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# System Analysis of the Interactive Behavior of a Pair of Flexible Cylinders under Unidirectional Wave Loading

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## ABSTRACT

This paper reports on some of the results of an experimental test program on the dynamic response and interactive behavior of marine risers and tendons, which was conducted in the deep water basin at the Offshore Technology Research Center (OTRC), Texas A&M University. Specifically, the paper focuses on the nonlinear system analysis of the interactive behavior of a pair of long tensioned cylinders in close proximity exposed to unidirectional random wave loading. The Volterra series approach was utilized, since it is a nonparametric method and thus does not require a priori information about the unknown physical system to be modeled. By using well developed high-order statistical signal processing techniques, the Volterra kernels can be determined by calculating the various higher-order moments of the excitation-response measurements. Time-series data of wave elevations, displacements, and reaction forces for a pair of cylinders spaced 3.5 diameters apart and in tandem array with respect to the wave flow were organized as inputs-outputs of unknown systems and were modeled using nonlinear third-order Volterra models. Two different pretension levels of the cylinders were investigated, namely 9.815 MN ("low" pretension) and 13.8 MN ("high" pretension). It was found that the level of pretension influenced the interactive response of the cylinder pair considerably. In the high pretension case the cylinders can be considered as responding as a cluster, with comparable displacements and reaction forces, which are strongly linearly related, particularly over the high wave energy frequency band. In the low pretension case, the reaction forces for the two cylinders show some marked differences and are not strongly linearly related. Nevertheless, the third-order Volterra model can still capture 80-90% of the observed power of the downstream inline reaction force over all frequencies.

KEY WORDS: Pair of flexible cylinders; wave loading; interactive behavior; system modeling; Volterra model; coherence spectra.

## INTRODUCTION

The design of tendon and riser systems for offshore structures deployed in deep water, such as semi-submersibles, TLP and spar platforms requires serious attention. The tendon and riser clusters must withstand fatigue, extreme response behavior and damage from collisions among individual risers or tendons. A reliable predictive model for the hydrodynamic loading and dynamic response of each individual member in such groups of coupled long flexible structural elements exposed to wave and current flow is not currently available. This is mainly due to our limited understanding of the hydrodynamic interference ("group effect") among the constituent members in a group of risers or tendons.

Previous experimental studies of the fluid interaction with two or more cylinders in close proximity exposed to oscillating or wave flows were limited to either rigidly- or flexibly-mounted rigid cylinders (e.g. Sarpkaya and Isaacson, 1981; Borthwick and Herbert, 1988; Blevins, 1990; Haritos and He, 1993). The emphasis in these studies was on obtaining coefficients for use in the Morison equation that account for the shielding between cylinders. These studies indicated that flow interference had under certain conditions significant effects on force coefficients of each individual member in a riser group. They also indicated that the interference phenomenon is in general a function of flow characteristics, cylinder flexibility, as well as spacing and orientation with respect to flow direction.

A series of experimental studies on the wave- and current-induced response of long, flexible risers and tendons in close proximity, conducted at the Offshore Technology Research Center (OTRC) of the Texas A&M University, involved single and clusters of two or three long, slender, flexible, vertical cylinders, using a distorted scale technique (Duggal and Niedzwecki, 1993; Guérandel, 1994), or a single scale (Niedzwecki et al., 1995; Rijken, 1997). Duggal and Niedzwecki (1993) focused on the analysis of the collision behavior of a pair of cylinders in tandem orientation. Experimentally obtained relative displacements, which were computed from strain gauge measurements along the inside of the hollow cylinders, were compared to those obtained from finite element simulations. It was found that the relative displacement magnitude was controlled by: differences in pretension between the two cylinders leading to changes in structural response amplitude, frequency and phase; and hydrodynamic interference between the two cylinders caused by flow separation and vortex shedding from the cylinders, which modified the wave-induced loading in amplitude, frequency and phase. Rijken (1997) used direct displacements measurements, by utilizing underwater optical tracking techniques and an algorithm to resolve the displacements of multiple submerged objects from multiple cameras (Rijken and Niedzwecki, 1997). He also developed a finite element formulation for a cluster of cylinders under a variety of hydrodynamic loading conditions, based on the work of Paulling and Webster (1986) for a single riser or tendon. Large differences between numerically and experimentally obtained results in clearance estimates were attributed to the lack of an interactive module in the numerical formulation, based on the experimentally observed interaction behavior.

The purpose of the present study was to expand on Rijken's work, by applying a nonlinear system identification and modeling approach to the analysis of the dynamic response behavior of riser/tendon groups subject to random waves. Specifically, select data sets from Rijken's experiments on the random wave interaction with a pair of long flexible cylinders in tandem and for two levels of cylinder pretension were analyzed utilizing Volterra series modeling techniques. The experimental data were organized in pairs representing the input-output (excitation-response) of physical systems, such as the wave flow-upstream (or downstream) cylinder interaction system, and the upstream-downstream cylinder interaction system. These systems were modeled via third-order (i.e., cubic) orthogonal Volterra-like models valid for nonGaussian random excitation. The objective was to verify the presence of nonlinearities in the input-output (excitation-response) relationships and their order (i.e., quadratic, cubic or higher order), and to quantify the strength of the nonlinear interactions between different frequency components of the input (excitation). A very important part of this modeling technique was the estimation of the orthogonal coherence spectra, which allowed for the decomposition of the observed response power spectra into their constituent linear, quadratic, and cubic components.

The paper is organized as follows. In the next section the experimental set-up and the data used in this study are detailed. The following section contains a brief presentation of the Volterra modeling technique. It is followed by a section describing the organization of the data sets for the system analysis. A discussion of the results of the Volterra-model based analysis of the experimental data and the conclusions are presented in the final two sections of the paper.

## EXPERIMENTAL SETUP

As previously mentioned, Rijken's experiments were the fourth in a series of experimental investigations into the dynamic behavior of risers/tendons under wave loading or under combined wave and current loading. The experiments were conducted at the Offshore Technology Research Center's model test basin in College Station, Texas, which is 30.5 m (100 ft) wide and 45.7 m (150 ft) long with a depth of 5.79 m (19 ft). The riser/tendon models were placed inside a 6.10 m by 9.14 m (20 by 30 ft) pit at the center of the basin, which is 16.76 m (55 ft) deep. The cylinder models consisted of an ABS tube around a tightly central fitting steel wire core in the center. The steel wire core carried the tension load while the ABS tube created the hydrodynamic diameter. In this paper data for a pair of vertical cylinders in tandem configuration, with a cylinder center to center spacing of 3.5 cylinder diameters were analyzed. The cylinder diameter was 710 mm (prototype scale) and the water depth was 850 m. Measurements recorded for each individual cylinder included wave elevation, reaction forces and cylinder displacements and the cylinder tension. Wave elevation measurements were made via five capacitance wave probes. Reaction forces at the top and bottom of each cylinder were obtained from shear strain measurements. Cylinder displacement was characterized by the motion of a single target on each cylinder, which was simply a white tape attached to the cylinder at a point 298 m below the water surface. The two-dimensional position of this target was established from two cameras and a computer algorithm for obtaining the global coordinates of the underwater target based upon digitized data from the cameras (Rijken and Niedzwecki, 1998). The wave environment for the selected data sets represented a 1 hour realization of a 100 year storm in the North Atlantic, characterized by a Jonswap spectrum with Hs=14.0 m, Tp=16.3 s and $\gamma$=2.0.

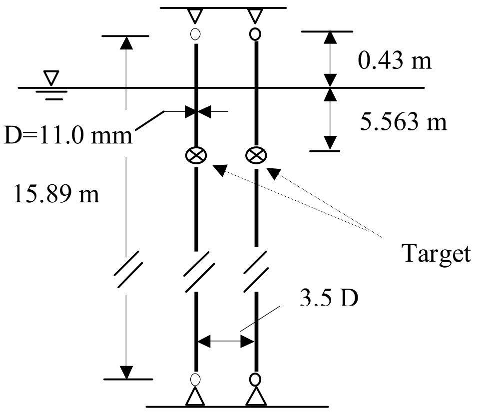

Fig. 1. Schematic of cylinders' dimensions.

## SYSTEM MODELING AND ORTHOGONAL THIRD-ORDER VOLTERRA MODELS

If we let *x(t)* represent "input" data measured at an upstream point and *y(t)* represent "output" data taken at a downstream point, then the objective of the nonlinear system modeling problem is, given *x(t)* and *y(t)*, to calculate the linear transfer function (LTF) $H_1(f)$, the quadratic transfer function (QTF) $H_2(f_1,f_2)$, and the cubic transfer function (CTF) $H_3(f_1,f_2,f_3)$ (Fig. 1), so that when the nonlinear system is excited by the input *x(t)*, the output of the model closely approximates the actual output *y(t)*.

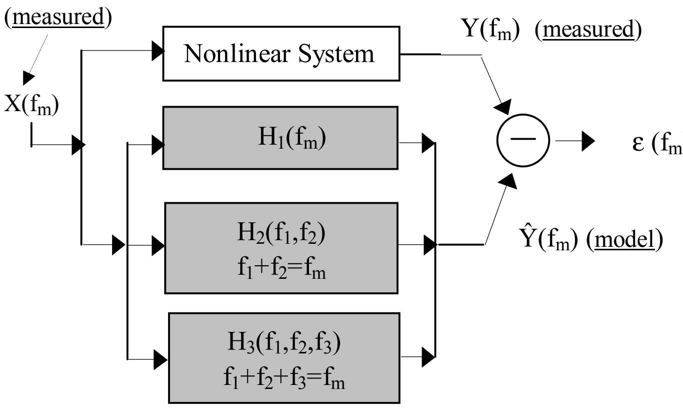

Fig.2 Frequency domain Volterra model for a cubically nonlinear system.

A frequency domain Volterra series representation of the model shown in Fig. 2 can be written as follows:

$$\hat{Y}(f) = H_1(f)X(f) + \sum\sum_{\substack{f_1\,f_2\\ f_1+f_2=f}} H_2(f_1,f_2)X(f_1)X(f_2) + \sum\sum\sum_{\substack{f_1\,f_2\,f_3\\ f_1+f_2+f_3=f}} H_3(f_1,f_2,f_3)X(f_1)X(f_2)X(f_3) \tag{1}$$

and

$$Y(f) = \hat{Y}(f) + \varepsilon(f) \tag{2}$$

where $Y(f)$ and $X(f)$ denote the Fourier transforms of x(t) and y(t), $\hat{Y}(f)$ is the Volterra model output, and $\varepsilon(f)$ is the modeling error. Eqn. 1 states that the model output at frequency $f$ comprises of a linear, a quadratic, and a cubic component. The second term of the right hand side models the contributions of all frequency pairs present in the input which, as a result of quadratic mixing, add (or subtract) to frequency $f$ in the output. The contribution of any particular frequency pair is given by the product of the complex amplitudes $X(f_1)X(f_2)$ times the QTF $H_2(f_1, f_2)$. Similarly, the third term models the contributions of all frequency triplets present in the input which, as a result of cubic mixing, add (or subtract) to frequency $f$ in the output. The contribution of any particular frequency triplet is given by the product of the complex amplitudes $X(f_1)X(f_2)X(f_3)$ times the CTF $H_3(f_1,f_2,f_3)$.

It is important to note that the outputs of the linear, quadratic, and cubic parts are complex quantities. Thus, the contributions from each transfer function at a given output frequency may act constructively or destructively with one another in the summation of the RHS of Eqn. 1, according to the relative phases of the outputs from the linear, quadratic and cubic components. The result is the appearance of "interference" terms when using the model of Eqn. 1 to calculate the output power spectrum.

To avoid the interference effects, which greatly hinder the interpretation of linear and nonlinear phenomena, Im and Powers developed an orthogonalized version of the Volterra model by orthogonalizing the input vector using the Gram-Schmidt procedure. The process is detailed in Im and Powers (1996a, 1996b) and Sibetheros et al. (2000) papers. In the orthogonalized model, the output power spectrum predicted by the model can be decomposed into orthogonal linear, quadratic, and cubic power spectra. In this case the orthogonal higher order coherence $\gamma^2(f)$, which by definition is equal to ratio of the model output power spectrum divided by the actual (measured) output power spectrum at frequency $f$, can be decomposed into the orthogonal linear, quadratic and cubic coherences:

$$\gamma^2(f) = \gamma_L^2(f) + \gamma_Q^2(f) + \gamma_C^2(f) \tag{3}$$

$\gamma^2_L(f)$ is the orthogonal linear coherence, which represents the fraction of the observed response power spectrum at frequency $f$ which can be accounted for by the linear component of the orthogonalized model; $\gamma^2_Q(f)$ is the orthogonal quadratic coherence, which represents the fraction of the observed response power spectrum at frequency $f$ which can be accounted for by the quadratic component of the orthogonalized model; and $\gamma^2_C(f)$, the orthogonal cubic coherence, represents the fraction which can be accounted for by the cubic component. It is important to note that each of the orthogonal coherences in Eq. 3 is bounded by zero and unity, and that the closer the total coherence $\gamma^2(f)$ is to unity the better the Volterra model approximation of the system output power.

## SYSTEM ANALYSIS OF THE EXPERIMENTAL DATA

Following the Volterra model system identification approach described above, the wave excitation-pair of cylinders response data were organized in system input-output terms. Specifically, wave elevation and upstream cylinder response (inline displacements/reaction forces) data comprised the input and output, respectively of System I (Fig. 3). Upstream cylinder and downstream cylinder response data comprised the input and output, respectively of System II. The Volterra analysis was used to identify the presence and strength of nonlinearities in the wave excitation-cylinder response relationship of System I. Such information is important in the design of parametric models for the cylinder response under wave loading, which ideally should generate the same nonlinearities identified and quantified by the Volterra models. The Volterra analysis of System II, on the other hand, was intended to identify the nature of the interaction and interference between the two cylinders.

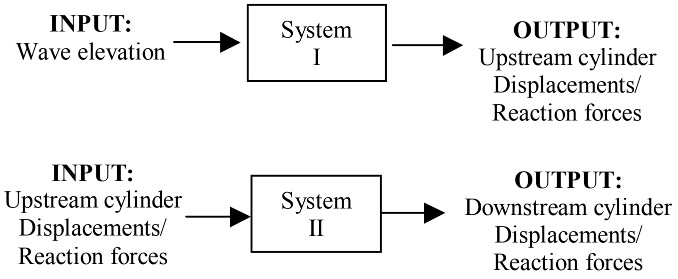

Fig. 3 System representation of wave-cylinder and cylinder- cylinder interaction (**Important note**: Reaction forces=sum of top & bottom reaction forces).

There were no assumptions made regarding the structural model of the cylinders or their interactive response behavior. The only assumption made was that the input-output relationships of systems I and II could be reasonably described by up to third order Volterra series.

The analysis focused on the effect of the pretension level in the linear/nonlinear wave-cylinder and cylinder-cylinder relationships. For this matter, data were used from experiments with two different levels of pretension for the each of the cylinder pair, namely T=9.815 MN, hereafter referred to as "low" pretension, and T=13.8 MN ("high" pretension).

## DISCUSSION OF RESULTS

The results of the frequency domain Volterra model-based analysis of systems I and II are presented in this section in terms of two kinds of plots : 1) *Auto-power spectra* of the input-output (measured) data sets; 2) *Orthogonal coherence spectra* for the Volterra models applied to the input-output data pairs. The coherence spectra were constructed by estimating each of the orthogonal coherence functions in Eq. 3 for the Volterra models of Systems I and II and by plotting them against the frequency. From these plots, the fractions of the observed response (output) power accounted for by the Volterra model as a whole and by each of its orthogonal linear, quadratic, and cubic components, respectively, can be read and compared to each other.

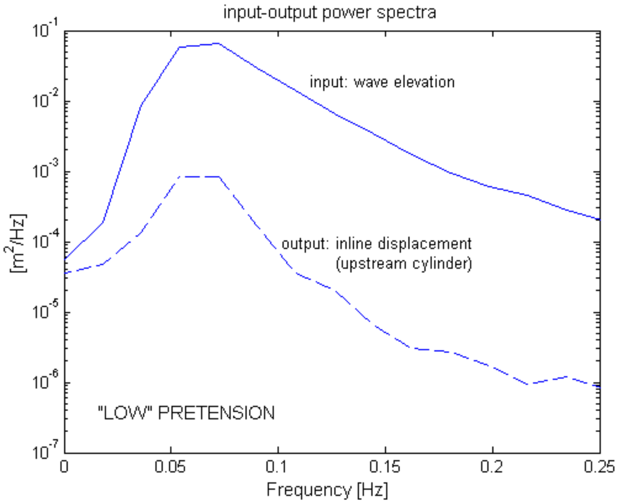

Fig. 4 Power spectra of wave elevation –cylinder inline displacement system; low pretension

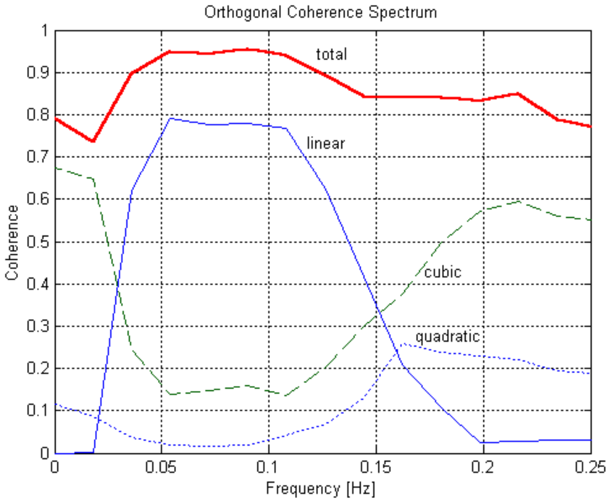

Fig. 5 Orthogonal coherence spectrum for the Volterra model of the wave elevation-cylinder inline displacement system; low pretension.

## System I: Wave-cylinder System

Two sets of excitation-response data for the wave-upstream cylinder system were analyzed, namely the wave elevation-inline displacement and wave elevation-inline reaction force sets for the two pretension levels. Figs. 4~7 display the wave elevation-upstream cylinder inline displacement for low and high pretension levels. As expected, the inline displacements decrease as the pretension increases. (Figs. 4 and 6). The orthogonal coherence spectra in Figs. 5 and 7 show that in the [0.05-0.1 Hz] frequency band the total model coherence is approximately equal to 0.95 in the low pretension case and 0.92 in the high pretension case. In other words, the Volterra model accounts for approximately 95% of the measured displacement power in the low pretension case, and approximately 92% in the high pretension case, over the frequency band where most of the excitation (wave) energy is concentrated. The linear component of the Volterra model accounts for 75-80%, and the cubic component for approximately 15% of the observed displacement power over the same frequency band in both pretension cases. The cubic component's contribution increases in both cases at low frequencies (<0.03Hz, approximately) and high frequencies (>0.15Hz, approximately). The quadratic component's contribution is small over all frequencies.

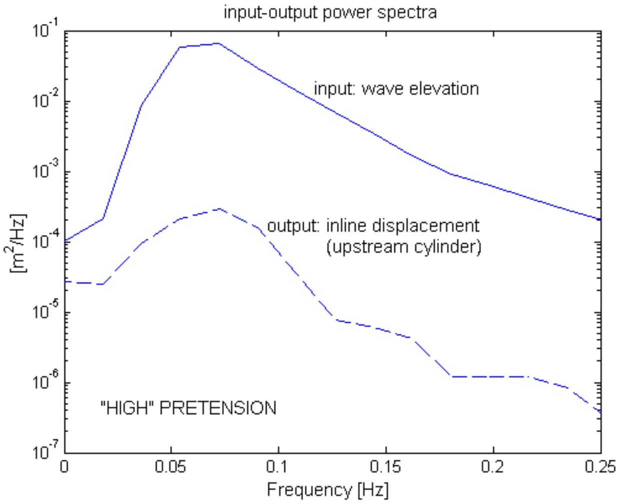

Fig. 6 Power spectra of wave elevation –cylinder inline displacement system; high pretension.

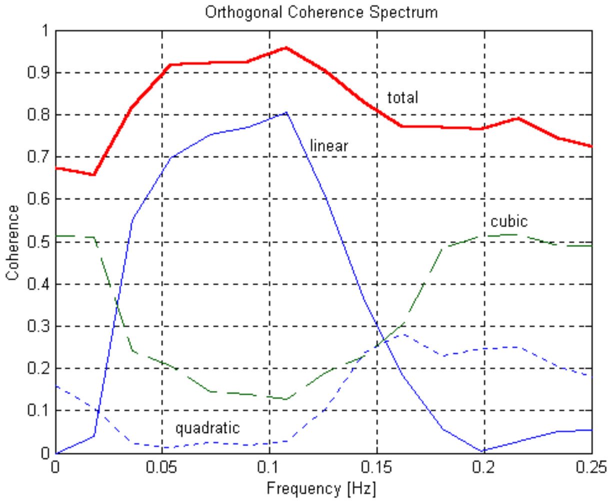

Fig. 7 Orthogonal coherence spectrum for the Volterra model of the wave elevation-cylinder inline displacement system; high pretension.

Figs. 8~13 display the wave elevation-inline reaction force analysis results. In this case, too, the reaction force decreases as the pretension level increases (Figs. 8 and 10). It is interesting to note that in the high pretension case the reaction force power spectrum has two equal peaks, at $f$=0 and in the high wave energy frequency band. The orthogonal coherence spectra (Figs. 9 and 11) display similar characteristics to the wave-displacement coherence spectra in Figs. 5 and 7, that is a strong linear component in the [0.05-0.1Hz] frequency band and significant cubic components at low and high frequencies. The total coherence, however, is higher in the high pretension case than in the low pretension case.

The main conclusion that can be drawn from the analysis of System I is that, given the wave excitation, the Volterra model can accurately approximate the upstream cylinder response, particularly in the high pretension case. The pretension level affects the wave-displacement and wave-reaction force correlations differently. It has little effect on

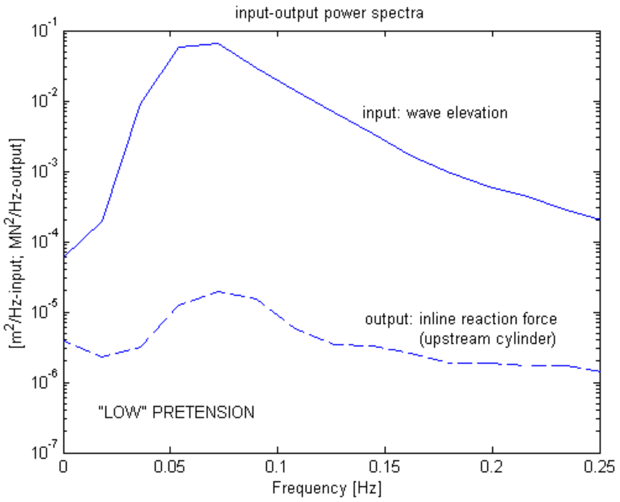

Fig. 8 Power spectra of wave elevation –cylinder inline reaction force system; low pretension.

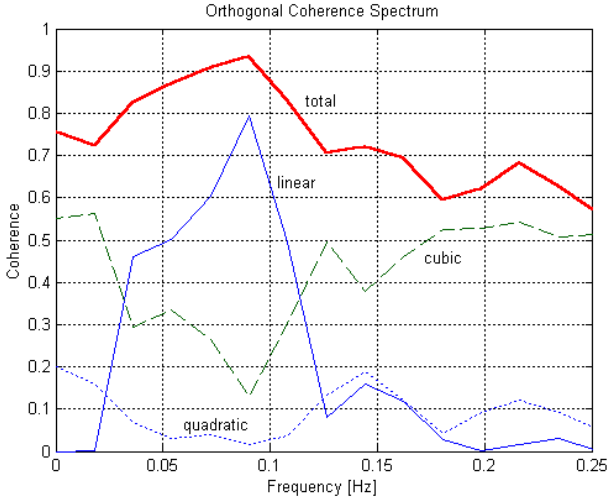

Fig. 9 Orthogonal coherence spectrum for the Volterra model of the wave elevation-cylinder inline reaction force system; low pretension.

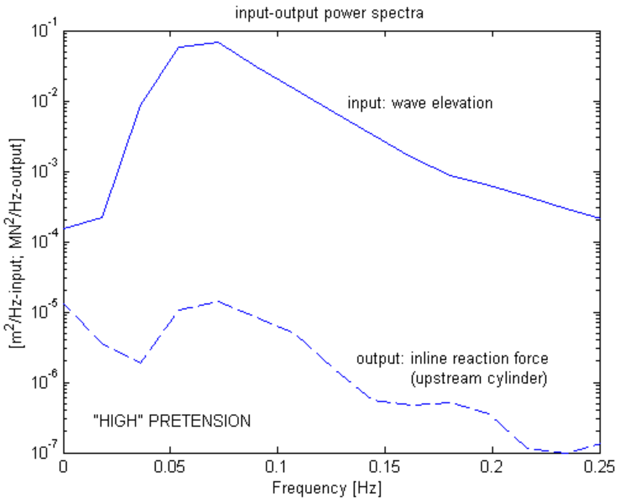

Fig. 10 Power spectra of wave elevation –cylinder inline reaction force system; high pretension.

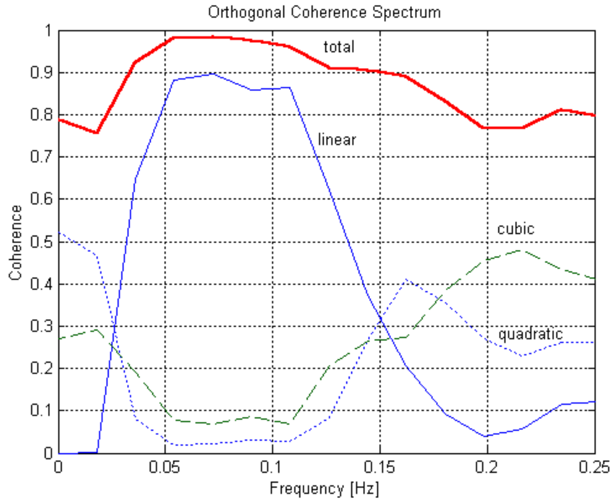

Fig. 11 Orthogonal coherence spectrum for the Volterra model of the wave elevation-cylinder inline reaction force system; high pretension.

the wave- displacement correlation. On the other hand, higher pretension results in a very strong wave-reaction force correlation, which is mainly linear in the high wave energy frequency band. This effect must be taken into account in the design of parametric models for the cylinder response under wave loading.

## System II: Upstream-downstream cylinder system

The upstream-downstream cylinder system was analyzed by using two sets of data, namely the cylinders inline displacements and cylinders inline reaction forces, respectively,for the two pretension levels. Since the analyzed data sets contained displacement and reaction force measurements for both inline and transverse directions, transverse displacement and reaction force power spectra are also presented, for completeness purposes.

Figs. 12~17 display the results of the upstream-downstream displacements analysis. The power spectra of the upstream and downstream cylinder inline displacements are very similar to each other for both pretension cases (Figs. 12 and 15). It is interesting to note the high spectrum component at $f$=0 (steady term) for the downstream cylinder displacement in the high pretension case (Fig. 15). The orthogonal coherence spectra in Figs. 13 and 16 are characterized by very high total coherence values (>95%) in the [0.05-0.15 Hz] frequency band for both pretension levels, which are mainly due to the linear component of the Volterra model. The cubic component's contribution to the total coherence in both plots is important only at low and high frequencies, as in System I.

The transverse displacements power spectra depicted in Figs. 14 and 17 reveal a relatively larger effect of the pretension level on the transverse than on the inline displacements (Figs. 12 and 15). For the lower pretension case, the upstream cylinder transverse displacement has a very similar in shape power spectrum to the inline one, albeit of smaller amplitude. However, the downstream transverse displacement power spectrum has a different shape than the inline, with its peak at zero frequency and not in the [0.05-0.1 Hz] frequency band. For the higher pretension case, the upstream and downstream power spectra for the transverse displacements are similar to each other (Fig. 17), but different than their inline counterparts (Fig. 15). They, too, have their main peak at zero frequency, rather than in the [0.05-0.1 Hz] frequency band. These changes in the spectral shapes indicate an energy transfer from the high energy frequency band to the lower frequencies, which can only happen through nonlinear mechanisms.

The results of the upstream-downstream reaction forces system analysis (Fig. 18~23) demonstrate more clearly the effect of the pretension level on the paired cylinder response. In the low pretension case, the downstream inline power spectrum is narrower than the upstream one (Fig. 18). In the high pretension case, the reaction forces have almost identical power spectra, with two spectral peaks of similar amplitude at $f$=0 and in the high wave energy frequency range (Fig. 21). The linear coherence in the high pretension case approaches unity in the [0.05-0.1Hz] range, compared to a linear coherence equal to 0.6-0.7 over the same frequency band in the low pretension case (Figs. 19 and 22). However, the higher cubic coherence in the low pretension case compensates for the lower linear one, resulting in a total model coherence with values over 0.9 in the [0.05-0.1Hz] range

The effect of the pretension level can more clearly seen in the transverse reaction forces power spectra for the two pretension cases (Figs. 20 and 23). In the low pretension case, the transverse reaction force power spectra have very similar shapes but the downstream one has larger power over all frequencies. In the high pretension case the two power spectra are very similar, with most of the power concentrated at very low frequencies. In both pretension cases, there appears to be a nonlinear energy transfer from the high wave energy frequency band to the very low frequencies.

The Volterra analysis of System II showed the third-order Volterra model can accurately approximate the inline response of the downstream cylinder, based on the upstream cylinder response input. Furthermore, it is the linear component of the Volterra model that contributes the most in the high wave energy frequency band, particularly in the high pretension case. In other words, the downstream cylinder response can be modeled with reasonable accuracy, depending on the pretension level, by applying linear transfer functions to the upstream cylinder response.

## CONCLUSIONS

The objective of this paper was to demonstrate the usefulness of the nonlinear system identification and analysis approach in tackling a difficult wave-structure interaction problem. The frequency domain Volterra model analysis of the experimental data in the wave-pair of cylinders interaction enabled the identification and quantification of the linear/nonlinear correlation between the excitation and response variables. Such an assessment of the significance of the various linear/nonlinear mechanisms is important in the design of practical parametric models predicting the response of each of the two cylinders. Furthermore, the Volterra model analysis also revealed the effect of the pretension level on the linear/nonlinear system representing the interaction between the two cylinders. Clearly, there is a need for further assessment and parameterization of the effect of the pretension level on the paired cylinder response.

## ACKNOWLEDGEMENTS

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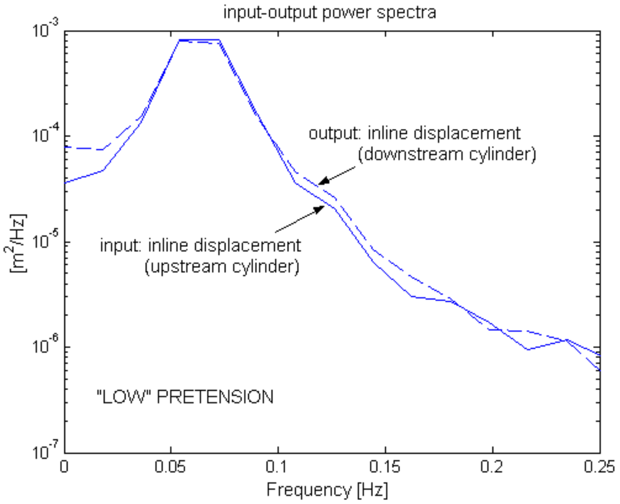

Fig. 12 Power spectra of paired cylinder <u>inline displacements</u>; low pretension.

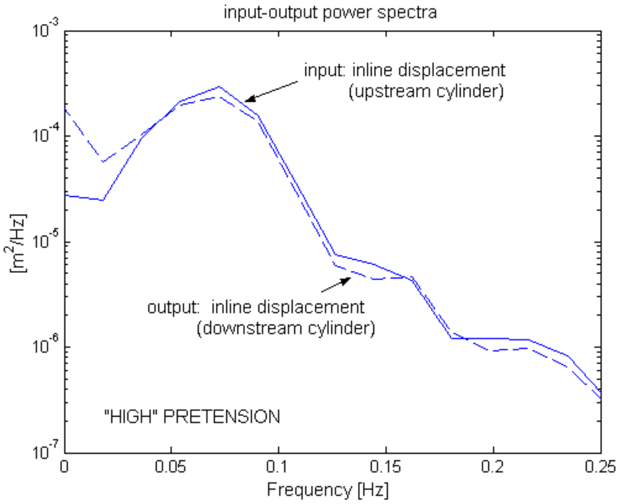

Fig. 15 Power spectra of paired cylinder <u>inline displacements</u>; high pretension.

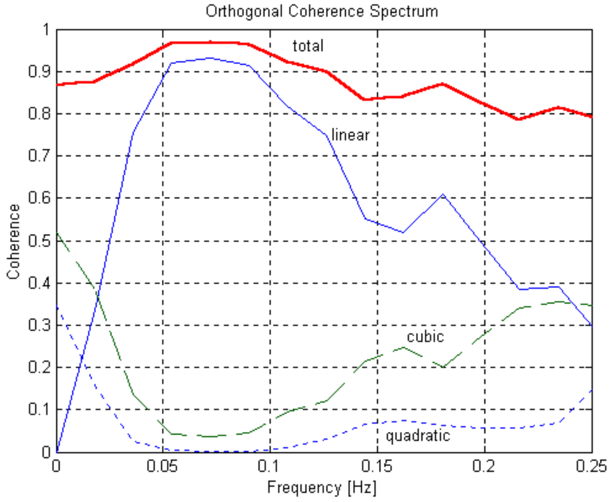

Fig. 13 Orthogonal coherence spectrum for the Volterra model of the paired cylinder <u>inline displacements</u> system; low pretension.

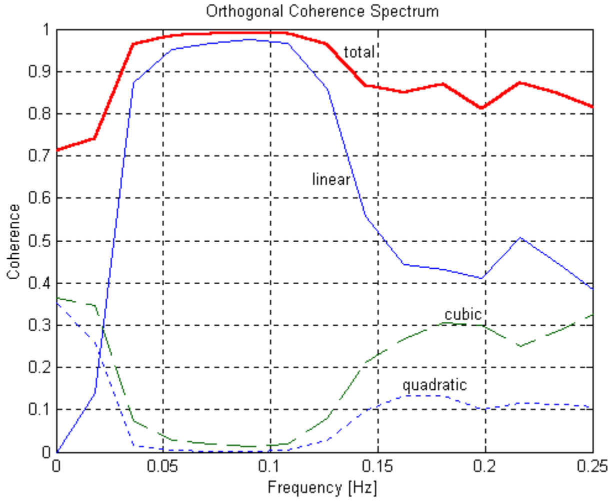

Fig. 16 Orthogonal coherence spectrum for the Volterra model of the paired cylinder <u>inline displacements</u> system; high pretension.

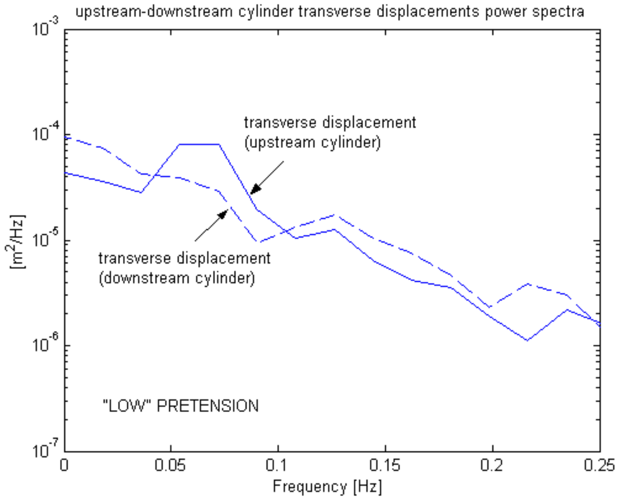

Fig. 14 Power spectra of paired cylinder <u>transverse displacements</u>; low pretension.

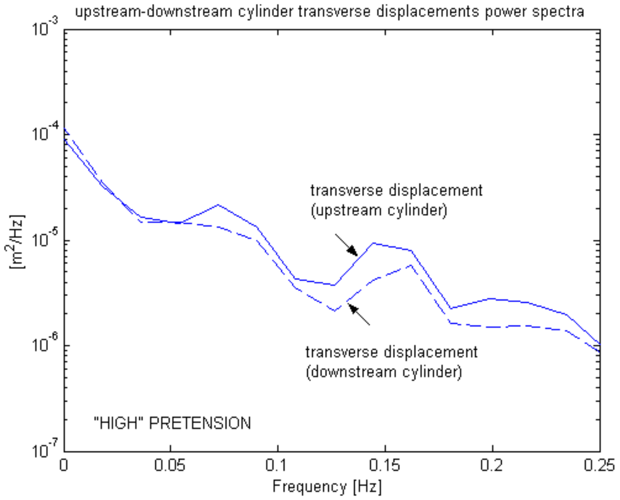

Fig. 17 Power spectra of paired cylinder <u>transverse displacements</u>; high pretension.

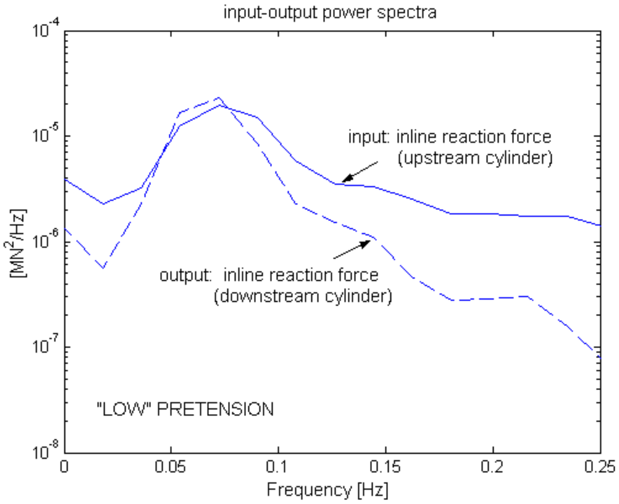

Fig. 18 Power spectra of paired cylinder inline reaction forces system; low pretension.

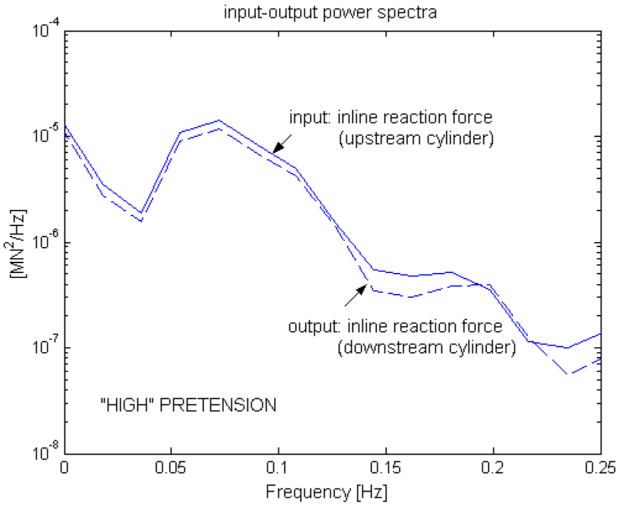

Fig. 21 Power spectra of paired cylinder inline reaction forces system; high pretension.

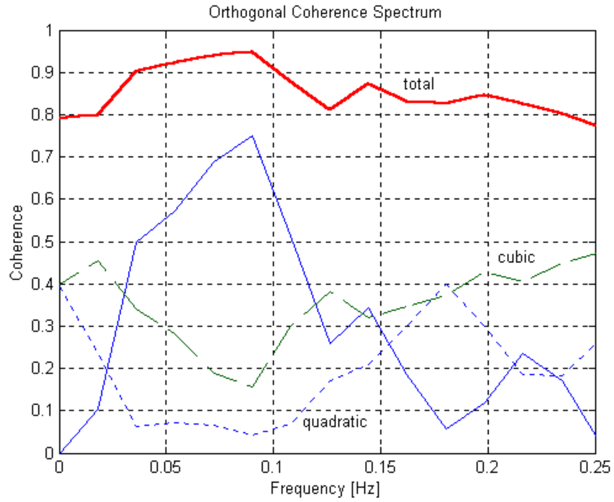

Fig. 19 Orthogonal coherence spectrum for the Volterra model of the paired cylinder inline reaction forces system; low pretension.

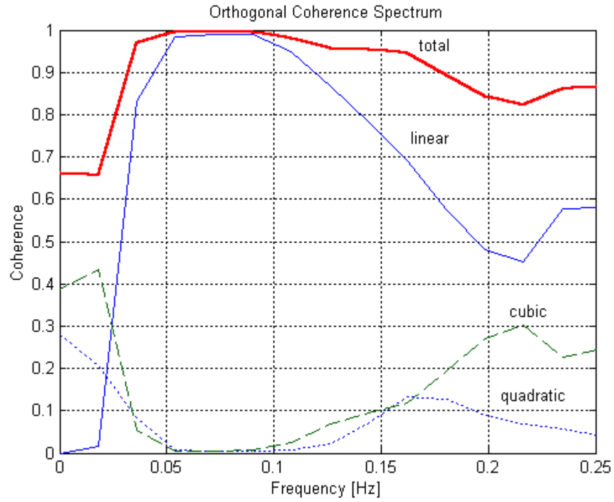

Fig. 22 Orthogonal coherence spectrum for the Volterra model of the paired cylinder inline reaction forces system; high pretension.

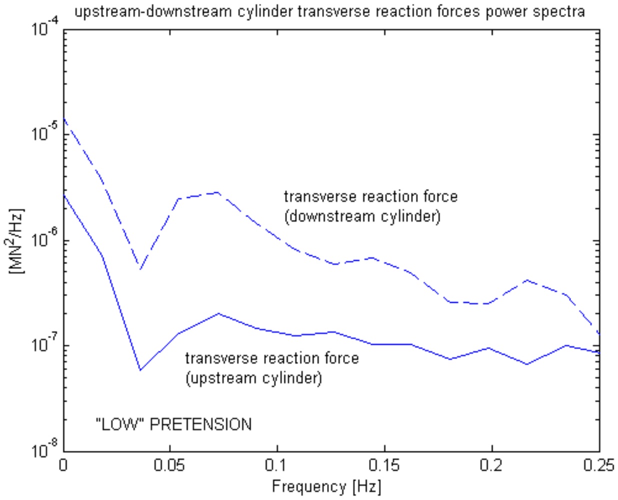

Fig. 20 Power spectra of paired cylinder transverse reaction forces; low pretension.

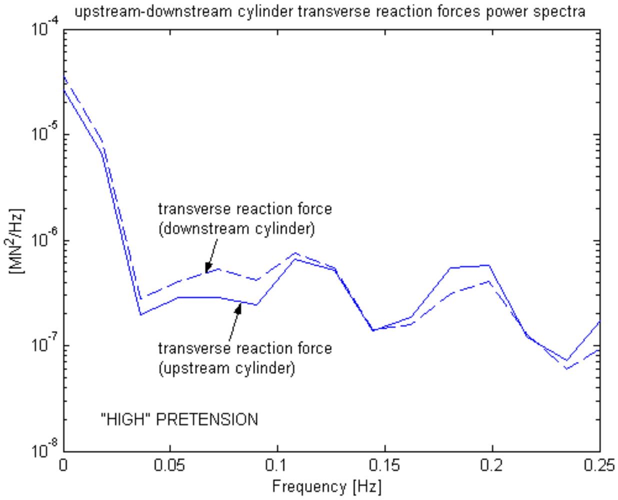

Fig. 23 Power spectra of paired cylinder transverse reaction forces; high pretension.