CEAC-TR-96-0103

LONG-TERM CYCLIC FATIGUE STRENGTH PREDICTION METHODOLOGY FOR FIBER-COMPOSITE LAMINATES UNDER MULTIAXIAL LOADING

By

S. S. Wang* and M. Karayaka**

*Professor and Director, CEAC and Mechanical Engineering Department
**Research Scientist, CEAC

December, 1996

COMPOSITES ENGINEERING AND APPLICATIONS CENTER
FOR PETROLEUM EXPLORATION AND PRODUCTION

UNIVERSITY OF HOUSTON
HOUSTON, TX 77204-0900

230 E

CEAC-TR-96-0103

(Report #3 of the Final Report Series to American Petroleum Institute and Amoco Corporation for the Project on Long-Term Multiaxial Strength of Fiberglass Tubing)

LONG-TERM CYCLIC FATIGUE STRENGTH PREDICTION METHODOLOGY FOR FIBER-COMPOSITE LAMINATES UNDER MULTIAXIAL LOADING

by

S. S. Wang* and M. Karayaka**

Composites Engineering and Applications Center, and
Department of Mechanical Engineering
University of Houston
4800 Calhoun Road
Houston, TX 77204-0900

December, 1996

---

* Professor and Director, CEAC and Mechanical Engineering Department
** Research Scientist, CEAC

Forward

This report is the third part in the final report series for the project on long-term multiaxial strength of fiberglass tubing, conducted during the period of 1994-96 by researchers of the Composites Engineering and Applications Center (CEAC) for Petroleum Exploration and Production at the University of Houston, Houston, TX. The research was funded mainly by a contract from the American Petroleum Institute (API), Washington D. C. Owing to the broad scope of work and the depth of the investigation, supplementary support was also provided by a grant from the Amoco Corporation, Chicago, IL and by CEAC internal funds.

The overall objectives of the research program, as requested by API, were to:
(1). Examine the validity of the assumptions and hypotheses of the proposed API rating methodology for long-term strength of fiberglass composite tubing under multiaxial loading;
(2). Provide rigorous understanding of progressive leakage failure mechanisms and mechanics of FRP tubing subjected to combined internal pressure and axial loading, and
(3). Identify the limitations of the proposed API rating methodology.

The current study has been directed to focus on the following critical issues of the leakage failure in fiberglass composite tubing used in oil and gas exploration and production operations:
(1). Progressive, leakage failure modes;
(2). Long-term and short-term leakage failure envelopes;
(3). Safety (or service) factors in composite tubing design, and
(4). Load sequence effects.
Both composite tube bodies and threaded fiber-composite joints were studied. Two types of composite tubing were considered; one for downhole applications and the other for typical line pipe applications. The effect of different multiaxial loading modes, including short-term loading, long-term creep and cyclic fatigue, on the composite tubing leakage was investigated.

The first report in the final report series addresses the aforementioned critical issues of leakage failure in a fiberglass *composite tube body* . In the second report, the complicated leakage failure of *threaded fiber-composite tubular joints* is investigated. This third report covers the important problem of *long-term cyclic fatigue strength prediction methodology* for fiber composite laminates *under multiaxial cyclic fatigue loading*. In all these reports, the analytical and experimental methods developed for the studies are described in detail to ensure a clear understanding of the advanced level of the approach used in the investigation.

TABLE OF CONTENTS

1. Introduction

Cyclic fatigue of fiber composite laminates has been an important technical issue ever since composites were introduced to engineering applications decades ago. The ability to accurately predict the long-term fatigue strength and life of a composite laminate structure subjected to multiaxial loading is critically needed. It is essential in the design, performance evaluation and reliability assessment of composite structures in service conditions. It is recognized that the development of an accurate fatigue life prediction capability requires clear understanding of cyclic deformation and associated damage evolution in the composite material and the establishment of a quantitative, mechanics-based fatigue failure theory, involving both microscopic material parameters, and macroscopic lamination and structural variables as well as external loading modes.

The fatigue life prediction problem for a composite laminate is obviously complicated since many issues at different scales are involved. These may include identification of various failure modes, evaluation of damage evolution and associated property degradation, establishment of local multiaxial failure criteria at the ply level, accurate determination of ply deformation and stresses, and investigation of cyclic loading effects on the composite damage and failure.

It becomes obvious that the aforementioned complexities are generally difficult to address in a quantitative manner, since they require in-depth knowledge of composite material constitutive equations with evolving microstructural changes, heterogeneous composite microstructure, continuum material damage mechanics theory, unknown fatigue failure modes and associated strength criteria, and a suitable life prediction methodology for the composite laminates subjected to complex multiaxial loading.

Extensive studies on cyclic fatigue of fiber-reinforced polymer-matrix composites have been reported in the literature. For example, Hashin and Rotem[1] and Awerbuch and Hashin[2] have given a large amount of fatigue test data on cyclic behavior of glass/epoxy and carbon/epoxy composites. Empirical and semi-empirical methods have been used to interpret the fatigue test data. Analytical attempts, such as the one proposed by Hashin [3] on fatigue of unidirectional composites, have also been introduced to rationalize the complex phenomena of cyclic fatigue failure in multilayer composite laminates, but only have limited success. Owing to the importance of the fatigue issues in the long-term composite structural safety and performance, many studies have been and are being conducted. However, the development of a suitable fatigue life prediction methodology, based on physically observed failure modes with rigorous fatigue mechanics formulation, remains to be accomplished.

In next section, a brief review of the literature is made to assess the current status of the subject. The objectives and the scope of work in this investigation are defined in Section 3. Based on the fundamental fatigue failure modes and local failure criteria obtained in several related studies, a physical mechanism based fatigue life formulation is developed in Section 4 for fiber composite laminates under general loading. In Section 5, a series of studies have been made to utilize the currently developed method to study fatigue failure and life prediction of unidirectional composites under off-axis loading and multilayer fiber composite laminates under in-plane cyclic loading. Also, a major effort has been made here to address fatigue lives of fiber composite laminate tubular components subjected to combined cyclic axial loading and internal pressure. The results clearly demonstrate that the present approach are capable to evaluate the composite fatigue failure strength and life with significant accuracy and confidence.

2. Literature Review

Current design methodologies for internally and externally pressurized composite tubulars used in petroleum E & P and related industries have been developed, mainly, based on the experience from monolithic metals. These methodologies have serious drawbacks as the well known heterogeneous and directionally oriented microstructure of fiber composite laminates promote different failure modes to operate simultaneously in the composites during their service lives. Current qualification and quality control procedures [4-6], developed to qualify multilayer fiber composite tubulars, fail to address the interactive relations among fatigue damage growth, failure modes and strength, and the inherent multiaxial loading.

It has been well recognized that the ASTM D-2992 standard[7] is commonly used to guide the current test and design of fiber reinforced plastic (FRP) pipes and fittings subjected to an internal pressure. Two recommended procedures have been introduced: (1) Procedure A for cyclic loading, where $R=0$ at a frequency of 25 cpm, and (2) Procedure B for static loading. However, these procedures fail to address several important issues concerning the safe design of composite pipe in long-term loading conditions. Specifically, first, the proposed test procedures include only two hoop-to-axial-stress ratios, i.e., 1/0 and 2/1. It is noted that the two biaxial stress ratios may not be the most critical loading. In fact, recent experimental studies[8-11] covering a wide range of applied multiaxial stress ratios have indicated that failure envelopes of multilayer fiber-composite laminate tubes subjected to various multiaxial loading modes are highly distorted because of the distinct failure modes involved. Second, simulation of long-term cyclic and static fatigue loading can be made only for a limited time period in a laboratory environment, owing to the cost, time and the availability of equipment. The ASTM D-2992 procedure proposes tentative extrapolating strategies for estimation of the long-term composite strength from short-term experimental data without a rigorous basis of modern composite mechanics and fatigue failure theories.

Several researchers have reported studies on composite cyclic and static fatigue, based on the ASTM D-2992 and suggested models to rectify this standard. Experimental results obtained by Shell[6], for example, on multilayer glass fiber composite line pipe are shown in Fig. 1. Note that the burst strength is estimated by extrapolating the long-term fatigue data. The extrapolation scheme assumes that the fatigue failure is governed by the same failure mechanism, possibly due to elastic strains, in both high-cycle and low-cycle fatigue regimes. Such an extrapolation does not correlate well with the burst strength in the case of large inelastic deformation.

In Fig. 2, a large number of fatigue data reported by Shell and Battelle [5] are shown. These experiments were conducted for a wide range of R ratios. As pointed out in Ref. [5], fatigue crack growth in the composite matrix is the predominant failure mechanism for all R ratios. An effective stress parameter, based on a "fracture mechanics" approach has been introduced. However, as shown in Fig. 2 , the approach fails to predict the fatigue lives of the composite in the experiments with failure cycles $N_f < 10^5$.

Kujawski, et al. [4] have presented a fatigue life prediction method and made use of the experimental data reported by Battelle. In an attempt to simulate field conditions more realistically than those suggested by ASTM D-2992, the study has modified the Procedure A in the following two significant ways:

1). Experiments have been conducted at several R ratios with a constant peak pressure. For a given stress range, this condition results in fatigue experiments with higher mean stress values than those of the original Procedure A.

2). Experiments have been conducted at a cyclic frequency of 200 cycles per minute. Due to the high test frequency as compared to the original Procedure, longer-time experiments have been conducted (up to 100 million cycles). Although it is not extensively discussed, the fiber-composite laminate tubes subjected to fatigue loading at the higher frequency (200 cpm) exhibit longer lives than those subjected to the lower frequency (25 cpm).

A motivation of the study has been to investigate the contribution of mean stress to fatigue lives of fiber composite tubes. The experimental data (stress range vs. cycles to failure) reported by Battelle are presented in Figs. 3 and 4. The large data scatter in the figures, due to different mean stresses, affirms the complexities and the need of further investigation. A multiaxial fatigue parameter, based on the energy approach, to account for the mean stress contribution was proposed. The equivalent stress parameter is defined as $\Delta\tilde{\sigma} = \Delta\sigma \sqrt{f(\frac{\sigma_m}{\sigma_a}; m)}$, where $\Delta\sigma$ is the hoop stress range, and f is a mean stress function defined by $f^2 - m \frac{\sigma_m}{\sigma_a} f - 1 = 0$. The constant, m, is determined by experiments.

The aforementioned life prediction methodologies generally suffer from several shortcomings. First, the nominal stress in both the energy based and the "fracture mechanics" based approaches, may not govern the local fatigue failure modes in the composite laminates under multiaxial loading. Second, the proposed models do not have a underlying physical basis. Actual failure modes and associated fatigue crack growth directions could not be identified with these approaches. Third, the proposed fatigue parameters fail to address the interactions among the multiaxial stresses. For example, the models suggest the same damage contribution from tension and from torsion loading - a direct violation of the anisotropic strength properties in the composite.

3. Objectives and Scope of Work

Given the very large number of lamination variables, geometric configurations and loading modes on fiber composite laminates, it may be unrelealistic to expect that fatigue failure of the fiber composites could be fully determined or predicted to any degree of generality through experiments only. Thus, an advanced physical-mechanism-based model needs to be introduced to address fatigue degradation and failure modes in the fiber composites and to provide a predictive capability for any combination of material, lamination and loading parameters.

The objective of this study is to develop a life prediction methodology for multilayer fiber-composite laminates (panels and tubular) based on physically observed fatigue crack initiation and growth mechanisms under general multiaxial loading.

Specifically the scope of the work includes the following tasks:

(1). Developing an analytical cyclic fatigue failure model for multilayer fiber composite laminates, based on constituent ply fatigue failure modes.

(2). Establishing a rigorous life prediction methodology on the basis of the fatigue failure model(s) introduced to include both fiber- and matrix-dominated fatigue damage mechanisms and their interactions.

(3). Evaluating the effects of various fatigue loading parameters, such as multiaxial loading and mean stress (i.e., the R ratio), and the off-axis fiber angles in unidirectional and multilayer fiber composite laminate panels and tubes.

4. Theoretical Development

It has been well recognized that inherently weak (i.e., critical) planes, susceptible to fatigue damage, exist in fiber reinforced composites due to strength anisotropy, heterogeneous material microstructure, and the basic nature of different failure modes. Consider a multilayer fiber composite laminate subjected to multiaxial cyclic loading, $\Delta\sigma_1^\infty$ and $\Delta\sigma_2^\infty$, as illustrated in Fig. 5. (We note that when the composite tube has an internal or external liner, failure criteria of the tubular are governed by fiber-dominated failure modes.) The commonly observed failure modes set the stage for the study of the matrix- and fiber-dominated fatigue failure and to determine the critical planes for a general fatigue analysis and life prediction of the fiber composite laminate panels and tubes.

The current approach is formulated, based on the observed physical mechanisms of fatigue damage development. The primary cause of the fatigue crack growth in a fiber composite laminate is assumed to be governed by the cyclic shear stress, $\Delta\tau_{LT}$, which is parallel to the fatigue crack growth direction (Fig. 5b). The contribution of the normal stress during a fatigue cycle on the multiaxial failure process is accounted for mainly by the maximum transverse stress, $\sigma_T^{\max}$, which acts normal to the fatigue crack growth plane. The longitudinal (fiber) stress, σ_L, is assumed to have minimum effect on the matrix crack growth. According to Findley [12], a multiaxial stress interaction parameter k may be introduced to combine the contributions of the cyclic shear and transverse stresses in the multiaxial fatigue failure process in a fiber composite:

$$\Delta\tau_{LT} + k\,\sigma_T^{\max} = f\left(N_f\right) = \sigma'\,N_f^m \qquad (1)$$

where N_f is the number of cycles to failure, and σ' and m are material constants to be determined from properly conducted experiments. The critical fatigue stress function $f(N_f)$ is most important to be dealt with in the cyclic fatigue life evaluation.

The multiaxial stress interaction parameter k* for a fiber reinforced composite may be determined from the consideration of material strength anisotropy and composite fatigue experiments. The physical significance of k in a fiber-composite laminate fatigue problem may be viewed differently from that for a monolithic metal. When a composite laminate fails in a matrix-dominated mode, fatigue crack growth is generally confined in the weak plane regardless of the loading multiaxiality. Accordingly, the composite properties which govern the strength of the weak plane in a matrix-dominant failure, such as transverse and in-plane shear strengths, would have the most influence on the interaction parameter.

Consider a unidirectional composite subjected to off-axis cyclic loading. The involvement of k on the critical fatigue stress function (Fig. 6) in the material coordinates may be introduced as

---

* In a monolithic material, orientation of the critical plane for crack growth is influenced by the level of inelastic strain during a multiaxial fatigue cycle. For ductile materials undergoing large inelastic strains fatigue damage development is shear dominated and the k assumes small values. In a brittle material with small inelastic strains, damage development is governed by tensile stress and the k assumes a high value. Based on experimental observations[12], the value of k lies between 0 and 1 for analyzing the multiaxial crack growth in a monolithic material.

$$\Delta\tau_{LT} + k\ \sigma_T^{\max} = \left((1-R)Cos\theta\ + k\ Sin\theta\right)Sin\theta\ \sigma^{\max} \qquad (2)$$

where $\sigma^{\max}$ is the maximum nominal stress in global coordinates during a fatigue cycle.

Figure 7 illustrates the nature of the critical fatigue stress function, the effect of the off-axis angle, and the multiaxial stress interaction parameter for a biaxial fatigue case with R=0.

The emphasis in the fatigue failure theory development here has been so far placed on the critical fatigue stress function. For the convenience of discussion, in Fig. 7 the vertical axis is normalized by the critical cyclic stress parameter value at θ=90° in order to compare the composite systems with different *k*'s. The relative fatigue strength of a composite is governed by the fatigue strength coefficient σ' (Eqn. 1). Three regions, based on cyclic fatigue failure modes, may be noted in Fig. 7:

(1) Fiber-dominated failure regime;

(2) Shear-dominated (matrix) failure regime, and

(3) Transverse stress dominated (matrix) failure regime.

The extend of the fiber-dominated region may depend on longitudinal and transverse failure strain ratios of the composite laminate and will be discussed later. Along the demarcation line between two matrix-controlled failure regimes, the cyclic shear stress range and the maximum transverse stress equally contribute to the critical cyclic stress function. The extent of the shear-dominated region increases with smaller *k* values. High *k* values suggest that the transverse stress governs the matrix failure for a wide range of off-axis loading cases.

The value of *k* also has important implications on the critical fiber orientation in off-axis cyclic fatigue. High *k* values suggest similar fatigue lives for large off-axis angles due to shallowness of the critical cyclic stress function. For a low *k* case, the critical cyclic stress function assumes a bell shape, suggesting that the most detrimental fiber orientation may occur at an off-axis angle.

5. Results and Discussion

5.1 Fatigue Failure of Unidirectional Composites under Off-axis Cyclic Loading

In this section the currently developed fatigue theory is employed to investigate the off-axis cyclic fatigue failure of unidirectional glass/epoxy and graphite/epoxy, using the available experimental data in the literature. A common feature in the experimental studies is the use of the off-axis fiber orientation as a parameter to introduce multiaxial fatigue stresses on the unidirectional composites.

The first case investigated by the current approach is the fatigue test data obtained by Hashin and Rotem[1] on off-axis fatigue of a unidirectional E-glass/epoxy with an R ratio of 0.1. Fatigue test results from the experiments are summarized in Fig. 8 in their original form. The transition from fiber- to matrix-dominated failure is noted to occur in a manner similar to that of monotonic strength at a very small off-axis loading angle. In the case with a large off-axis cyclic loading the fatigue failure is governed by matrix/interface crack growth along the weak plane, i.e., the fiber direction.

Based on the present critical fatigue stress function approach, fatigue lives of the composite subjected to all off-axis loading cases are shown in Fig. 9, and the results may be expressed by

$$\Delta\tau_{LT} + k\ \sigma_T^{\max} = 43\ N_f^{-0.0624} \tag{3}$$

Note here that the value of k =1 gives the most consistent description of the widely scattered fatigue data shown in the previous figure. The k value used here is also compatible with the shape of the critical cyclic stress function when

Hashin and Rotem's data are viewed in conjunction with Fig. 7. Clearly, all the fatigue data from the experiments under, 60°, 30°, 20°, 15°, and 10° off-axis loading collapse into a single, well-behaved "master" curve in Fig. 9. Slight deviation of the data from the 5° off-axis experiment is noted, and this may result from transition from a matrix- to fiber-dominated failure mode.

The test results from the off-axis fatigue experiments on a carbon/epoxy composite conducted by Awerbuch and Hahn[2]* are shown in Fig. 10. Similar to the glass/epoxy cases, fatigue failure was observed in weak planes normal to the fiber direction for all cases subjected to off-axis cyclic loading. Contrary to Hashin and Rotem's study on the glass/epoxy, the carbon/epoxy composite exhibited comparable fatigue strength when subjected to cyclic off-axis loading larger than 45°. Accordingly, a shallow critical cyclic stress function for high off-axis loading cases, such as the ones given here, with a high k value is suitable. As shown in Fig. 11, $k = 1.3$ gives the best fit for Awerbuch and Hahn's data for the carbon/epoxy composite fatigue with

$$\Delta\tau_{LT} + k\ \sigma_T^{\max} = 83\ N_f^{-0.0419} \tag{4}$$

Another critical issue in evaluating cyclic fatigue of fiber composites is the influence of the sign and magnitude of mean stress during fatigue loading. For an off-axis unidirectional composite subjected to a stress with an amplitude $\Delta\sigma$ and different R ratios, the critical cyclic stress function may be expressed as

---

* Through extensive microscopic observations of fracture surfaces Awerbush and Hahn identified failure mechanisms such as fiber/matrix interface failure and matrix failure in the form of serrations, cleavage, and transverse microcrack formation. Of these failure mechanisms the degree of serration, mainly governed by shear stresses, was found to be more extensive for the specimens tested at small off-axes angles. Cleavage mechanism, typical to brittle fracture, was noted to be extensive in the resin-rich regions of the specimens tested at high off-axes loading orientations, Although these failure mechanisms are clearly identified for the small, <20°, and large, >45°, off-axes angle orientations, the transition angle for these mechanisms could not be obtained from microscopy observations. For a multiaxial interaction parameter value of 1.3 the critical stress parameter approach predicts transition from transverse stress- to shear stress-dominant matrix failure mechanisms at fiber orientation angle of 34°, which is consistent with the fractography observations of fatigue damage.

$$\Delta\tau_{LT} + k\ \sigma_T^{\max} = \left(Cos\theta\ + \frac{k}{1-R}\ Sin\theta \right) Sin\theta\ \Delta\sigma \tag{5}$$

In the current approach, the contribution of mean stress to fatigue life is governed by the ratio $k/(1\text{-}R)$. The effect of R ratio on the critical cyclic stress function for a glass/epoxy composite with k=1 is illustrated in Fig. 12. Three regions of fatigue failure are noted. The demarcation between $\Delta\tau$ and σ_T dominated regions changes with the mean stress level. As the R ratio decreases fatigue strength of the fiber composite increases for all off-axis cyclic loading cases. Note that the present critical cyclic stress function approach predicts a more detrimental mean-stress contribution to composite fatigue life under higher-angle off-axis loading.

Fawaz and Ellyin [13] has investigated the effect of R ratio on cyclic fatigue behavior of a unidirectional glass/epoxy. In Fig. 13 a comparison of fatigue lives under R=0 and R=0.5 is shown for the composite in various off-axis cyclic stress conditions. As expected, the fatigue life of the unidirectional composite decreases with increasing tensile mean stress. Experiments conducted with R=0.5 have revealed shorter fatigue lives than those with R=0. As the off-axis loading angle increases the effect of mean stress on composite fatigue life becomes more detrimental, which is consistent with the predictions in Fig. 12.

The multiaxial stress interaction parameter of k=1 provides most consistent fit for Fawaz and Ellyin's data on the glass/epoxy with

$$\Delta\tau + k\ \sigma_T^{\max} = 77\ (N_f)^{-0.047} \tag{6}$$

Note here again that with the current approach the experimental data for the cases of R=0.5 (dark data points) and R =0 (open data points) all collapse into a single master curve as shown in Fig. 14.

5.2 Fatigue Failure of Multilayer Fiber Composite Laminates

In fatigue failure of a multilayer fiber composite laminate, the situation is obviously more complex, involving both matrix- and fiber-dominated cracking, which are governed by basic ply properties and ply failure criteria as well as the interactions between adjacent plies. In the current fatigue failure modeling, accurate determination of ply-level stresses and associated ply failure modes during cyclic loading is essential. Analytical methods with different levels of sophication may be taken for this purpose. The governing cyclic stress function can then be evaluated for each individual ply in the progressive failure process.

Extensive experimental data on angle-ply composite laminates are available in the literature. Thus, the current approach may be conveniently used to evaluate cyclic fatigue lives of the multilayer composite laminates. Consider first the extensive experiments conducted by Rotem and Hashin[14] on angle-ply glass/epoxy laminates. In Fig. 15, cyclic fatigue lives of the angle-ply glass/epoxy with different fiber orientations are shown. Depending on the fiber angle placed in the laminates, several distinct failure modes are observed in the composite laminate fatigue failure. For the laminate with $\theta<45°$, delaminations start from laminate edges and grow into a critical size to cause fatigue failure; for $\theta = 45°$, the dominant failure mode is matrix cracking and accumulation along the weak plane normal to the fiber direction. For the cases with $\theta>45°$, sudden crack growth along the fiber direction results in the laminate fatigue failure. Using the current approach, one can properly evaluate the cyclic fatigue failure lives of the composite laminates. Taking the value of k=1 and the cyclic nominal loading applied, the fatigue lives of all angle-ply glass/epoxy composite laminates studied in [14] are found to follow the expression,

$$\Delta\tau + k\,\sigma_T^{\max} = 101\ (N_f)^{-0.073} \tag{7}$$

5.3 Fatigue of Fiber Composite Laminate Tubular under Cyclic Internal Pressure

Extensive fatigue studies on multilayer composite laminate tubulars have been reported in the literature. As mentioned previously, Frost [5] summarizes the results (Fig. 2) of cyclic internal pressure experiments on glass/epoxy line pipe conducted by several laboratories for a wide range of R ratios. In these experiments, fatigue damage developments have been found to be predominantly governed by the matrix for all R ratios. As observed in the figure, the data scatter is rather significant and fatigue life prediction based on convention approaches has not be satisfactory.

Using the currently developed multiaxial fatigue model and the analytical procedure, one may easily place all the previously obtained data in a single master curve as shown in Fig. 17 with

$$\Delta\tau + k\,\sigma_T^{\max} = 114\ (N_f)^{-0.107} \tag{8}$$

Kujawski et al. [4] report experiments on FRP line pipe conducted by Battelle (Fig. 18). This study was conducted at 67° C for various R ratios. The present approach using the critical cyclic fatigue stress function gives excellent predictions. The results are shown in Fig. 19 and the following fatigue life equation is obtained:

$$\Delta\tau + k\,\sigma_T^{\max} = 52\,(N_f)^{-0.049} \qquad (9)$$

We note here that the fatigue life exponents obtained for the multilayer glass/epoxy composite laminates in these independent studies are in the range of 0.049-0.107, which is consistent with the value obtained from the aforementioned basic unidirectional composite studies. The strength reduction at an elevated temperature perhaps gives a lower fatigue strength coefficient for Kujawski's experiments than that of the studies conducted at room temperature. However, the fatigue strength coefficient (114 MPa) for the composite line pipe in Frost's study is comparable to Rotem and Hashin's (i.e., 101 MPa) for flat-panel angle-ply composite laminates.

5.4 Fatigue Life Prediction of Multilayer Fiber Composite Laminate Tubular under Combined Cyclic Axial Loading and Internal Pressure

Taking this method one step further, one may easily use it to determine the cyclic fatigue life of a composite laminate tube subjected to combined cyclic axial loading and internal pressure. In the context of the critical cyclic fatigue stress function approach, the multiaxial stress effect on the cyclic failure envelope must be determined on the basis of resolved ply stresses and the multiaxial, ply material fatigue failure criteria. In Figs. 20(a) and (b), isochronus fatigue lives are determined for ±45 and ±55 glass/epoxy multilayer laminate tubes subjected to combined cyclic internal pressure and axial tension with R=0 in a proportional loading mode. Similar to the static multiaxial failure envelope obtained in [9] under monotonic loading, the cyclic multiaxial fatigue failure envelopes are highly distorted, indicating a significant amount of interactions among the different participating stress components. For instance, in a combined biaxial tension-tension loading, each applied nominal stress component generates in-plane shear stresses in the individual plies in opposite directions. As the loading multiaxiality increases, the total shear stresses in the plies decrease, resulting in increased composite fatigue lives. At the tip of the distorted multiaxial failure envelope, the contribution of in-plane shear stress to the cyclic fatigue stress function eventually would vanish, and the fatigue failure is governed only by the transverse stresses. The shape of the multiaxial failure envelope depends not only on the anisotropic ply material properties, winding angle and loading multiaxiality, but also on fatigue loading parameters, such as the load path of the multiaxial stress components, the multiaxiality parameter k, and the R ratio. Accordingly, the optimum performance of a composite tubular vessel subjected to cyclic multiaxial loading may not be achieved when the design methodology is based only on monotonic deformation and failure behavior of the composite.

In Fig. 21 the effect of R ratio on the multiaxial cyclic failure envelope of a ±55 glass/epoxy laminate under proportional loading is shown. In this figure the fatigue lives are compared on the basis of the applied nominal stress range during a fatigue cycle. Distortion of the multiaxial fatigue failure envelope depends on the mean stress of the cyclic loading. The multiaxial failure envelope shrinks with an increasing mean stress (i.e., increasing R ratio).

5.5 Additional Remarks

The following remarks are also noted in understanding the mechanics basis for possible extension of this study. First, for a monolithic material, the justification for selection of a suitable stress multiaxiality parameter k is generally based on detailed observations of crack nucleation and growth[10]. For a fiber composite material, the present fatigue stress function approach could be erroneously viewed as a maximum shear theory for the case of k=0, or a maximum principal stress theory for k=1. Such an interpretation of the current approach for fiber-reinforced composites is not valid since the principal stress and strain directions generally do not coincide in the composites under multiaxial loading. It is recognized that in a fiber composite, critical (or weak) planes susceptible to fatigue damage initiation and propagation do exist due to material strength anisotropy and microstructural heterogeneity.

Second, other fatigue failure mechanisms may be also operative in the fiber-reinforced composite laminates under different loading. The present fatigue failure model is applicable only when the damage is confined along the weak planes normal to the fiber direction. In Figs. 7 and 12, a limitation of the current fatigue model due to fiber-dominated failure modes is indicated. Experimental studies have indicated that fiber-dominated failure modes may govern the fatigue failure up to a certain off-axis angle under tension-tension fatigue loading. Cyclic deformation and fatigue failure of unidirectional composites under tension-compression ($R<0$) and compression-compression fatigue are not well-understood. As depicted in Fig. 16, the delamination fatigue failure mode in an angle ply laminate introduces a different kind of complication since interlaminar stresses need to be included in the cyclic fatigue stress analysis. In general, detailed three-dimensional stress states must be obtained during the fatigue damage growth process. Such a task certainly requires advanced analytical mechanics development, which is yet to be established for this problem.

7. Conclusions

Based on the fatigue model and analytical procedure developed, and the results obtained in this study, the following conclusions may be drawn:

1. A cyclic fatigue damage model is proposed on the basis of physical failure mechanisms observed in fiber composites under cyclic loading and the governing cyclic stress components involved in damage growth in composites.

2. A fatigue life prediction methodology for fiber-composite laminates is established based on composite ply stresses introduced during fatigue and the ply fatigue failure criteria. The predictive capability of the proposed method is demonstrated for both unidirectional and multilayer fiber-composite laminates in panel and tube forms.

3. Conventional fatigue life prediction methodologies based on global nominal stresses may not be able to identify individual failure modes in multilayer composite laminates under cyclic multiaxial loading, and thus would not give satisfactory results in general. The cyclic fatigue failure of a fiber composite laminate is governed by ply stresses and associated ply failure criteria. Therefore the proposed fatigue life prediction methodology is formulated in accordance with the local ply failure model and associated ply stresses.

4. The proposed cyclic life prediction methodology has been used successfully to study the fatigue failure lives of composite panels and tubular subjected to several different multiaxial loading cases. The method is able to account for important fatigue loading parameters, such as loading multiaxiality, R ratio, and off-axis angle for both unidirectional and multilayer laminate composites.

5. The effects of loading multiaxiality and mean stress have been investigated, and they are found to influence the fatigue life of a composite laminate significantly.

8. References

[1] Hashin, Z. and Rotem, A., " A fatigue failure criterion for fiber reinforced materials," *Journal of Composite Materials*, Vol. 7, 1973, pp. 448-464.

[2] Awerbush, J and Hahn, H.T., "Off-axis fatigue of graphite/epoxy composite," *Fatigue of Fibrous Composite Materials, ASTM STP 723,* American Society for Testing Materials, 1981, pp. 243-273.

[3] Hashin, Z. "Failure criteria for unidirectional fiber composites," *ASME Journal of Applied Mechanics*, Vol. 47, 1980, pp. 329-334.

[4] Kujawski, D., Chiu, A.S. and Ellyin, F., "Predictive method for long-term fatigue strength of FRP pipe," Technical Report, Imperial Oil, Calgary, Canada, 1994.

[5] Frost, S.R., " The fatigue performance of glass fiber/epoxy matrix filament wound pipes", *Proc. of the Nineth International Conference of Composite Materials (ICCM-9),* Madrid, Spain, 1993

[6] Provost, E.T., "Extending the GRP option: A performance-based specification for GRP pipes and fittings (based on API-HR/API 15LR)," *The 12th International Conference of Offshore Mechanics and Arctic Engineering, OMAE*, Glasgow, Scotland, 1995.

[7] ASTM D-2992-91, "Standard practice for pressure design basis for fiberglass (Glass-fiber-reinforced thermosetting-resin) pipe and fittings", *Americal Society*

for Testing and Materials Standard, american Society for Testing and materials, 1991.

[8] Swanson, R.E., and Mindroui, V.E., "The stacked lamellar texture in fracture surfaces of fiber composites", *Journal of Materials Science*, Vol. 20, 1985, pp. 2801-2806.

[9] Swanson, S.R. and Christoforou, A.P., "Response of quasi-isotropic carbon/epoxy laminates to biaxial stresses", *Journal of Composite Materials*, Vol. 20, 1986, pp. 457-471.

[10] Lee, J., Soden, P.D., Kitching and Tse, P.C., "Strength of filament wound GRP tubers with axisymmetric steps", *Composites*, Vol. 20, 1989, pp. 234-243.

[11] Soden, P.D., Kitching, R., Tse, P.C., Tsavalas, Y. and Hinton, M.J., "Influence of einding angle on the strength and deformation of filament wound composite tubes subjected to uniaxial and biaxial loads", *Composites Science and Technology*, Vol. 46, 1993, pp. 363-378.

[12] Findley, W. N., " A theory for the effect of mean stress on fatigue of metals under combined torsion and axial loading or bending", *ASME Journal of Engineering for Industry*, 1959, pp. 301-306.

[13] Fawaz, Z. and Ellyin, F., "Fatigue failure model for fiber reinforced materials under general loading conditions", *Journal of Composite Materials*, Vol. 28, No. 15, 1994, pp. 1432-1451.

[14] Rotem, A. and Hashin, Z., "Fatigue failure of angle ply laminates", *AIAA Journal*, Vol 14, 1976, pp. 808-872.

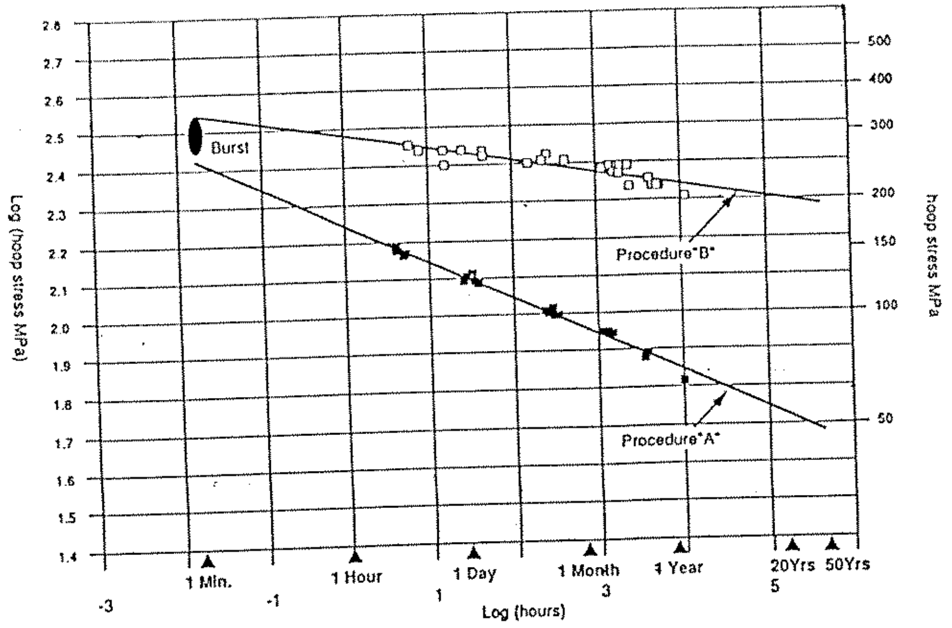

Figure 1: Static and cyclic fatigue data for FRP tubes (Provost, 1993)

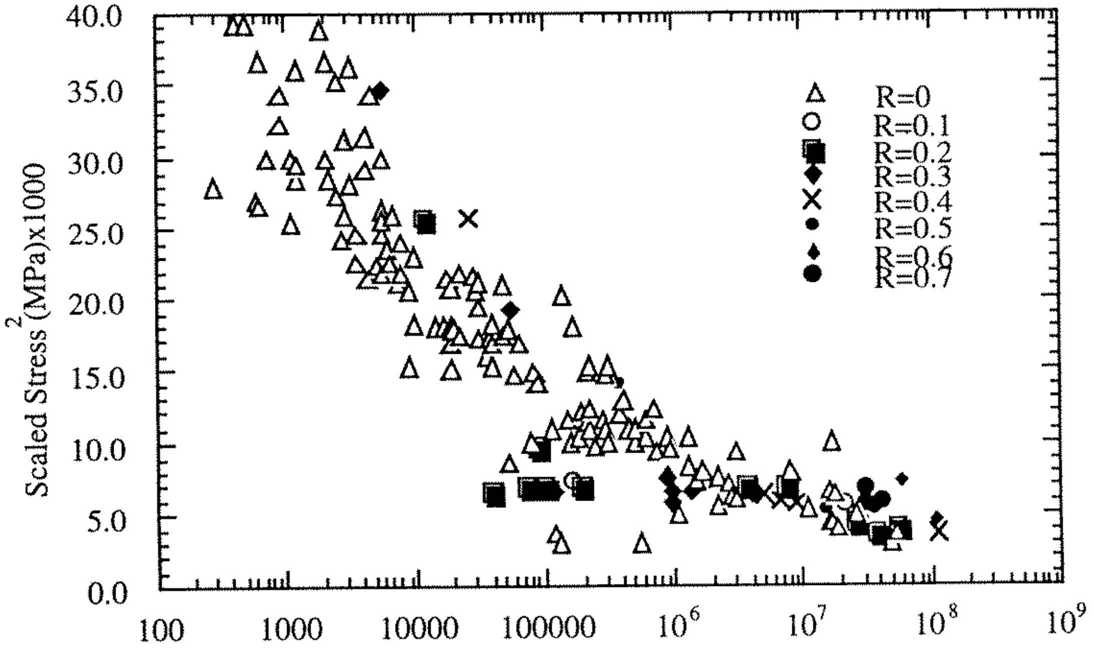

Figure 2: Fatigue-life data for fiber reinforced plastic pipe tested by ASTM D2992 (Frost, 1993)

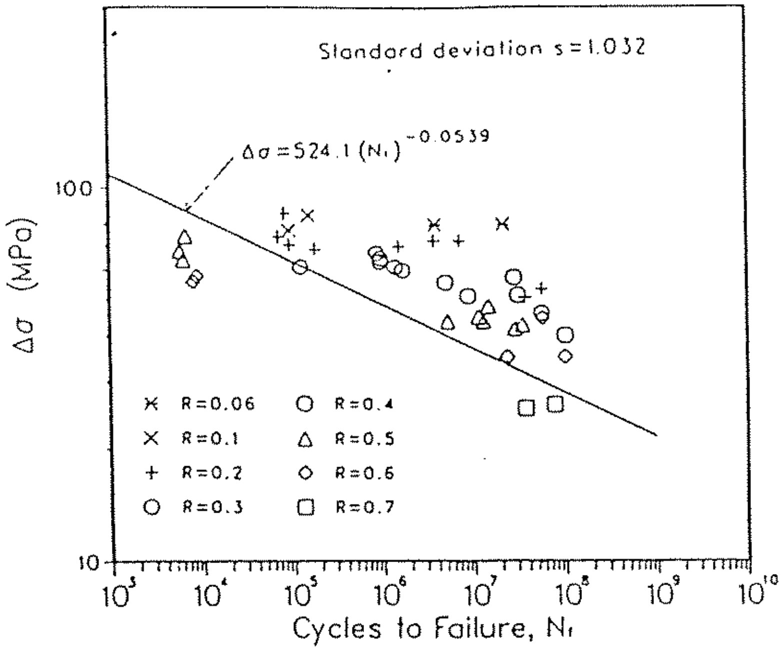

Figure 3: Battelle Lab's fatigue data on FRP tubes (Kujawski, et al, 1993)

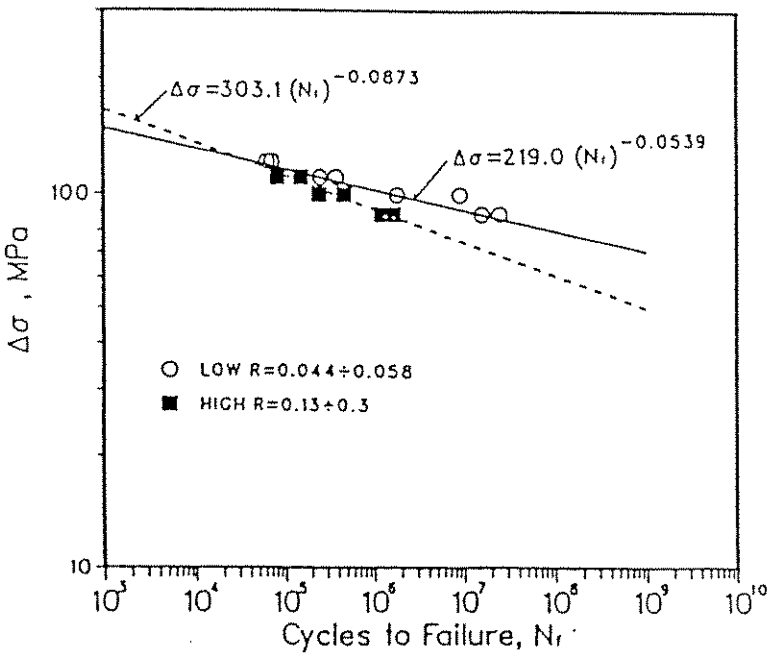

Figure 4: Battelle Lab's fatigue data on FRP tubes (Kujawski et al., 1993)

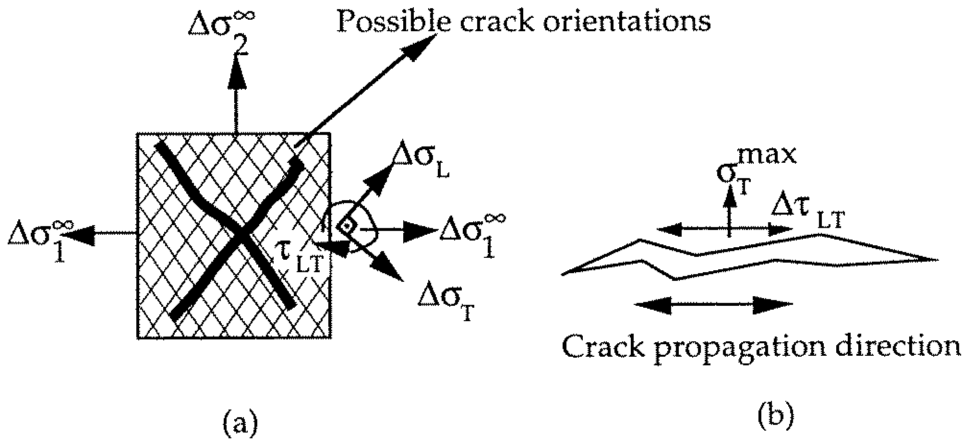

Figure 5: a) Fatigue cracking in a multilayer composite laminate, b) Governing fatigue stress components on a fatigue crack

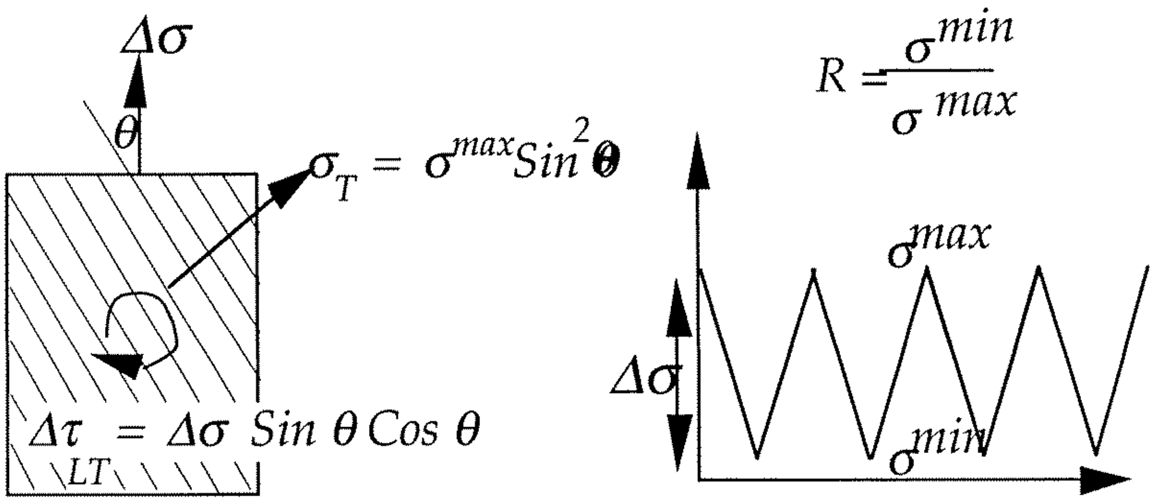

Figure 6: Cyclic stress parameters in fatigue of unidirectional composite

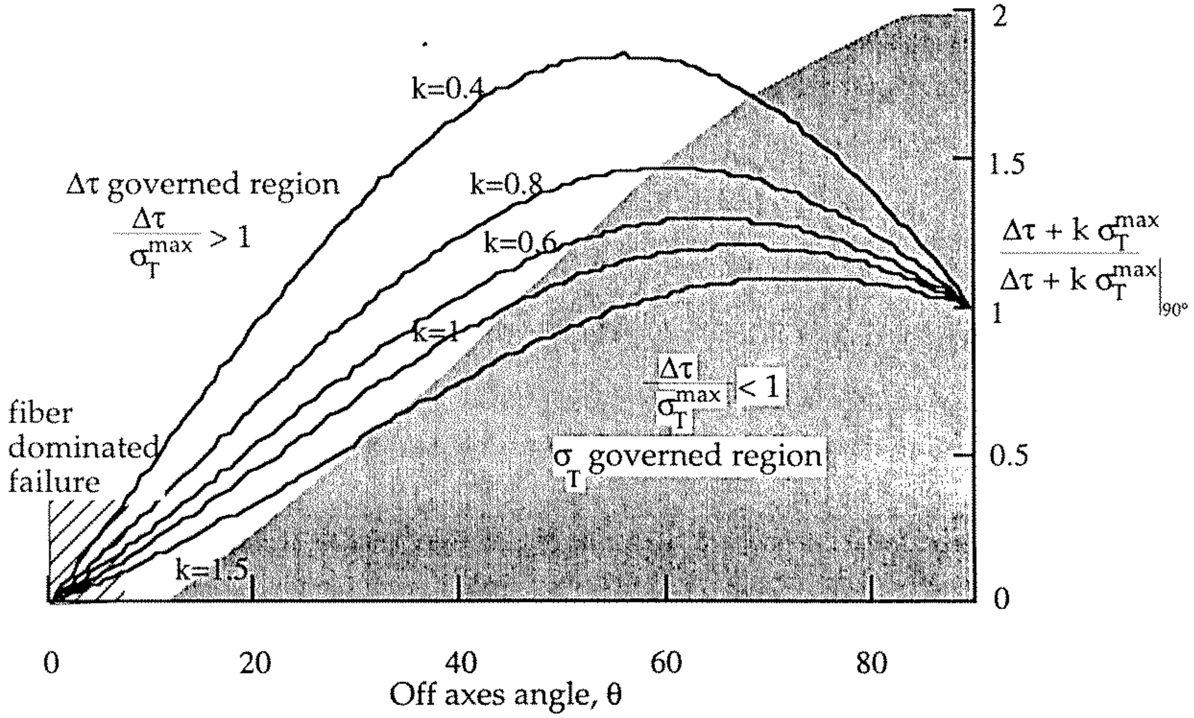

Figure 7: Variation of fatigue stress function for unidirectional composite, R=0

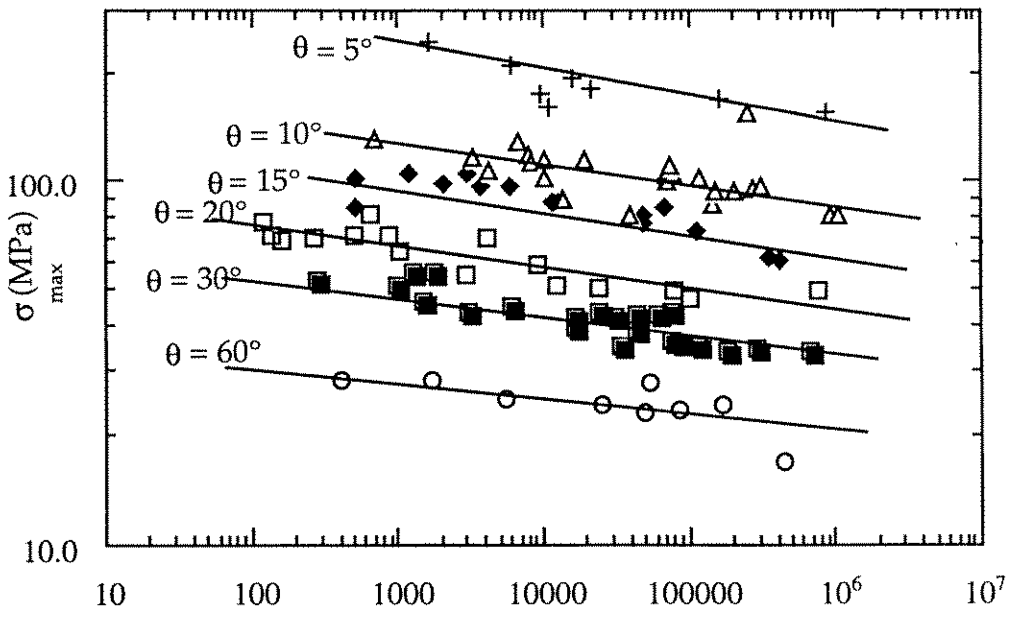

Figure 8: Fatigue life vs. off-axis cyclic stress on unidirectional glass/epoxy, R=0.1

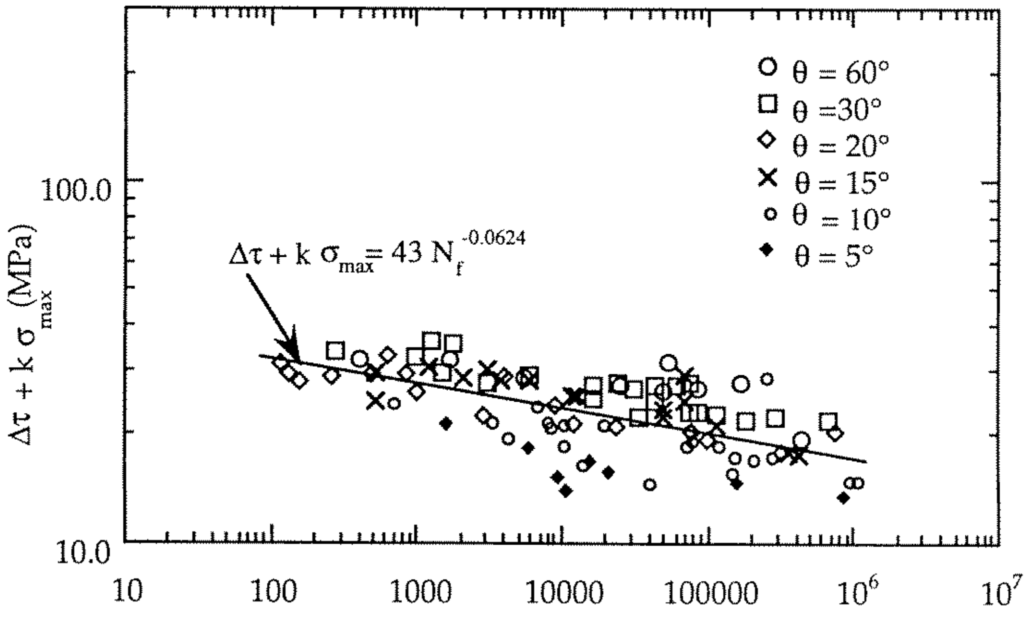

Figure 9: Fatigue life of unidirectional glass/epoxy subjected to off-axis cyclic loading.

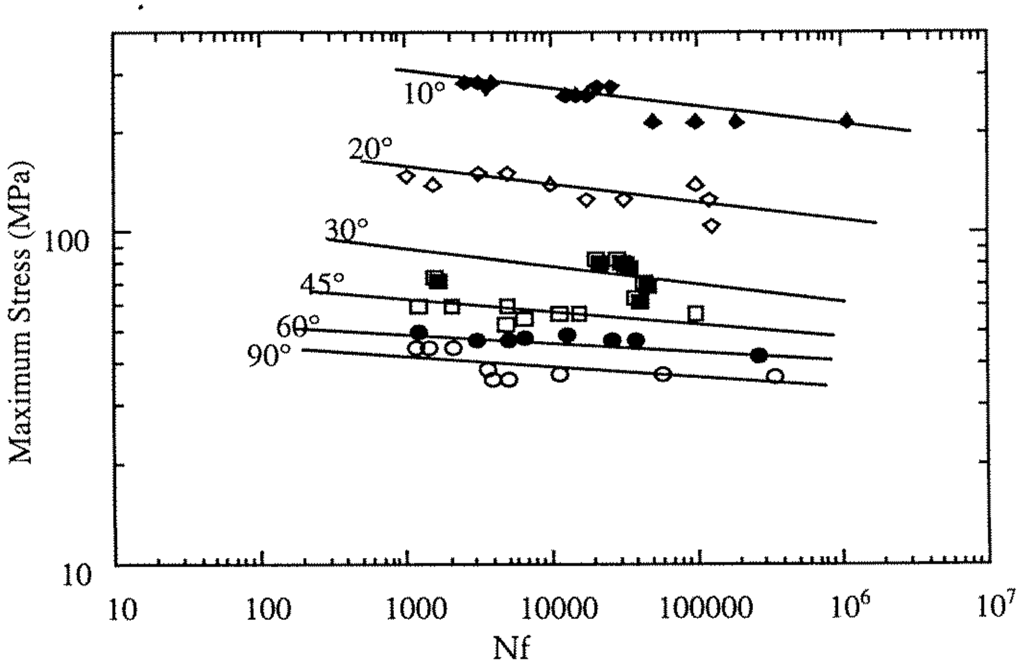

Figure 10: Fatigue-life curves of unidirectional carbon/epoxy under off-axis cyclic loading, R=0.1, (Awerbuch and Hahn)

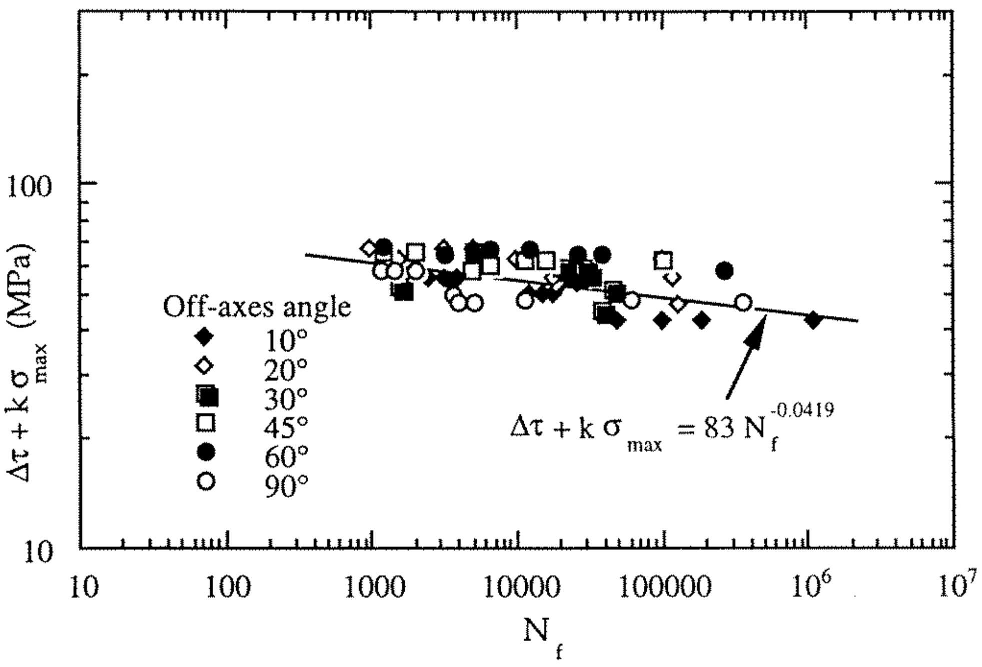

Figure 11: Cyclic fatigue stress function vs. fatigue life for unidirectional carbon/epoxy laminates, k=1.3

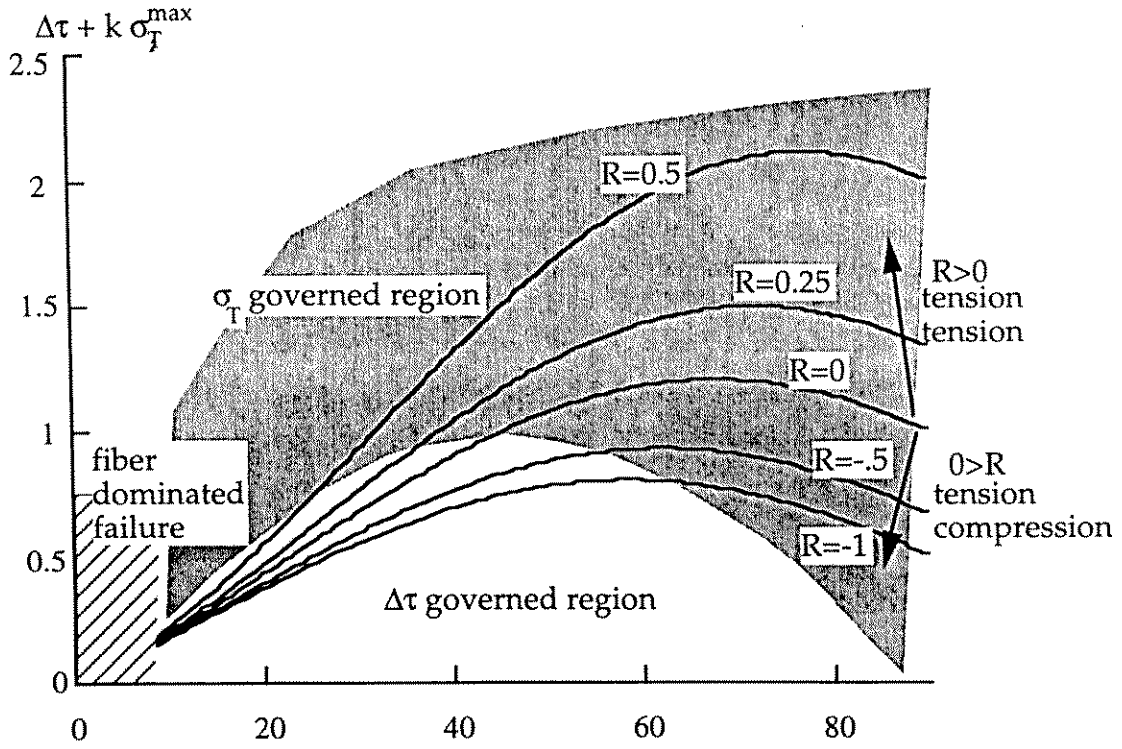

Figure 12: Effect of mean stress on fatigue life of unidirectional composite subjected to off-axis cyclic loading

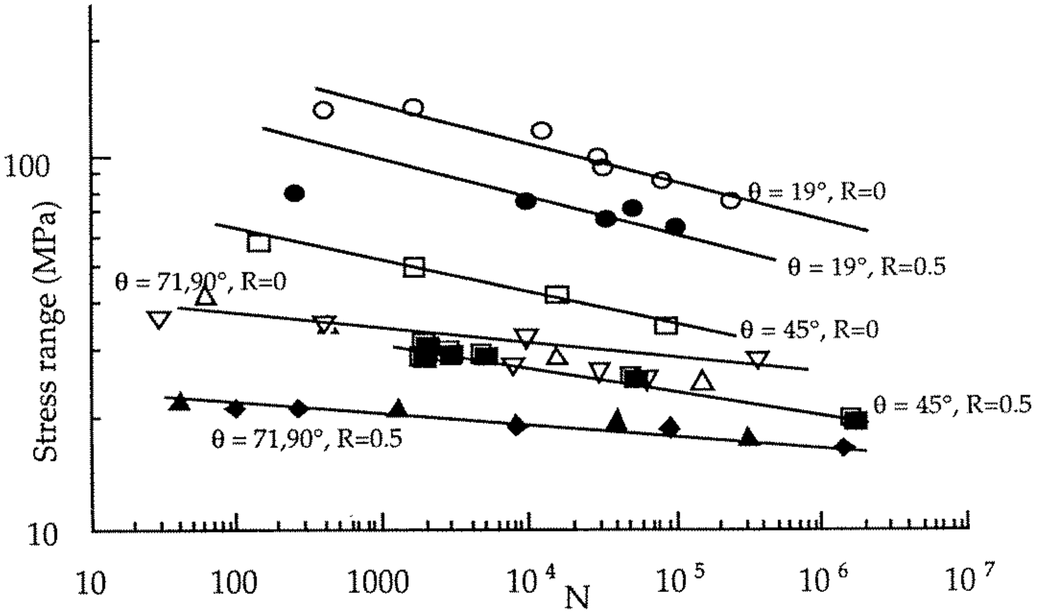

Figure 13: R-ratio effect on glass/epoxy composite under off-axis cyclic loading, (Fawaz and Ellyin, 1994)

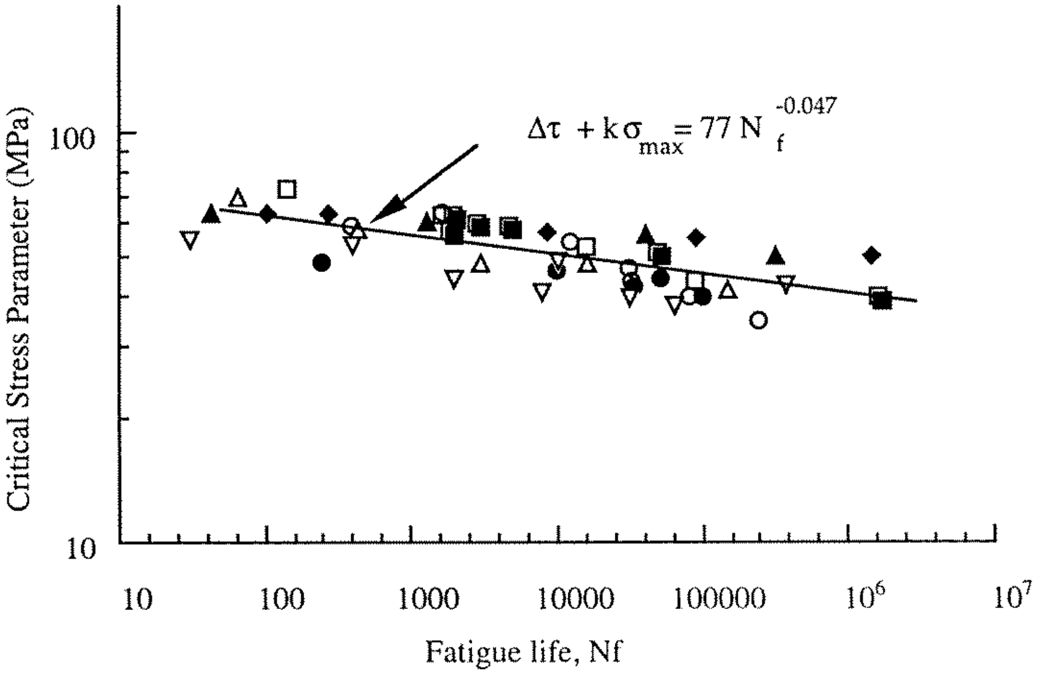

Figure 14: Fatigue life prediction of glass/epoxy composite determined by present fatigue stress function approach

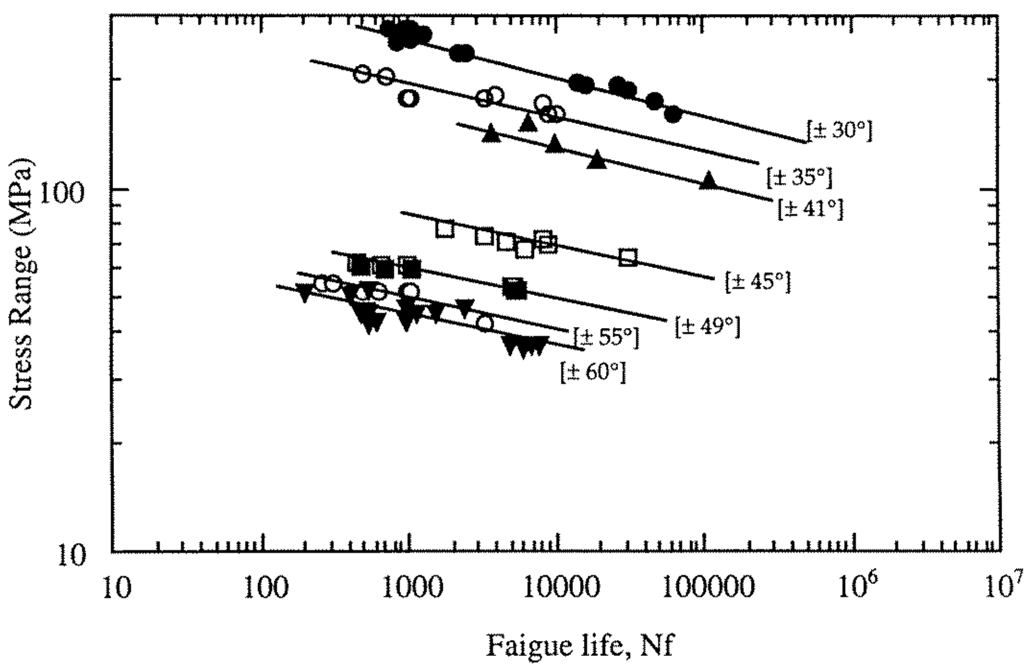

Figure 15: Fatigue life vs. cyclic stress range on angle-ply glass/epoxy laminate with R=0.1, (Rotem and Hashin, 1976)

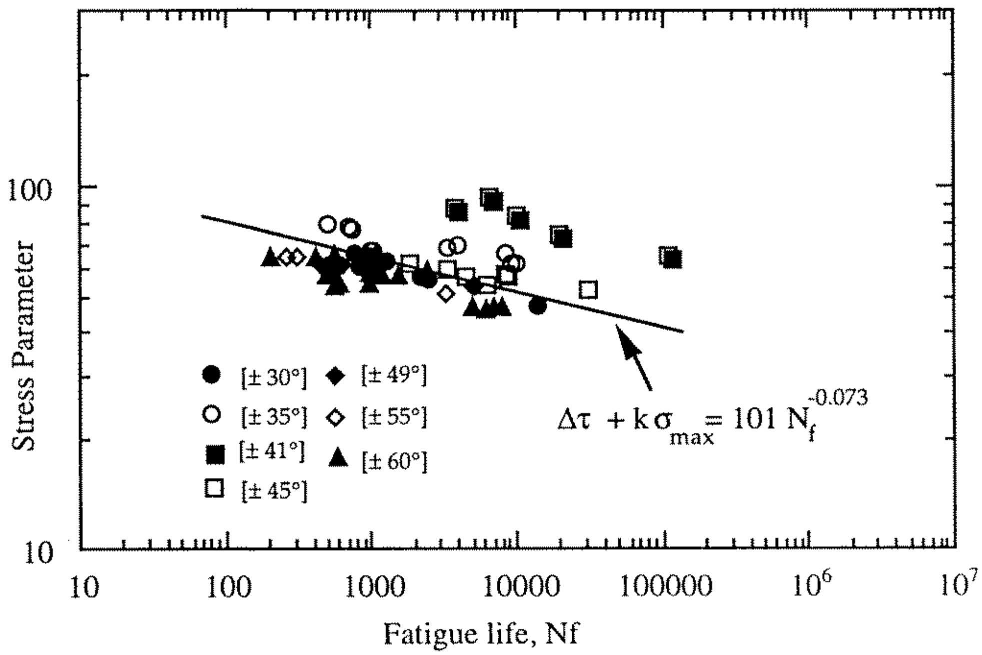

Figure 16: Cyclic fatigue stress function vs. fatigue life prediction for angle-ply lamiantes

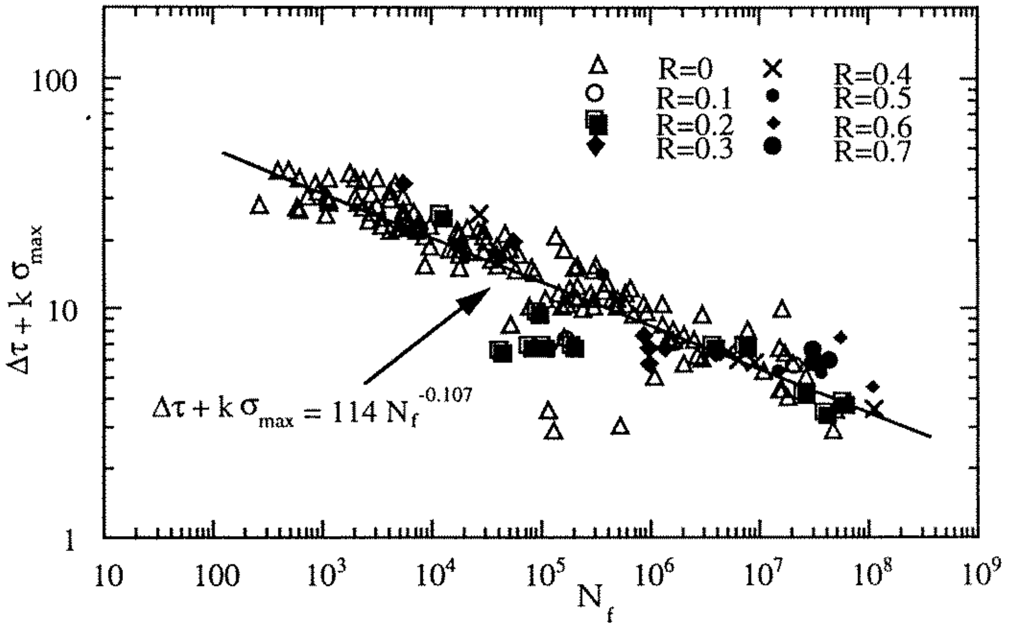

Figure 17: Fatigue life prediction based on cyclic fatigue stress function approach in multilayer glass/epoxy composite tubular.

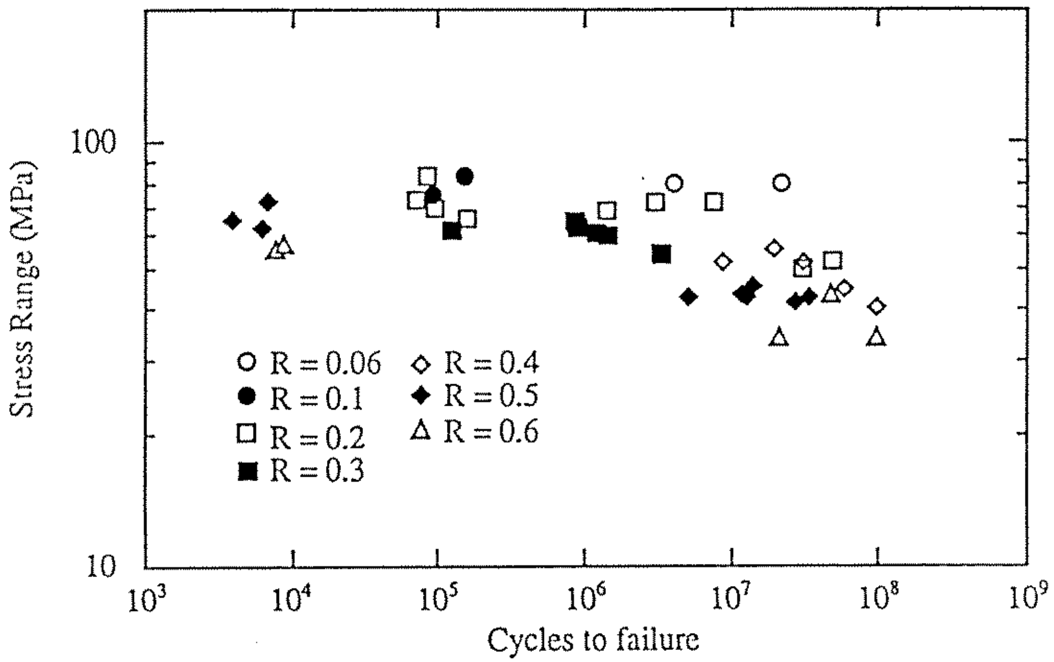

Figure 18: Fatigue failure of multilayer composite laminate tubular at elevated temperature, T=67°C, (Kujawski et al. 1994)

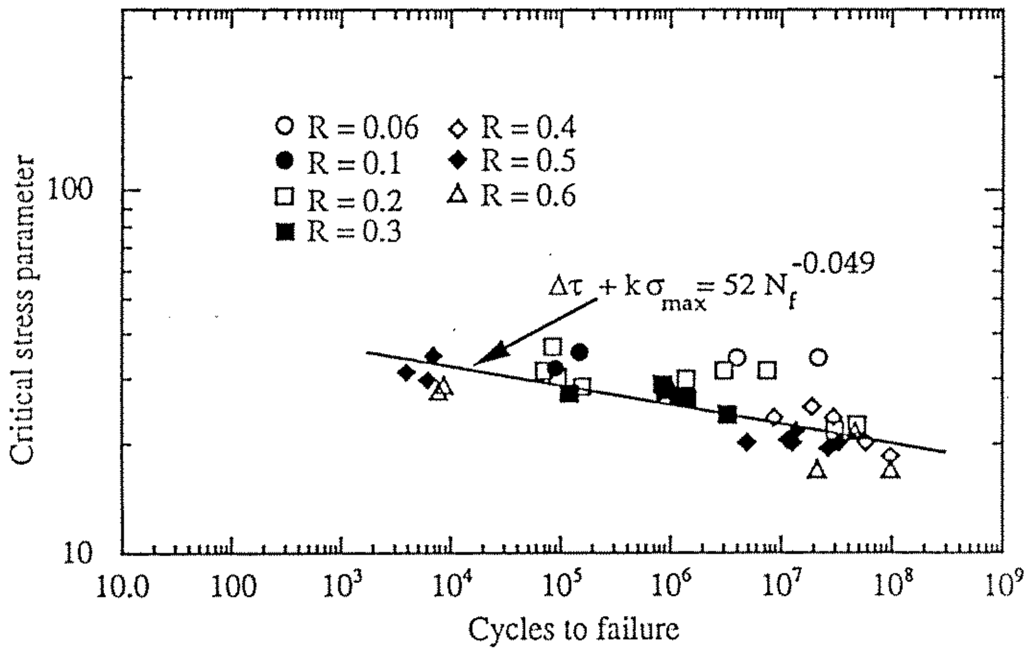

Figure 19: Fatigue life prediction of multilayer glass/epoxy laminates by cyclic fatigue stress function approach

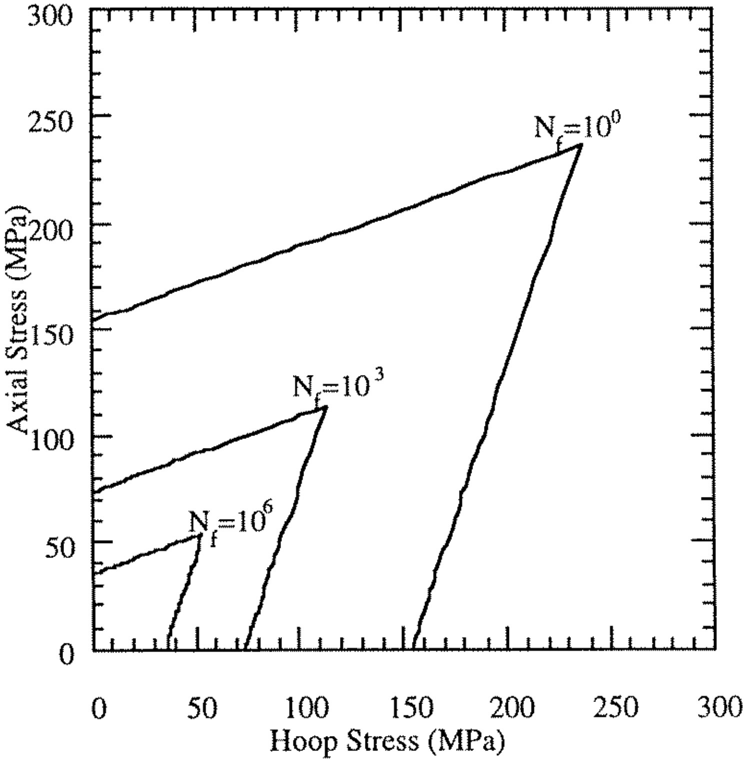

Figure 20a: Isochronus fatigue lives of ±55 glass/epoxy laminate tubular under multiaxial cyclic loading

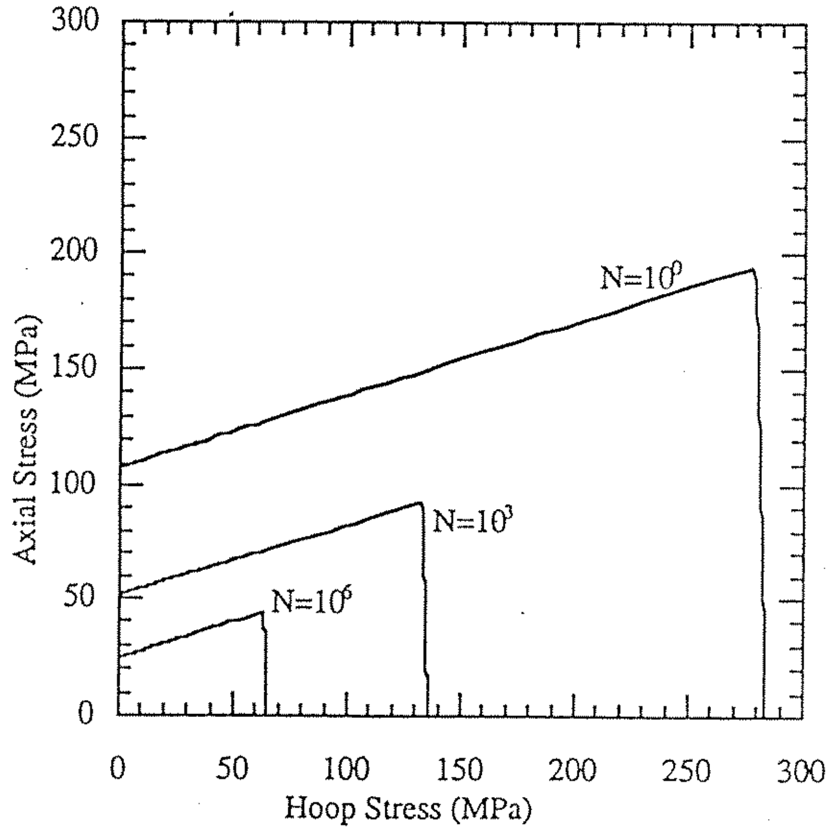

Figure 20b: Isochronus fatigue lives of ±45 glass/epoxy laminate tubular under multiaxial cyclic loading