and two-arm plume.

Although these two plume arrangements could only be tested by injecting the dye through the bubble plume pipe instead of the dye arm this should not have a noticeable effect since this flow rate is large enough to prevent differences in the data from procedures #1 and #2 when the one-arm bubble plume is tested.
6. Dimensional Analysis

The objective of the dimensional analysis is to find a relationship between $U$, the average concentration variation, and a dimensionless mixing time, $t/T$, where $t$ is the mixing time and $T$ is a characteristic system time. The characteristic time is a function of the independent physical variables of the mixing: the tank fluid properties, the fluid volume, the bubble plume properties, and the external forces.

Table 6.1: Independent Variables of the Mixing

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>gravity</td>
</tr>
<tr>
<td>$h$</td>
<td>fluid depth</td>
</tr>
<tr>
<td>$L$</td>
<td>tank length and width (square tank)</td>
</tr>
<tr>
<td>$L_{bp}$</td>
<td>effective horizontal length of the bubble plume</td>
</tr>
<tr>
<td>$p$</td>
<td>atmospheric pressure</td>
</tr>
<tr>
<td>$Q$</td>
<td>gas volume flow rate</td>
</tr>
<tr>
<td>$t$</td>
<td>time since the beginning of mixing</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>fluid density</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>gas density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>fluid-gas surface tension</td>
</tr>
<tr>
<td>$\nu$</td>
<td>fluid viscosity</td>
</tr>
</tbody>
</table>
The independent variables can be combined into five dimensionless groups: F, Froude number, the ratio of gravitational forces to inertial forces; \( P_r \), the ratio of total pressure (hydrodynamic and atmospheric) to atmospheric pressure; \( R \), Reynolds number, the ratio of viscous forces to inertia forces; \( t/T \), the dimensionless time; and \( W \), Weber number, the ratio of surface tension forces to inertial forces.

\[
F = \frac{Q}{\sqrt{gL^5}} \quad (6.1)
\]

\[
P_r = \frac{p + \rho gh}{p} \quad (6.2)
\]

\[
R = \frac{\nu}{\sqrt{gL^3}} \quad (6.3)
\]

\[
T = \frac{L^2}{Q/L_{bp}} \quad (6.4)
\]

\[
W = \frac{\sigma}{\rho gl^2} \quad (6.5)
\]

An important dependent dimensionless variable is \( k \), the ratio of plume width to plume height. Experimentally, for a constant gas flow rate, \( k \) has been found to be a constant independent of the height above the bubble plume except in the zone of flow establishment, which is immediately above the
origin of the bubble plume /1/. This zone is usually small in comparison to
the total bubble plume height so $k$ can be assumed constant over the entire
plume. In the literature $k$ is usually given a value in the range of 0.2 to 0.3
for an air plume in water without the influence of side walls /1/. The value
of $k$ may vary slightly in fluids other than water.

Another important dependent dimensionless variable is $V_r$, the ratio of
bubble plume volume to fluid volume.

$$V_r = \frac{hL^2}{kh^2L_{bp}}$$  \hspace{1cm} (6.6)

Ideally all of the independent groups should be included in the description of
the mixing but experimentally it is impossible to scale all the groups from
prototype to model and to vary the group values during experiments. To be
able to do the experiments, those groups which affect the mixing only
slightly have not been scaled.

The Reynolds number describes the viscous effects that control the
boundary layer flow at the tank walls and the microscale of the turbulent
flow, neither of which contribute greatly to the mixing unless $R$ is changed
by several orders of magnitude. Therefore it is not important to test at
prototype Reynolds number. However by coincidence if water is used as the
model tank fluid and crude oil is used as the prototype fluid and if the
length of the model is 1/10 of the prototype length, then the Reynolds
numbers of the model and prototype are roughly equivalent since crude oil is
about 30 times more viscous than water. So in this case the Reynolds number is approximately $5 \times 10^6$ and Reynolds number scaling can be achieved even though it is of minor importance. Interestingly, if a small scale test is done with crude oil, Reynolds number scaling is not achieved. Achieving it with crude oil requires a prototype size tank.

The pressure ratio has a small effect on the bubble size since it controls the expansion of the gas. The error in the pressure ratio is minimized by scaling the volumetric gas flow rate at the midheight of the tank.

The Weber number influences the ratio of bubble size to tank size. $W$ decreases as the tank size increases. The surface tension of crude oil is about $1/2$ that of water and the prototype length is 10 times larger than that of the model so the prototype Weber number is about two orders of magnitude smaller than the model Weber number.

It has been stated by various researchers that it is not the size and number of bubbles which control the bubble plume momentum and therefore the mixing, but that the overall volume of gas used is the dominating factor. If this is true a large change in the Weber number will only bring about a small change in the mixing.

Eliminating these three groups, the Froude number and purely geometric quantities remain. If the buoyant momentum of the bubbles is the most important contributor to the mixing then the Froude number is the most important dimensionless group. The volume ratio, $V_r$, is determined by choice of tank size, tank fluid, and bubble plume arrangement.
Variables remaining are $Q, L, L_{bp}, h,$ and $g$. The data reduction has been found to be most conveniently done in terms of two dimensionless times, $T_1$ and $T_2$; $T_1$ having the form of equation (6.4) and $T_2$ being a combination of $T_1$ and other dimensionless variables.

$T_2$ is proportional to the time it would take for all the tank fluid to pass through the mixing zone of the bubble plume once. The mixing zone is depicted in Figure 6.1. It is defined as the region in the tank in which the upward momentum of the gas bubbles entrains the tank fluid into the bubble plume and imparts a vertical velocity to the fluid. Its horizontal cross sectional area is called the bubble plume width and it is related to the height by the constant $k$. The volume of the mixing zone is:

$$v_{bp} = \int_0^h kh(z)L_{bp} \, dz$$

(6.7)

Since $k$ is constant,

$$v_{bp} = 0.5kh^2L_{bp}$$

(6.8)

Since $k$ is a constant, $V_r$, the ratio of the bubble plume volume to the fluid volume is

$$V_r = \frac{2L^2}{khL_{bp}}$$

(6.9)
Fluid is entrained into the plume along its height

Bubble plume width

Bubble plume pipe

Note: not drawn to scale

Figure 6.1: Mixing zone of the tank
If the fluid in the mixing zone has an upward velocity $V_f$ as it leaves the mixing zone, the time for the fluid to pass through the mixing zone is $h/V_f$. The characteristic time, $T_2$, for all of the tank fluid to pass through the mixing zone once, would be proportional to $h/V_f$ multiplied by the ratio of tank volume to mixing zone volume:

$$T_2 \approx \frac{h}{V_f} \times \frac{2L^2}{khL_{bp}} \approx \frac{2L^2}{khL_{bp}}$$

(6.10)

Assume that the fluid has negligible vertical momentum when it enters the bubble plume. Just before the fluid velocity becomes horizontal at the surface, the upward momentum of the fluid must then equal the buoyancy of the bubble plume. This buoyancy equals the volume of gas in the plume multiplied by the gravitational acceleration and by the density difference between the liquid and the gas. Since the time for a gas bubble to pass through the plume is proportional to $h/(V_f + V_s)$, then

$$M_{bp} = (\rho_f - \rho_g)gQ \times \frac{h}{V_f + V_s}$$

(6.11)

where the gas velocity, $V_g$, equals fluid velocity, $V_f$, plus gas slip speed, $V_s$. $V_s$ is not dependent on $V_f$. $V_s$ has been found to have a value of 0.3 m/s for air bubbles in water and it would differ only slightly for gas bubbles in crude oil.

The upward momentum flux of the fluid per unit time, $M_f$, is the mass flow rate multiplied by the velocity:
\[ M_f = \rho L_{bp} k h V_f \times V_f \quad (6.12) \]

If the momentum and buoyancy fluxes are equated and it is assumed that \( V_s \) is negligible compared to \( V_f \), and \( \rho_g \) is neglected since it is much smaller than \( \rho_f \), then

\[ V_f \propto \left( \frac{Qg}{L_{bp} k} \right)^{1/3} \quad (6.13) \]

so substituting for \( V_f \) and removing \( k \) since it is assumed constant for all conditions

\[ T_2 = \frac{L^2/L_{bp}}{(Qg/L_{bp})^{1/3}} \quad (6.14) \]

While the time for the fluid to rise through the mixing zone is directly proportional to the fluid depth, the ratio of mixing zone volume to fluid volume is inversely proportional and \( V_f \) is independent of depth, therefore \( T_2 \) is independent of fluid depth.

If \( L_{bp} \) is equal to \( L \) (the bubble plume is only along one wall of the tank), then

\[ T_1 = \frac{L^3}{Q} \quad (6.15) \]

\[ T_2 = \frac{L}{(Qg/L)^{1/3}} \quad (6.16) \]
In order to obtain a characteristic system time $T$ from $T_1$ and $T_2$, $T_1$ and $T_2$ are raised to exponential powers such that the sum of the exponents of $T_1$ and $T_2$ is one.

$$T = T_1^\alpha T_2^{1-\alpha}.$$ \hfill (6.17)

The dimensionless time then becomes $t/T$ where $t$ is the mixing time and $T$ is the characteristic system time. The value of the exponent $\alpha$ is determined from the experimental data. If $\alpha$ is chosen correctly then all the curves of $t/T$ versus $U$ for different $Q$, $L$, and $L_b$ will lie on top of one another. Once $\alpha$ is known then the relationship between average concentration variation, $U$, and $t/T$ can be found.

It is noted that $T$ can be written as:

$$T = T_2^{1+2\alpha} (L_b \frac{3g}{L^4})^\alpha.$$ \hfill (6.16)

To find the optimum value of the exponent $\alpha$ for the one-arm bubble plume mixing, a computer program analyzes the data files of the experiments and makes graphs of $t/T$ versus $U$ for any given value of $\alpha$. Thus by varying $\alpha$ and comparing the resulting graphs, it is found that values of $\alpha$ ranging from 0.15 to 0.3 give the best collapse of the dimensionless mixing curves. In Figure 6.2 $\alpha$ is given a value of 0.20 ($T = T_1^{0.20} T_2^{0.80}$ from Eqn. 6.17). A curve drawn through the data points in Figure 6.2 would be convex to the origin. The arrangement of the data points suggests an exponential relationship between the average concentration variation, $U$, and dimensionless mixing time, $t/T$. 
Figure 6.2: Average concentration variation, U, versus dimensionless mixing time, t/T. \( \alpha = 0.20 \)
To find the nature of this relationship, the one-arm bubble plume curves were graphed, with the aid of another computer program, in log-log form: log U versus log t/T. The individual curves plotted in a log-log form are not straight lines; rather they are kinked, usually at a value of U between 0.4 and 0.3. The downward slope of the curves changes after the second data point. This is true for the majority of the curves while a few curves simply have the steeper slope and are missing the kink.

The last data points of two of the log-log curves from the 2.44 m tank (Q = 0.48 and 1.1 l/s) cause a sharp kink in the curves at U = 0.3 and U = 0.4 respectively as shown in Figure 6.3, decreasing the downward slope. It is thought that this is caused by experimental error in the fluourometer measurements since the curve for the depth of 2.44 m and flow rate of 2.0 l/s is a straight line from U = 0.21 down to U = 0.03 (the last data point on the curve) and the curve of a separate experiment at h = 1.22 m and Q = 0.48 l/s, in which the tank fluid is sampled at times up to 240 seconds after the start of mixing, is straight until U = 0.02. Both of these curves are shown in Figure 6.4. In the particularly long experiment (previous experiments had only taken samples at 180 seconds) the samples were measured very carefully and the background fluorescence in the tank water was kept at a low level to decrease the experimental error. A build-up of background fluorescence is the main cause of error in the fluourometer measurements since the fluourometer must separate a small signal from a larger background signal. The resulting curve is a smooth, almost straight, from U = 0.10 to U = 0.02 with a slight kink at U = 0.02. The data points mentioned above (t = 180, U = 0.03 for h = 1.22 m, Q = 0.48 l/s; and t = 140, U = 0.04 for
Figure 6.3: Experimental error at low values of $U$ for experiments with $Q = 0.48 \, \text{l/s}$ and $1.1 \, \text{l/s}$, at $h = 2.44 \, \text{m}$. 

$\Delta h = 2.44 \, \text{m}, \, Q = 0.48 \, \text{l/s}$  
$\Box h = 2.44 \, \text{m}, \, Q = 1.1 \, \text{l/s}$
$h = 2.44 \text{ m}, Q = 2.0 \text{ l/s}$

$h = 1.22 \text{ m}, Q = 0.48 \text{ l/s}$

Figure 6.4: Values of $t/T$ at very low $U$
h=1.22 m, Q=1.1 l/s) must have experimental errors of the same order of magnitude as the actual values of U and they have been discarded.

For the same reasons, the last two points from the experiment h=1.22 m, Q=0.48 l/s (t=135, U=0.10, and t=180, U=0.08) and the last point from the experiment h=2.44 m, Q=0.32 l/s (t=240, U=0.03), both using procedure #1, have been discarded. These data points do not agree with the rest of the data at the same flow rates and depths and they probably contain large errors caused by poor flowmeter measurements.

The scatter of the data in the log-log curves is not particularly sensitive to the values of \( \alpha \) when \( \alpha \) is in the range of 0.15 to 0.30. Figure 6.5, 6.6, and 6.7 show the fit of the data to a curve when \( \alpha \) is 0.15, 0.20, and 0.25 respectively. In each Figure the log-log curve is broken into two straight lines meeting at \( U = 0.30 \). The equation for these two lines has been found by a least squares fit. From Figures 6.5-6.7, it can be seen that there is little to distinguish which has the best choice of \( \alpha \).

To determine the best value of \( \alpha \) the correlation coefficient for a least squares fit of a straight line to the data was calculated for \( \alpha = 0.15, 0.20, 0.25, 0.30 \). A correlation coefficient of -1.0 means the data points fall on a negatively sloped straight line while a value of 0.0 means that there is no correlation between the data and the least square fit. The correlation of the first part of the curve to the data is almost invariant at -0.97 for \( \alpha \) ranging from 0.15 to 0.30, while the correlation of the second half is best at \( \alpha = 0.20 \) compared to values at \( \alpha = 0.15, 0.25, \) and 0.30. The correlation coefficient is -0.916 when \( \alpha = 0.15, -0.944 \) when \( \alpha = 0.20, -0.940 \) when \( \alpha = 0.25, -0.944 \) when \( \alpha = 0.30 \).
Figure 6.5: Average concentration variation, $U$, versus dimensionless mixing time, $t/T$. $\alpha = 0.15$
Figure 6.6: Average concentration variation, $U$, versus dimensionless mixing time, $t/T$. $\alpha = 0.20$
Figure 6.7: Average concentration variation, $U$, versus dimensionless mixing time, $t/T$. $\alpha = 0.25$
0.25 and -0.930 when \( \alpha = 0.30 \). Overall the curve of \( \alpha = 0.20 \) has the best correlation to a least squares fit through the data.

The least squares fit through the data when \( \alpha = 0.20 \) is

\[
U = 0.98(t/T)^{-1.21} \quad t/T < 2.65 \quad (6.19a)
\]

\[
U = 3.63(t/T)^{-2.57} \quad t/T > 2.65 \quad (6.19b)
\]

or solving for \( t/T \)

\[
t/T = 0.98U^{-0.83} \quad U > 0.30 \quad (6.20a)
\]

\[
t/T = 1.65U^{-0.39} \quad U < 0.30 \quad (6.20b)
\]

Where \( T \) is found by substituting (6.4) and (6.14) into (6.17) and setting \( \alpha = 0.20 \) \((1/5)\) and solving for \( T \) in terms of \( Q, L, L_{bp}, \) and \( g \).

\[
T = \frac{L^2}{L_{bp}^{1/3} Q^{7/15} g^{4/15}} \quad (6.21)
\]

In Figure 6.8, equation (6.21) has been used to derive the characteristic system time, \( T \), for the "T" and two-arm bubble plumes. \( L_{bp} = 3L \) for the "T" plume and \( 2L \) for the two-arm plume. Also shown is the mixing predicted by (6.19). There is good agreement between the predicted and actual mixing although the predicted curve is more conservative. The conclusion is that
Figure 6.6: Two-arm and T plume dimensionless mixing curves compared to the predicted mixing from (6.19)
the equations derived for the one-arm bubble plume data can also be used to
determine the mixing for other line bubble plume arrangements but that
results predicted by (6.19) will likely be on the conservative side.

A physical interpretation of the change in slope at \( U = 0.30 \) is that once
most of the fluid has passed through the mixing zone once, the gross mixing
has been accomplished. \( U \) has fallen from values greater than 1.6 to less
than 0.4 in a relatively short period of time. The steeper slope starts when
partially mixed fluid is mixed still further. Once \( U \) reaches a value of 0.3
the real work in mixing the fluid has been accomplished.

This relationship of \( U \) and \( t/T \) has been found for Froude numbers ranging
from \( 5 \times 10^{-5} \) to \( 5 \times 10^{-4} \). Assuming that the characteristic prototype length is
12 m, the prototype flow rate, in order to be properly scaled, can have
values from 0.06 m\(^3\)/s to 0.8 m\(^3\)/s.

To find the mixing time for a prototype tank assume a value for \( U \), substitute
the values of \( Q, L, \) and \( L_{sp} \) into (6.21) and use (6.20a) and (6.20b) to solve for
\( t \). Figure 6.9 shows the mixing time calculated from (6.20) and (6.21) as a
function of gas flow rate for a 12 m tank.
Figure 6.9: Mixing time versus gas flow rate for $L = 12$ m, $L_{bp} = 12$ m.
7. Conclusions

The gas bubble plume was found to be an effective mixer. As expected increasing the gas flow rate increases the mixing but the increase is less than proportional. Varying the fluid depth did not have a effect on the mixing if the depth was greater than 1.0 m. At depths less than 1.0 m the effect of the plume origin and the bubble size increase the mixing but this decreases as the gas flow rate is increased. The low depth mixing was not further studied in this report although more research could be done in this area. Increasing the length of the line bubble plume also increased the mixing although both plumes did not function along part of their length so the mixing was probably less than it could have been.

Care had to be taken during the experiments to insure that the dye injection did not premix the tank fluid. The dye was injected into the tank over a short time interval once the bubble plume had started rather than injecting the dye before starting the plume since this has a significant effect on the mixing at low gas flow rates (less than 0.40 l/s)

Reducing the surface tension of the tank fluid by 46 % did not have any effect on the mixing but no firm conclusion can be drawn unless a tank with a much smaller Weber number is tested.

Two characteristic system times were identified using dimensional analysis and combined to form one characteristic system time, T. The best form of T was found by comparing the scatter of the data as the form of T was varied. This method worked quite well as the correlation of the data to
a straight line was relatively invariant with variations in $T$ so that small variations in the form of $T$ did not change the predicted mixing significantly.

The resulting form of dimensionless time plotted against the average concentration variation, $U$, in log-log form, shows that a curve drawn through the data would consist of a straight line with a knee at $U = 0.3$ after which the slope increases downwards. At this point on the curve the fluid is already partially mixed and further mixing decreases the log of $U$ more rapidly. This is useful in application of the gas bubble plume to actual processes. If dispersant is added in quantities that are 30% more than required, the tank will mix quickly and the additional mixing while the dispersant is being added will act as a safety factor to insure that sufficient amounts of the substance have reached all parts of the fluid.

From the data a relationship between $U$ and the independent variables of the system, given in equations (6.20) and (6.21), was found which can be used to predict the mixing of line bubble plumes. The relationship is valid for Froude numbers ranging from $5 \times 10^{-5}$ to $5 \times 10^{-4}$, ratios of mixing zone volume to fluid volume from 2 to 10, Reynolds numbers from $5 \times 10^{5}$ to $5 \times 10^{7}$, and a fluid depths greater than 1.0 m. The results are most likely valid over a greater range of Reynolds numbers than tested since the contribution of the Reynolds number is small. Further study with more viscous fluids would be useful to support this argument.

The data from experiments with the "T" bubble plume and two-arm bubble plume agreed well with the predicted mixing although the predicted curve is
more conservative since these two plumes did not function as well as planned. However for a given gas flow, the longer gas distribution lengths give faster mixing than the single-arm plume.

In conclusion the line bubble plume proved itself to be more than satisfactory in the mixing of rectangular tanks with the "T" plume being the fastest of the plume arrangements tested. It is predicted that a plume 12 m long operating at a gas flow rate of 0.1 m³/s would mix a tank of 12 m length and width to an average concentration variation of 0.3 within four and one-half minutes if the dispersant was added instantaneously into the bubble plume region of the tank. Extra time required to add the dispersant could act as a safety factor so that the total estimate of mixing time would be conservative.
8. Apparatus and Experimental Procedure

The tank and plume generation apparatus as described earlier were used. The tank was filled to a depth of 1.22 m with sweet Alberta crude oil. Plume gas and dispersant were introduced by procedure 2 with the straight distribution pipes. The dispersant was introduced above the gas at the mid-depth. We hasten to point out that this arrangement is not required to make the mixing procedure work in a ship. Any procedure which introduces the treating agent into the bubble plume will work. It is anticipated that the gas and treating agent would come from the same pipe in a ship tank.

To minimize the danger from hydrocarbon fumes the tests with oil were conducted out of doors and nitrogen was used as the plume gas. The outside temperature varied from 0 to +13 degrees C. The nitrogen flow of 0.6L/s was started first. After the plume was established, the radioactive dispersant was introduced over a period of 10 seconds by opening a valve from a pressurized dispersant tank. Mixing time was "started" 5 seconds after the valve was opened. The same procedure in sample taking as previously described was observed with a separate treating agent introduction for each height.

Two different fillings of the tank with oil were used. For each of these, there was no dispersant in the oil before the tests.

For the first filling of the tank with oil, a gravity-feed sampling system was used. It took 40 seconds to collect the sample size required by AECL, so the gravity-fed samples are representative of the mean concentration of each sampling location, but they are not at all representative of the unsteady concentration fluctuations. The reason is that over the relative long time of 40 seconds, many "regions" of fluid are swept past the entrance of a sampling tube by the flow.
For the second test, to minimize the effect of averaging due to sampling time, the sampling pumps from the water tests were used and the sampling size was reduced after consultation with AECL. With this arrangement samples were acquired over 3 seconds. Although this is an improvement, the samples were still influenced by spatial averaging since the typical flow speed in the tank is 10 to 20 cm/sec. For this filling there was insufficient radioactive tracer to do the experiment at all five sampling heights, so tests were done at just three heights: A, C, and E (see Figure A.3).

To obtain limited data without significant averaging due to sampling time, grab samples were taken near the top of the tank during tests with the second filling of oil.
9. Analysis of the Oil Experiments

The AECL staff determined radioactivity levels of all samples and reduced these measurements to percentages of fully mixed concentration. For the first tank filling (3-height data) with 60 points, we processed the data to determine \( U \), exactly as with the tests in water. For the second tank filling (3-height data) with 36 data points we used the same approach, but "assigned" to each point a volume \( V_i \) equal to 1/36th of the tank volume for the computation of \( U \).

For the 3-height data, the mean of each group of twelve samples did not always equal the fully mixed concentration, especially for the second and third sample collection times at height \( E \), and the results seemed to have a bias towards high concentration values. To correct this, \( U \) was calculated by using the mean concentration of each group rather than the fully mixed concentration.

Figure 9.1 shows the values of \( U \) for both the oil and a water test all at 0.6 L/s. Figure 9.2 is a log-log plot of \( U \) vs \( t' \), for all the straight pipe data in water at a 1.22 m depth and also the 3-height oil data. Equation 6.19 is also included in the plot.
FIGURE 9.1
REDUCED DATA (U vs t) FOR THE STRAIGHT DISTRIBUTION PIPE, GAS FLOW OF 0.6 L/s AND A FLUID DEPTH OF 1.22 METERS

FIGURE 9.2
LOG U vs LOG t' FOR 1.22 METER FLUID DEPTH
10. Discussion of Oil Results

The influence of averaging due to sampling time is apparent in the data. For short mixing times, calculated values of $U$ are smaller in the oil tests than in the water tests where sampled fluid was taken in a fraction of a second. The 5-height data with 40 second sample acquisition shows lower values of $U$ than the 3-height data with samples acquired in 3 seconds.

For large mixing times, the tests with oil show larger values of $U$ than the water tests. However, this does not imply that there is less mixing of treating agent into the oil. A point-by-point study of the data showed that at larger mixing times, larger than expected values of $U$ are mostly due to samples that contain considerably more dispersant than the fully mixed concentrations. So most of the samples have at least the fully mixed concentration of dispersant and many samples have even more. This could be the result of the small sample size of 36 compared to 60 samples taken during the water experiments or errors in the radioactivity measurements, but in any case all the samples have a considerable amount of dispersant in them.

Examining the results of the grab samples in Table 10.1, the same trend can be observed. During the initial part of the experiment the concentration variation of each sample from the fully mixed value decreases rapidly while at larger mixing times the variations are small but not decreasing noticeably. These variations are sufficiently small that use of the procedure on an actual ship would not be compromised.

The oil experiments have a Reynolds number approximately six times larger than that of the water experiments. The change in Reynolds number seems to have had little effect on the results. A full size tank (12 m by 12 m by 12 m) carrying crude oil would have a Reynolds number of not less than one third of the water experiments. Since a sixfold increase in the Reynolds number did not have an effect on the mixing it is unlikely that the mixing will be affected by such a small decrease in the Reynolds number.
Table 10.1 Percentage of fully Mixed Concentration of the Grab Samples.

<table>
<thead>
<tr>
<th>Time (Sec)</th>
<th>Percentage of Fully Mixed Concentration</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>165 129 157</td>
</tr>
<tr>
<td>60</td>
<td>87   68  89</td>
</tr>
<tr>
<td>90</td>
<td>99   77  79</td>
</tr>
<tr>
<td>120</td>
<td>90   87  93</td>
</tr>
<tr>
<td>150</td>
<td>100  93  100</td>
</tr>
<tr>
<td>180</td>
<td>96   95  100</td>
</tr>
<tr>
<td>210</td>
<td>99   95  104</td>
</tr>
<tr>
<td>240</td>
<td>99   94  100</td>
</tr>
<tr>
<td>270</td>
<td>97   96  100</td>
</tr>
<tr>
<td>300</td>
<td>97   90  99</td>
</tr>
</tbody>
</table>

Each group of samples were taken when a separate height was tested.
11. Conclusions

The rapid mixing which can be achieved with bubble plume can be characterized by equation 6.19 for square based tanks and a considerable range of liquid heights, gas flows and plume distribution pipe configurations. The only conditions under which the characterization of mixing vs the dimensionless time, $t^{-}$, failed to be accurate in our experiments are when the fluid depth and gas flow are both small; conditions not likely to be found in applications. The most likely cause of this is the artifact of pre-mixing by the method of dye introduction. Since equation 6.17 predicts the mixing for three different overall flow patterns the equation may be applicable to a considerable broader range of conditions than we tested. For example, it might be applied to rectangular base tanks by substituting the base area for $L^2$.

The characteristic mixing time $T$ is a combination of the time it takes to supply on tank volume of gas, $T_1$ and the time it takes to circulate a tank volume of fluid through the plume, $T_2$. On a logarithmic scale, $T_1$ contributes 20% of the mixing time and $T_2$ contributes 80%. $T_2$ is the more influential factor.

The steeper slope in $U$ vs $t^{-}$ for $t^{-} < 2.65$ can be interpreted as an increase in mixing that occurs once the fluid has passed through the plume several times and the large scale fluctuations have been eliminated.

Although the oil tests suffered from long sample acquisition times and a small excess dispersant concentration at long mixing times, the findings indicate that there are not any significant differences between oil mixing and water mixing. This applies to the high Reynolds numbers such as those used in both the water and the oil tests.

The values of $U$ from the oil experiments agree with the results of the water experiments when $t'$ is less than 10 and for larger dimensionless times $U$ has a value of 0.037. This may be the result of not taking enough samples to determine $U$ accurately or of errors in radioactivity measurements.
However, any value of \( U \) less than 0.05 can be considered low enough to be ignored since in the anticipated mixing applications, the excess dispersant used would exceed 5% of the theoretical dispersant requirement.

The bubble plume proved itself to be an efficient mixer of rectangular tanks with the "T" plume being the fastest of the plume arrangements tested. On the basis of the experimental results, it is predicted that a plume 12 m long operating at a gas flow rate of 0.1 m\(^3\)/s would mix a tank of 12 m length and width to an average concentration variation of 30% within four and one-half minutes if the dispersant was added instantaneously into the bubble plume region of the tank. Extra time required to add the dispersant could act as a safety factor so that the total estimate of mixing time would be conservative.
References


Appendix A: Derivation of Normalized Weights for Sample Volumes

To calculate the normalized weight of any given volume, multiply the sample areas found from Figure A.1 or A.2 by the vertical length of the sample volume from Figure A.3 or A.4 and divide by the tank volume. Total volume is 0.982 if \( h = 0.66 \text{ m}^3 \), 1.812 m\(^3\) if \( h = 1.22 \text{ m} \), and 3.348 m\(^3\), if \( h = 2.44 \text{ m} \).
Figure A.1: Sampling areas on the horizontal cross-section for 0.66 m and 1.22 m fluid depth
Figure A.2: Sampling areas on the horizontal cross-section for 2.44 m fluid depth

Note: All dimensions are in centimeters
- sampling point
**Figure A.3**: Vertical length of sample volumes for 1.22 m and 0.66 m fluid depth. Dimensions are in cm. • indicates sampling point.
Figure A.4 Vertical length of sample volumes for 2.44 m fluid depth. Dimensions in cm. • indicates sampling point.