Riser Interaction Model: A Combined Time/Frequency Domain Approach

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Executive Summary

This research investigation focused upon the development of a new approach to interpret complex hydrodynamic phenomena using data from experimental studies or field measurements. A promising system identification technique was further developed to address nonlinear and frequency dependent aspects common to marine riser and ocean platform dynamics. The sequential development of analytical formulations and the use of computer simulations and model basin experiments are documented herein and in archival refereed publications.

Initially, a distributed parameter formulation incorporating the reverse system identification technique for multiple input/ single output nonlinear problems was investigated. This combined time/frequency domain method was used to illustrate the propagation of various types of hydrodynamic nonlinearities along the length of a marine riser. The marine riser response model introduced the combined method of normal modes and system identification procedures. Numerical simulations using this new approach demonstrated that the parameters of interest were convergent for each of the modes that were included. Further, the sensitivity of this methodology to predict the selected parameters over a range of frequencies and the degree of variation that could be expected under ideal simulation conditions, and the propagation of nonlinearities along a riser were illustrated.

The further development of this time/frequency domain system identification technique addressed the precise evaluation of the frequency dependence of parameters such as added mass, stiffness and damping, as well as the use of fully correlated signals in the process of parameter estimation. Although some multiple riser data was available, it was not suitable for this aspect of the study and a recent series of rigid and compliant mini-TLP model tests was used. Practical issues regarding the application of this approach, the utilization of force and moment measurements, the over specification of non-linearities in a predictive model and the ordering of non-linear contributions by their importance were addressed. In general the results demonstrate that the methodology is quite robust and yields predictions that are most accurate for the parameters associated with the largest motions of the ocean system being investigated. At this point the methodology is sufficiently developed to where it can be applied to investigate a wide range of marine riser, ocean structures and floating platform systems.

1. Introduction

The design of deepwater marine riser systems and compliant offshore structures requires engineers to address many difficult challenges which include defining and modeling the local offshore environment, specifying the associated combined global loading on innovative platform designs, and numerical simulation and model test verification of the platform response characteristics. The focus of this research investigation is the recovery of key parameters from time series measured during an industry type model basin test program using new model developments incorporating the reverse multiple input/single output (R-MI/SO) technique.

The reverse multiple input/single output (R-MI/SO) technique as originally developed by Bendat (1990,1993) and Bendat, Coppolino and Palo (1995) is a frequency domain system identification technique that is suited to address a wide range of problems in science and engineering. The methodology, which is intended to extract information about the system parameters directly from recorded time series, requires measurement of the excitation (cause) and the corresponding response data (effect). In the formulation the causality is reversed, meaning that the system kinematics (displacements, velocities, etc…) that are typically the output due to the applied loading inputs (forces and moments) are switched, that is the outputs become the inputs and vice versa. A conditioning procedure is then applied to successively eliminate the linear contents between the inputs and the output. In more complex systems, these operations decompose the nonlinear system into a number of linear sub-systems. The auto- and cross-spectra are used to obtain the linear transfer functions of each sub-system, and subsequently by examining these transfer functions and the ordinary and cumulative coherence functions for each linear and nonlinear term, one can interpret the importance and magnitudes of the desired system parameters. It is worth noting here that should additional nonlinear terms be introduced into the problem definition, the procedure will yield coefficients whose value is essentially zero and effectively the terms will drop out. This method is therefore particularly well suited for the identification of parameters appearing in nonlinear second-order differential equations of motion with constant or frequency-dependent coefficients, which are commonly encountered in offshore mechanics. The R-MI/SO method has been applied to single-degree-of freedom models of compliant offshore platform (Panneer-Selvam and Bhattacharyya 2001).

In the sections that follow, the sequential development of the model is presented in a compact manner. Additional details can be found in the archival publications listed in Section 5.

2. Marine Risers

Many engineering problems are best modeled as distributed-parameter systems. The governing equations describing the dynamic behavior of these systems require derivatives of the response with respect to two or more independent variables, usually time and position or angle. Mathematically describing their behavior leads to either a single partial differential equation or to a coupled system of partial differential equations with constant coefficients. The objective of system identification is evaluation of the key problem parameters from time series data, based upon the form of the governing equation or equations. Linear and nonlinear system identification is an extensively developed subject where very efficient methods combining time and frequency domain methods have been developed to extract information about key system parameters from measured records of excitation and response data (see for example Imai et al. 1989, Bendat 1990, Rice and Fitzpatrick 1991, and Bendat 1998).

A reverse dynamic nonlinear systems identification technique for multiple-input/single-output (MI/SO) problems described by means of ordinary differential equations was presented by Bendat (1990, 1998). The power of the remarkable reverse MI/SO technique is that a nonlinear system model with feedback can be transformed into an equivalent reverse dynamic MI/SO linear model without feedback. The resulting system is then decomposed into a number of linear sub-systems that involve the computation of various conditioned (residual) spectral density functions that successively eliminate the linear contents between the inputs and the output. Using this procedure typical system parameters including, the mass, stiffness and damping, as well as, the coefficients associated with a general nonlinear damping-restoring term can be evaluated from the frequency domain results. Application of this approach to investigate a variety of two-degree of freedom nonlinear systems can be found in the technical book by Bendat and Piersol (1992). Later Bendat and Piersol (1993) pointed out that estimation procedures based on frequency response functions for single-input/single-output (SI/SO) systems can easily be extended to arbitrary distributed-parameter systems subjected to distributed inputs if the system can be described in terms of its normal modes. In 1998 Bendat showed how to replace six degree of freedom (DOF) nonlinear models for ocean engineering applications with equivalent reverse linear models than can be solved by the linear data analysis procedures.

The parameters of physical systems and engineering problems of interest are generally distributed in space, and thus system identification methods must be extended to deal with distributed-parameter systems. Banks and Kunisch (1989) published a monograph which summarized their development efforts on parameter identification analyses of distributed-parameter systems. The monograph focus is on approximation methods for least squares inverse problems governed by partial differential equations and addresses issues of the identifiability and stability of the estimated parameters. Specific results dealing with the approximation and estimation of coefficients in linear elliptic equations were presented and discussed.

Some previous studies have addressed aspects that are connected with the approach taken in this study of marine riser dynamics. Stansby et al. (1992) investigated different forms of the extended Morison equation including extra terms such as a Duffing type force. The inclusion of the extra terms in the force was used to address specific consideration of vortices rather than the more general view of non-linearities taken in this present study. Jones et al. (1995) indicated that standard decay tests for the evaluation of the damping are not readily available for large structures and that the only economical approach is to use ambient vibration data. Based upon similar logic, it seems reasonable that the system identification approach presented herein addresses the use of field or laboratory excitation of marine risers by ocean wave and currents. A compliant single degree of freedom system was studied by Panneer-Selvam and Bhattacharyya (2001). They considered four different data combination scenarios and developed an iterative scheme for the identification of the hydrodynamic coefficients in a Morison type excitation model and included in their analysis a non-linear stiffness parameter (Duffing coefficient). Their analysis procedure used reverse MI/SO technique and their findings showed that the approach was robust for both weak and strongly nonlinear systems.

In this study, a production riser for a deepwater structure is considered and Bendat's MI/SO reverse identification technique is extended to address distributed-parameter multi-DOF systems that include general nonlinear damping-restoring terms. It is assumed that the physical properties of the marine riser, e.g. mass, stiffness, etc., are constant along the length of the riser. Thus, the resulting coupled partial differential equations involve two independent variables, time and location along the axis. The discretization of the marine riser is carried out with the objective of accurately modeling the excitation and obtaining accurate modal responses to compare with data that measured displacement at a single elevation in the laboratory tests. The analysis illustrates the use of modal analysis and the nature of the convergence of modal parameter estimates for random ocean wave excitation of the marine riser.

2.1 Normal Mode Formulation

As one begins to think about the range of sources for non-linear behavior that are possible, other approaches to modeling the non-linear response behavior need to be considered. The governing partial differential equation for a marine riser has been developed and discussed in many technical articles; see for example McIver and Olson (1981). In this study, it is suggested that the nonlinear drag force be treated more generally in a form consistent with nonlinear system identification,

$$\begin{aligned} EI\frac{\partial^4 v(x,t)}{\partial x^4} - T_e(x,t)\frac{\partial^2 v(x,t)}{\partial x^2} - w(x,t)\frac{\partial v(x,t)}{\partial x} \\ + M(x)\frac{\partial^2 v(x,t)}{\partial t^2} + c(x,t)\frac{\partial v(x,t)}{\partial t} + p(x,\dot{x},\ldots,t) = f_L(x,t) \end{aligned} \tag{1}$$

where,

$$f_L(x,t) = C_M\frac{\rho_w \pi D^2}{4}\frac{\partial u(x,t)}{\partial t} + C_D\frac{\rho_w D}{2}\sqrt{\frac{8}{\pi}}\sigma_{\dot{r}}\ u(x,t) \tag{2}$$

$$p(x,\dot{x},...,t) = k_3\, v^3(x,t) + \frac{c_3}{3}\frac{\partial v^3(x,t)}{\partial t} \tag{3}$$

In general the notation used throughout this report is consistent with that adopted by offshore engineers (see for example Sarpkaya and Isaacson 1981), however, others are not and will be defined. Note that k_3 is the Duffing coefficient and c_3 is the Van der Pol coefficient. The Duffing non-linearity acts as an artificial spring with variable positive stiffness, $k_3(x)\,v^2(x,t)$, that increases as the displacement, $v(x,t)$, gets larger. Such a spring grows stiffer as the riser differential elements move away from their equilibrium position, but it recovers its original value when the segments return to their original position. Thus, high-amplitude excursions should oscillate faster than low-amplitude ones and the sinusoidal shapes should be "pinched in" at their peaks. The Van der Pol non-linearity acts as an additional damper.

In the normal mode approach a transformation to a generalized orthogonal space can be accomplished using the eigen properties of the problem, see for example Clough and Penzien (1993). Following this approach the displacement of the riser at any location x and time t can be expressed as a linear combination of N modes, specifically

$$v(x,t) \cong \sum_{i=1}^{N} \phi_i(x)\, P_i(t) \tag{4}$$

where $\phi_i(x)$ and $P_i(t)$ are the usual mode shape and generalized coordinate for the i^{th} normal mode. Similarly it is assumed that the cubic term can be expanded in terms of the same mode shapes, $\phi_i(x)$, and the new generalized coordinate $Q_i(t)$ of the i^{th} normal mode

$$v^3(x,t) \cong \sum_{i=1}^{N} \phi_i(x)\, Q_i(t) \tag{5}$$

By virtue of the orthogonal properties of the modes, all terms in each of the summations vanish except the one term for which $i=n$. Consequently the mathematical model for the marine riser for each mode, n, can be expressed as,

$$M_n \ddot{P}_n(t) + C_n \dot{P}_n(t) + K_n P_n(t) + D_n Q_n(t) + V_n \dot{Q}_n(t) = F_n(t) \qquad n = 1, 2, \ldots, N \tag{6}$$

For a pin-pin connected beam or string, the vibration shape of the n^{th} mode can be expressed as (Clough and Penzien 1993)

$$\phi_n(x) = sin\left(\frac{n\pi x}{L}\right) \tag{7}$$

where L represents the length of the marine riser. The corresponding generalized expressions are

$$M_n = \frac{mL}{2} \tag{8}$$

$$C_n = \frac{cL}{2} \tag{9}$$

$$K_n = \frac{EIL}{2}\left(\frac{n\pi}{L}\right)^4 + \frac{TL}{2}\left(\frac{n\pi}{L}\right)^2 \qquad n = 1, 2, 3, \ldots \tag{10}$$

$$D_n = \frac{k_3 L}{2} \tag{11}$$

$$V_n = \frac{c_3 L}{6} \tag{12}$$

$$P_n(t) = \frac{2}{L}\int_0^L v(x,t)\sin\left(\frac{n\pi x}{L}\right)dx \tag{13}$$

$$Q_n(t) = \frac{2}{L}\int_0^L v^3(x,t)\sin\left(\frac{n\pi x}{L}\right)dx \tag{14}$$

In the reverse dynamic model, $F_n(t)$ is the mathematical output, and $P_n(t)$ and $Q_n(t)$ are the required inputs to the system. The two inputs are computed from the dynamic response of this nonlinear system, and can be non-Gaussian and correlated to some extent. From knowledge of $P_n(t)$, $Q_n(t)$ and $F_n(t)$, without restrictions on their probability or spectral properties, the TI/SO (Two Inputs/Single Output) linear system can be solved using the reverse MI/SO technique to identify the two frequency response functions, from which the system parameters can be recovered. Thus, all that remains in the formulation is the development of the frequency domain equations.

Based on the computation of these three quantities, it is straightforward to compute the two frequency response functions $E_n(f)$ and $F_n(f)$

$$E_n(f) = K_n - (2\pi f)^2 M_n + j(2\pi f)C_n \tag{15}$$

$$F_n(f) = D_n + j(2\pi f)V_n \tag{16}$$

The generalized stiffness, mass and damping for the n^{th} mode of vibration are given by

$$K_n = \lim_{f \to 0} \mathrm{Re}\left(E_n(f)\right) \tag{17}$$

$$M_n = \frac{K_n - \mathrm{Re}\left(E_n(f)\right)}{(2\pi f)^2} \tag{18}$$

$$C_n = \frac{\mathrm{Im}\left(E_n(f)\right)}{2\pi f} \tag{19}$$

$$D_n = \mathrm{Re}\left(F_n(f)\right) \tag{20}$$

$$V_n = \frac{\mathrm{Im}\left(F_n(f)\right)}{2\pi f} \tag{21}$$

Recalling Equations (8), (9), (11) and (12), the mass, damping, Duffing and Van der Pol terms can be computed from any mode of vibration and the following parameters recovered

$$m = \frac{2}{L} M_n \qquad \forall n = 1, 2, 3, \ldots \tag{22}$$

$$c = \frac{2}{L} C_n \qquad \forall n = 1, 2, 3, \ldots \tag{23}$$

$$k_3 = \frac{2}{L} D_n \qquad \forall n = 1, 2, 3, \ldots \tag{24}$$

$$c_3 = \frac{6}{L} V_n \qquad \forall n = 1, 2, 3, \ldots \tag{25}$$

On the other hand, the determination and partition of the flexural stiffness and tension require the value of the generalized stiffness for at least two different modes of vibration. Knowing the generalized stiffness for two different modes of vibration *p* and *q* and recalling Equation (10), we get a linear system with two equations and two unknowns. Solving for the tension and the flexural, or bending, stiffness yields

$$T = \frac{q^4 K_p - p^4 K_q}{\frac{L}{2}\left(q^4 \left(\frac{p\pi}{L} \right)^2 - p^4 \left(\frac{q\pi}{L} \right)^2 \right)} \qquad \forall p \neq q \tag{26}$$

$$EI = \frac{q^2 K_p - p^2 K_q}{\frac{L}{2}\left(q^2 \left(\frac{p\pi}{L} \right)^4 - p^2 \left(\frac{q\pi}{L} \right)^4 \right)} \qquad \forall p \neq q \tag{27}$$

2.2 Convergence of Identified Parameter Values

For this example, a marine riser, 873m long, with the properties summarized in Table 1 was studied. The marine riser was assumed to be pin-pined with the top hinge positioned 23 m above the still water level and was subject to Gaussian random wave excitation. Linear wave theory was used to generate the wave kinematics in the computation of the the wave loads. The initial spatial discretization of the marine riser model was 1 meter and the riser motions were computed at 8192 time steps, which is equivalent to a test duration of 27.3 minutes.

The mass per unit length for the first five modes of vibration identified using the reverse system identification model are presented in Figure 1. Recall that the frequency range of interest is 0 to 1 Hz. This graph illustrates that the marine riser can be thought

	Units	Specified value
Riser Length	m	873
Outer radius	m	0.7
Mass per unit length (Including added mass)	kg/m	912
Linear viscous drag coefficient	Ns/m	120
Duffing coefficient	N/m^3	8000
Van der Pol coefficient	Ns/m^3	5000
Tension	N	7×10^6
Bending stiffness	Nm^2	10^7

Table 1. Particulars for the Marine Riser numerical example.

of as a low pass filter since the estimates of mass per unit length do not cover the white noise frequency range in its entirety. On the other hand, as can be observed in the figure, the specified mass $(912\,kg/m)$ can be recovered with good accuracy from the generalized mass of any mode of vibration. Moreover, it seems that higher modes tend to give a better accuracy over the whole frequency range. The estimates of the linear viscous damping coefficient for the first five modes of vibration are presented in Figure 2. It can be observed that the value of the linear viscous damping is recovered with acceptable accuracy over the 0 to 0.5 Hz frequency range. In the 0.5 to 1 Hz frequency range, the scattering of the results is perhaps a consequence of the filtering action of this marine riser system. The estimates of the Duffing coefficient for the first five modes of vibration are shown in Figure 3. Again it is observed that the estimates are accurate over only a portion for the frequency range of interest (0 - 1 Hz). This is similar to the case for the mass per unit length, and it is concluded that these results reflect the same signs of low pass filtering by the marine riser system. The estimates of the Van der Pol coefficient for the first five modes of vibration are shown in Figure 4. This modal coefficient is less well behaved at very small frequencies. Omitting this range a reasonable estimate can be obtained for the coefficient.

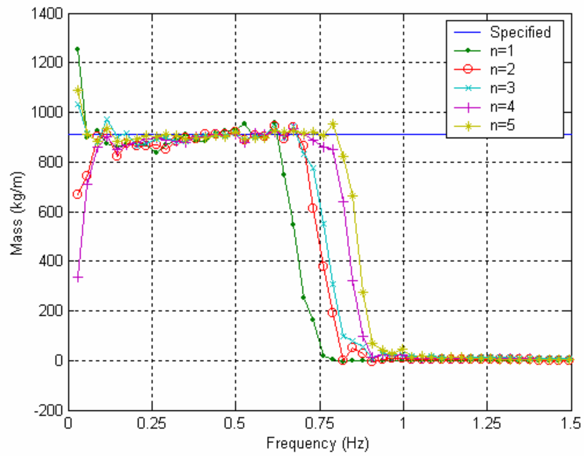


Figure 1. Modal estimates of the mass per unit length obtained using reverse system identification.

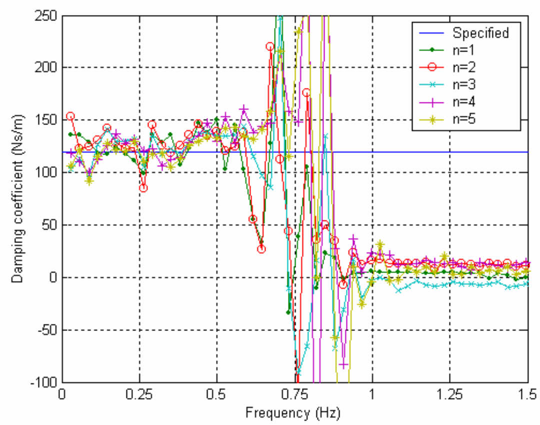


Figure 2. Modal estimates of the viscous damping coefficient obtained using reverse system identification

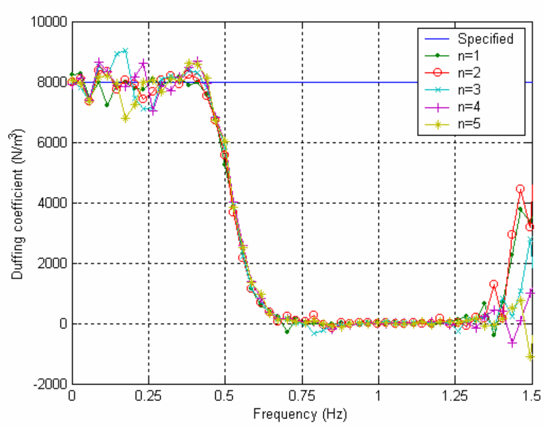


Figure 3. Modal estimates of the Duffing coefficient obtained using reverse system identification.

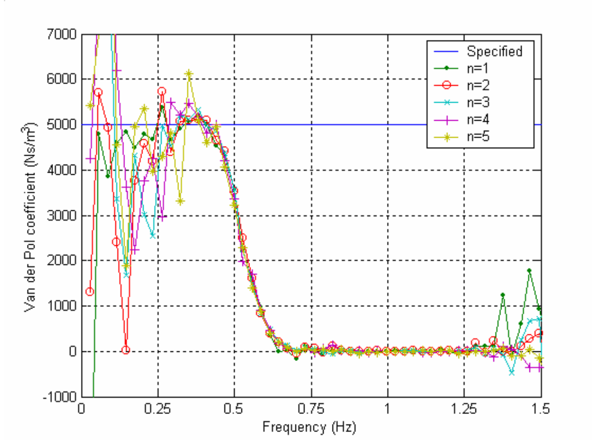
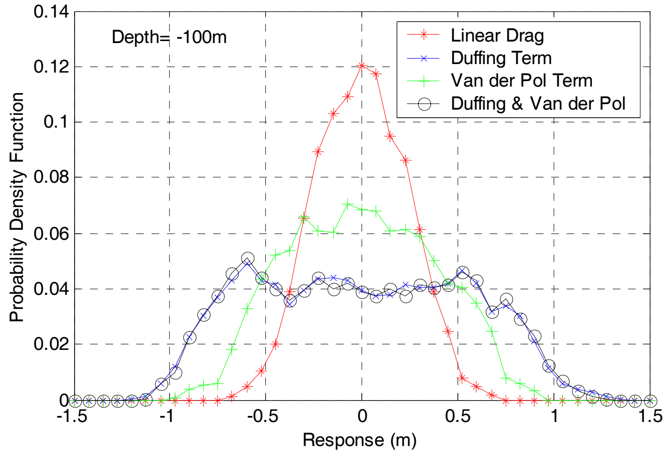


Figure 4. Modal estimates of the Van der Pol coefficient obtained using reverse system identification.

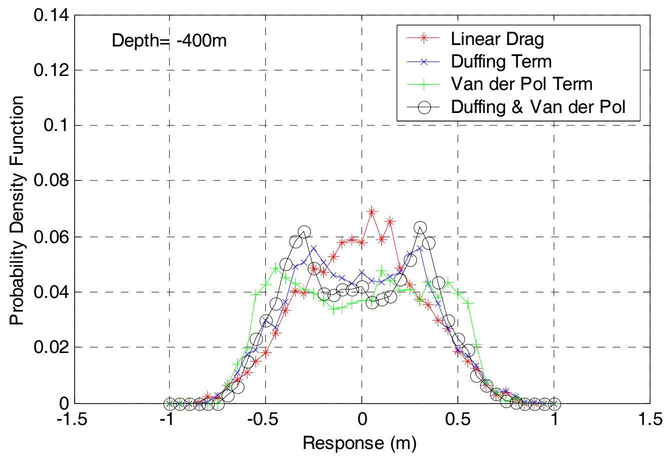
2.3 Propagation of Nonlinearities along a Marine Riser

It is expected that where Gaussian excitation is applied as an input to a linear system, here the linear riser model with a linear force model, the output, i.e. response will also be Gaussian. The random sea excitation consisted of a wave train of 117 waves with a minimum wave elevation of –8.44m and a maximum wave elevation of 12.53m. An analysis of the wave input indicated that the design seas had a mean of 0.27 m, a variance of 11.92 m^2, a skewness of 0.303 and a kurtosis of 3.02.

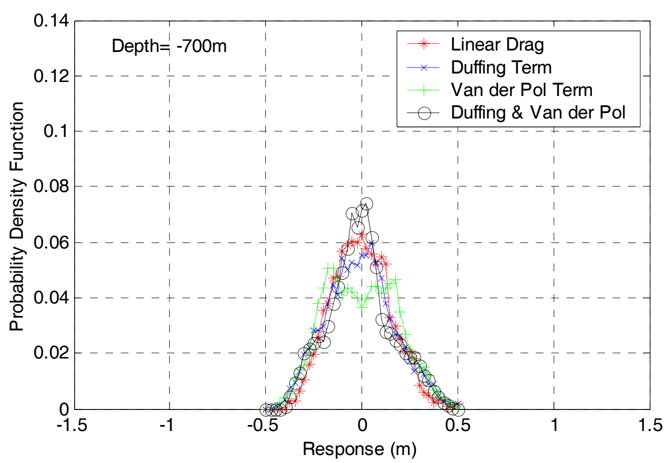
If one considers the graphs of the distributions shown in Figure 5, some interesting behavior is observed that is not obvious from the values of the skewness and kurtosis at the various water depths. In Figure 5a, the linear system behavior can be observed in the shape of the distribution, which appears to be Gaussian. The influence of the nonlinearities is also captured in the reduction of the peak values and broadening of the response behavior distributions. In the combined Duffing – Van der Pol model the response is dominated by the Duffing non-linearity. Since the excitation is concentrated in the near surface, decaying exponentially for deep water, this combined with the fluid damping along the riser will also diminish the response behavior with depth and this should be translated into reduced peaks and distribution width. This is indeed the case for the 700m-water depth, and the coalescing of the distribution width leaving only the peaks to be modified by the nonlinear models is consistent with one's intuition about the anticipated response behavior. However, at the 400m water depth the bi-modal behavior of the distribution peaks for the Duffing and combined nonlinear models is quite interesting as is the continued domination of the response by the combined nonlinear Duffing - Van der Pol model.



(a)



(b)



(c)

Figure 5. Probability density function profiles at three different depths.

3. More Complex Ocean Systems

In previous studies of fluid-structure interaction problems (Chung and Kim 1995, Niedzwecki and Liagre 2002), computer-generated numerical data from simulated nonlinear systems were used to demonstrate that the R-MI/SO technique provides an accurate practical method to identify the dynamic properties of various desired nonlinear systems by converting nonlinear models into equivalent linear models. These studies were quite focused and did not take complete advantage of the R-MI/SO technique to help identify the best nonlinear integrodifferential equation of motion. Attention was directed at specified constant parameters used to synthesize the systems' dynamic response where the identified system parameters were numerically estimated from the spectral mean of the transfer functions over a determined frequency range, such as, the response frequency range of the system. Using a more extended approach, Bhattacharyya and Panneer Selvam (2001) applied the R-MI/SO to a synthesized dataset describing the response of a large moored floating body to ocean waves, which included frequency dependent added-mass and damping coefficients as well as linear and nonlinear mooring line stiffness coefficients. The application of the R-MI/SO method to a two input/single output model of the system showed that the recovery of the added-mass coefficient's frequency dependence is feasible and reasonably accurate as long as the linear stiffness coefficient is assumed to be constant. The frequency dependence of the damping coefficient was recovered quite accurately.

This phase of the study builds upon the R-MI/SO method as presently reported in the open literature but develops the methodology to address realistic systems with both constant and frequency dependent parameters. Of particular interest is the identification of the frequency dependence of the hydrodynamic added-mass and damping coefficients, since these coefficients are essential for understanding of the nonlinear effects influencing the dynamic behavior of compliant deepwater offshore structures. Large-scale model test data from a recent model test program of a mini-TLP (Faltinsen 1990, Narayanan and Yim 1998, Niedzwecki et. al. 2001), is used as the basis to investigate the application of the R-MI/SO method. This model test data was selected since it contains data on both the rigidly restrained hull and the corresponding compliant platform under identical wave conditions. This data set was selected for this reason, since it is impractical to measure the forces acting on a compliant structure without affecting its motions. The validity of assuming equivalency of forces and moments is investigated and discussed. The wave forces and moments were measured directly on the rigidly restrained hull model using a six degree-of-freedom load cell. An infrared optical tracking system was used to monitor the compliant model response behavior. Again, each model was subjected to the same wave conditions. The restoring forces due to the vertical mooring system, the tendons, also commonly referred to as tethers, and risers were post-computed from the time series recordings of the mini-TLP motions.

3.1 Mini-TLP Formulation

Commonly, the first issue in the implementation of any system identification technique is the selection of the governing equation or equations. The selection of the appropriate mathematical model must reflect both physical insight and the known mechanical properties of the system. In the present study, Newton's second law of motion governs the mini-TLP motions and the choice of a nonlinear second-order differential equation of motion asserts itself naturally (Newman 1977, Faltinsen 1990). A nonlinear stiffness (Duffing) term as well as a nonlinear damping term is added to this equation in order to take into account the nonlinear restoring forces due to the vertical mooring system and the hydrodynamic damping due to the wave-structure interactions. The system of coupled nonlinear differential equations governing the six degree-of-freedom (DOF) mini-TLP motions can be expressed in a manner consistent with Bendat (1998) as

$$\left[M + A\right]\ddot{q}(t) + B\,\dot{q}(t) + K\,q(t) + R\,q^3(t) + D\,\dot{q}(t)\left|\dot{q}(t)\right| = f(t) \tag{28}$$

where, M is the mass matrix, A is the hydrodynamic added-mass matrix, B is the linear damping matrix, K is the linear stiffness matrix, R is a nonlinear Duffing type stiffness matrix, and D the nonlinear damping matrix. The nonlinearity of the horizontal plane restoring forces due to the mooring system is accounted for by the cubic term. This first odd nonlinear function captures the change of direction of the restoring forces when surge or sway motions change of sign. The vertical restoring forces are believed to be dominated by linear behavior.

The general kinematic and environmental excitation vectors are defined as

$$\begin{aligned}
&q(t)=\left[x(t),y(t),z(t),\phi(t),\theta(t),\psi(t)\right]^T\\
&q^3(t)=\left[x^3(t),y^3(t),z^3(t),\phi^3(t),\theta^3(t),\psi^3(t)\right]^T\\
&\dot{q}(t)\left|\dot{q}(t)\right|=\left[\dot{x}(t)\left|\dot{x}(t)\right|,\dot{y}(t)\left|\dot{y}(t)\right|,\dot{z}(t)\left|\dot{z}(t)\right|,\dot{\phi}(t)\left|\dot{\phi}(t)\right|,\dot{\theta}(t)\left|\dot{\theta}(t)\right|,\dot{\psi}(t)\left|\dot{\psi}(t)\right|\right]^T\\
&f(t)=\left[f_x(t),f_y(t),f_z(t),m_x(t),m_y(t),m_z(t)\right]^T
\end{aligned} \tag{29}$$

For each degree of freedom (Bendat 1998), the components of $q(t)$, $q^3(t)$ and $\dot{q}(t)\left|q(t)\right|$ are taken as the inputs of the equivalent R-MI/SO model, while the component of $f(t)$ is taken as the output of the mathematical model. The platform kinematics vector, $q(t)$, encompasses the following translation and rotational components: $x(t)$the surge motion response (inline), $y(t)$ the sway motion (transverse), $z(t)$ the heave motion (vertical), $\phi(t)$ the roll rotation response about the x axis, $\theta(t)$ the pitch rotation response about the y axis, and $\psi(t)$ the yaw rotation response about the z axis. The general excitation vector $f(t)$ contains the following total external forces and moments applied on the hull of the compliant mini-TLP: $f_x(t)$ the inline force excitation, $f_y(t)$ the transverse force excitation, $f_z(t)$ the vertical force excitation, $m_x(t)$ the roll moment excitation, $m_y(t)$ the pitch moment excitation and $m_z(t)$ the yaw moment excitation.

The matrices M, A, B, K, R, and D are each 6×6 matrices and except for the system mass matrix all the other matrices could be considered to be potentially frequency dependent. The platform has two planes of symmetry and the origin of the axes on the platform is assumed to coincide with the center of gravity of the platform. Therefore the mass matrix, M, is a diagonal matrix. The first three entries on the diagonal are equal to the system structural mass, m, and the last three entries are the principal moments of inertia about the x, y and z axes, denoted I_{xx}, I_{yy} and I_{zz} respectively.

In each problem being analyzed, each potential source of coupling between the degrees of freedom in the mathematical model must be considered and only the most significant terms are generally retained for analyses. A study of an unrestrained ship reported by Mulk and Falzarano (1994) concluded that all six-degrees of motion affect one another and that one needs to be careful in assessing the importance of the various coupling terms. Unlike free-floating vessels, tension-leg platforms have well defined couplings between certain degrees of freedom, due to the platform geometry and vertical taut mooring system design. For the mini-TLP, based upon the geometry and mooring system restraints, it could be anticipated that the most significant sources of motion coupling include surge/heave, surge/pitch, sway/roll and heave/pitch. However, through further investigation of the model test data it was concluded that the surge/heave and heave/pitch coupling terms could be neglected for this particular platform, and only the coupling corresponding to the terms associated with the surge/pitch and sway/ roll will be included in this study. More specifically, this will be reflected in the added-mass, linear damping and linear stiffness matrices. For a symmetric platform such as the mini-TLP, neglecting the second-order terms will result in the nonlinear stiffness matrix, R, and the nonlinear damping matrix, D, being diagonal matrices.

The system of coupled nonlinear differential equations presented in Eq. (28) can be rewritten in the following indicial form

$$m_{ii}\ddot{q}_i(t)+\sum_{n=1}^{6}\left[a_{in}\ddot{q}_n(t)+b_{in}\dot{q}_n(t)+k_{in}q_n(t)\right]+r_{ii}q_i^3(t)+d_{ii}\dot{q}_i(t)\left|\dot{q}_i(t)\right|=f_i(t) \tag{30}$$

with $i=1,2,...,6$. Upon taking the Fourier transform of both sides of Eq. (30), the equivalent mathematical reverse dynamics input/output models are obtained

$$\underbrace{H_i^l(\omega)U_i(\omega)}_{\text{Linear terms}}+\underbrace{\sum_{\substack{n=1\\n\neq i}}^{6}H_n^c(\omega)U_n(\omega)}_{\text{Linear coupling terms}}+\underbrace{H_i^r(\omega)V_i(\omega)}_{\text{Nonlinear stiffness}}+\underbrace{H_i^d(\omega)W_i(\omega)}_{\text{Nonlinear damping}}=F_i(\omega) \tag{31}$$

where, $i = 1, 2, ..., 6$, and

$$\begin{aligned} U_i(\omega) &= \Im\left[q_i(t)\right] \\ V_i(\omega) &= \Im\left[q_i^3(t)\right] \\ W_i(\omega) &= \Im\left[\dot{q}_i(t)\left|\dot{q}_i(t)\right|\right] \\ F_i(\omega) &= \Im\left[f_i(t)\right] \end{aligned} \tag{32}$$

In addition, $\omega = 2\pi f$, is the radian frequency, f is the frequency in Hz, and $\Im[\ ]$ indicates a Fourier transform of the quantity in brackets. The relations between the transfer functions, H, and the Fourier transforms of the system parameters are found to be of the form

$$H_i^l(\omega) = K_{ii}(\omega) - \omega^2\left(M_{ii} + A_{ii}(\omega)\right) + j\omega B_{ii}(\omega) \tag{33}$$

$$H_n^c(\omega) = K_{in}(\omega) - \omega^2 A_{in}(\omega) + j\omega B_{in}(\omega) \tag{34}$$

$$H_i^r(\omega) = R_{ii}(\omega) \tag{35}$$

$$H_i^d(\omega) = D_{ii}(\omega) \tag{36}$$

The Fourier transforms $B_{ii}(\omega)$ and $B_{in}(\omega)$ of the linear damping coefficients b_{ii} and b_{in} appear by themselves respectively in the imaginary part of the transfer functions $H_i^l(\omega)$ and $H_n^c(\omega)$. Likewise, the Fourier transforms $R_{ii}(\omega)$ and $D_{ii}(\omega)$ of the nonlinear stiffness coefficients r_{ii} and nonlinear damping coefficients d_{ii} are found in the real part of two different transfer functions $H_i^r(\omega)$ and $H_i^d(\omega)$. This does not cause a problem for their evaluation from the transfer functions. However, the Fourier transforms $M_{ii}(\omega)$, $A_{ii}(\omega)$ and $K_{ii}(\omega)$, as well as $A_{in}(\omega)$ and $K_{in}(\omega)$, of the elements of the mass matrix M, the added-mass matrix A and linear stiffness matrix K turn up respectively in the real parts of the transfer functions $H_i^l(\omega)$ and $H_n^c(\omega)$. To avoid the problem of partitioning the structural system mass, the hydrodynamic added-mass and the linear stiffness terms, the formulation is modified in the following way. The acceleration and displacement signals are used as separate inputs. Given that the velocity is automatically accounted for by the imaginary part of the transfer function associated with the displacement, it does not need to be used as an input. Since the acceleration and displacement input signals are highly correlated, i.e. the acceleration is the second derivative with respect to time of the displacement; a method was developed to reduce the input signals' correlation before applying the conditioning procedure. Hence the mass appears alone in a dedicated transfer function, while the linear stiffness and linear damping appear respectively in the real and imaginary part of the second transfer function.

The most general equivalent mathematical reverse dynamics multiple input/multiple output (MI/MO) linear model has twenty-four inputs, the original six physical inputs $q(t)$ with its second derivative $\ddot{q}(t)$, plus the six associated $q^3(t)$ terms and the six associated $\dot{q}(t)|\dot{q}(t)|$ terms, and six outputs, the original three physical forces and the three physical moments. Whenever possible, it is always preferable to split the MI/MO model into a series of MI/SO models since the conditioning procedure of a large number of inputs and outputs (Sridhar, Mulder, van Staveren 1994) can introduce significant computational errors. In the present case, MI/SO models can be employed because the coupling between motions is principally due to the mechanical properties of the mooring system of the mini-TLP. These effects are included in the general excitation vector, $f(t)$. As mentioned earlier, it is preferable to consider the displacement and acceleration components as two different inputs so that there is no ambiguity in the determination of the mass and stiffness coefficients. Indeed, if only the displacements $q(t)$ are used as inputs to represent the linear part of the system, the mass and stiffness terms both appear in the real part of the same transfer function, which makes them impossible to discern. Utilizing the symmetry conditions of the mini-TLP and the assumption that only surge/pitch and sway/roll are the most important coupling effects, the 24-input/6-output MI/MO linear model can be recast as six MI/SO models, where each of these models requires either four (in the case of heave and yaw motions) or five (all the other modes of motion) different frequency response functions.

If the measured inputs are not correlated to each other, the contribution of each input to the output can be obtained from the frequency response and ordinary coherence functions between the various inputs and the output. On the other hand, if the measured inputs are partially correlated, a new set of uncorrelated inputs must be obtained by conditioning the inputs before calculating the contribution of each input. To ensure that the conditioning analysis is successful, correct priorities of the correlated inputs must be determined somehow before conditioning. One reasonable approach is to put the correlated inputs in the order of their magnitude based upon the integral of the original coherence functions over a specified frequency range of interest. The reader is referred to Liagre and Niedzwecki (2003) for additional details.

3.2 Mini-TLP Model Test Particulars

The mini-TLP was designed as an unmanned deepwater compliant platform for use off of West Africa (Teigen and Niedzwecki 1998). In order to accommodate testing in a wider range of sea states the hull deck elevation was increased by 5 m by increasing the vertical column heights.. Thus, the dynamic behavior is somewhat different than the original design and the sea states reported are somewhat different. The mini-TLP hull consists of four vertical columns connected by four submerged rectangular pontoons. The experiments were conducted using the 1:40 scale model. All the results presented herein are reported at prototype scale. Both the rigidly constrained and compliant model tests were run with head and quartering sea platform orientations and the identical ocean wave simulations. For the rigidly constrained model tests, the mini-TLP hull was not fitted with either tethers or risers. The six degree-of-freedom load cell was bolted to the model deck and to the access bridge. The load cell was carefully selected to provide adequate stiffness so that the vibrations were minimized at the testing frequency. The access bridge, which also can be used as a towing carriage, was lifted off its wheels and supported on blocks, and stiffened by installing adjustable bracing connected between the bridge structure and the model basin walls. In an earlier set of tests, involving a truncated cylinder whose displaced mass was almost equivalent to that of the mini-TLP model, the dynamic amplification due to the load cell flexibility and system damping were evaluated. It was concluded that the inertial forces were negligible and that all the values in the cross-talk matrix of this instrument are less than five percent. Based upon evaluations from other earlier tests with the compliant model, the mini-TLP was installed using an equivalent four-tethers/four riser system instead of the eight-tethers/twelve risers in the original design. The three restoring force components due to the tethers and risers at the connection points on the platform are computed using the top tensions and mini-TLP motions measured during the compliant model tests. The total restoring forces and moments are computed at the center of gravity of the platform and combined with the wave loads measured during the rigidly restrained hull tests to produce the components of the general excitation vector, $f(t)$.

The test program included a static offset test to demonstrate the stiffness behavior of the mooring system designed for the experiments. The results of this test indicated that the as-built system had been accurately scaled and provided restoring forces that were in accordance with the prototype specifications. Free-decay tests to determine natural periods and damping factors were also performed. The mini-TLP was tested in specific design storm environments, as well as several unanticipated storm environments for academic purposes. In the present study, only the data recorded during the simulations of the West Africa (JONSWAP spectrum with $H_s = 4\,\text{m}, T_p = 16\,\text{s}$ and $\gamma = 2$) and Gulf of Mexico (JONSWAP spectrum with $H_s = 13.1\,\text{m}, T_p = 14\,\text{s}$ and $\gamma = 2$) storms with the platform in head seas configuration will be used. The tests were recorded for 1707 seconds (three hours prototype scale) at a sampling frequency of 40 Hz (6.32 Hz prototype scale) and filtered with a hardware low pass filter at 10 Hz (1.58 Hz model scale). This resulted in records containing 68280 data points in each time series.

3.3 Non-linear Frequency Dependent Behavior

The introduction of partial and cumulative coherence functions to guide the selection and rank the importance of various non-linear terms is an important aspect of this methodology. An example is presented in Figure 6, where the suggested terms in the equations of motion have been rank ordered and their contribution over the selected frequency range illustrated.

The behavior of the hydrodynamic added-mass and damping coefficients as a function of frequency was simulated for the mini-TLP using an industry standard radiation-diffraction software package (WAMIT 2002). The resulting numerical predictions were used in evaluating the accuracy of some of the key problem parameters. Examples of the frequency dependence of the system added mass and damping are presented in Figures 7 and 8. In general the results demonstrate that the methodology as modified in this study is quite robust and yields predictions that are most accurate for the parameters associated with the largest motions of the platform. Practical issues regarding the application of this system identification approach including the utilization of both force and moment measurements and observed strengths and weaknesses in dealing with data regardless of its source must be carefully considered in applying this methodology.

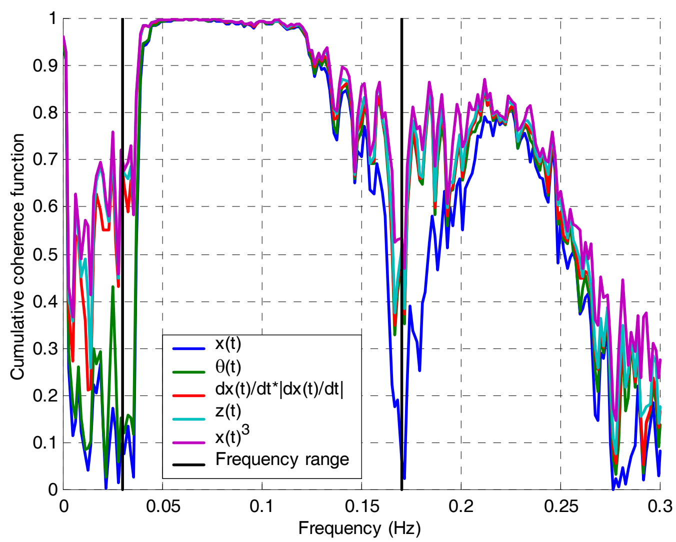


Figure 6. Cumulative coherence functions for the R-MI/SO model using the inline force, $f_x(t)$, as output.

The identified coupled surge/pitch added-mass, a_{15}, is presented in Figure 7. The R-MI/SO model results and numerical predictions have surprisingly comparable trends throughout the whole frequency range, and within the frequency range of interest, the two evaluations have the same order of magnitude.

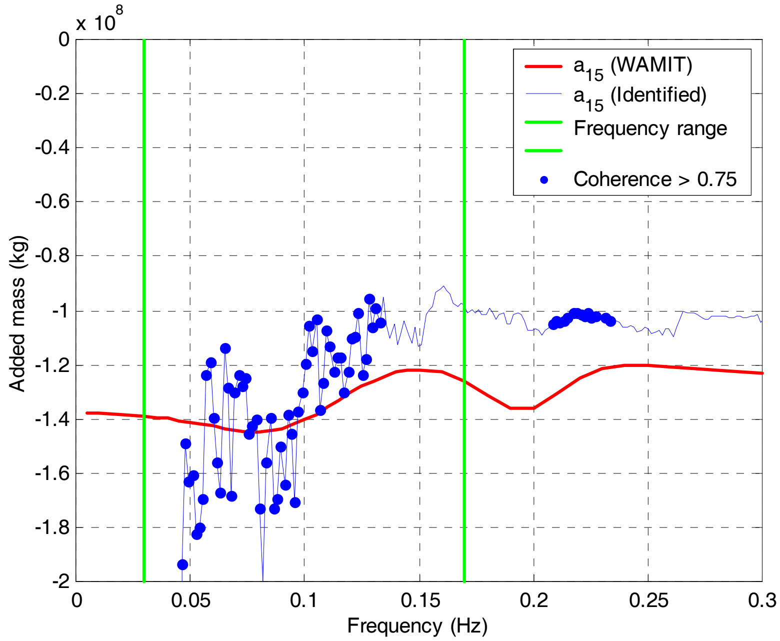


Figure 7. Comparison of the frequency-dependent coupled surge/pitch added-mass, a_{15}.

The identified surge linear damping, b_{11}, and the numerical predictions are presented in Figure 8. In this case the agreement between the two procedures is excellent in the $[0.04\,\mathrm{Hz}, 0.11\,\mathrm{Hz}]$ frequency range where the largest amount of energy is dissipated by the system through its surge motion. The surge linear damping coefficient computed using the damping ratio was equal to 9.75×10^{6} kg/s, as reported by Liagre and Niedzwecki (2003). This value is greater than the results obtained using system identification, but nonetheless of the same order of magnitude.

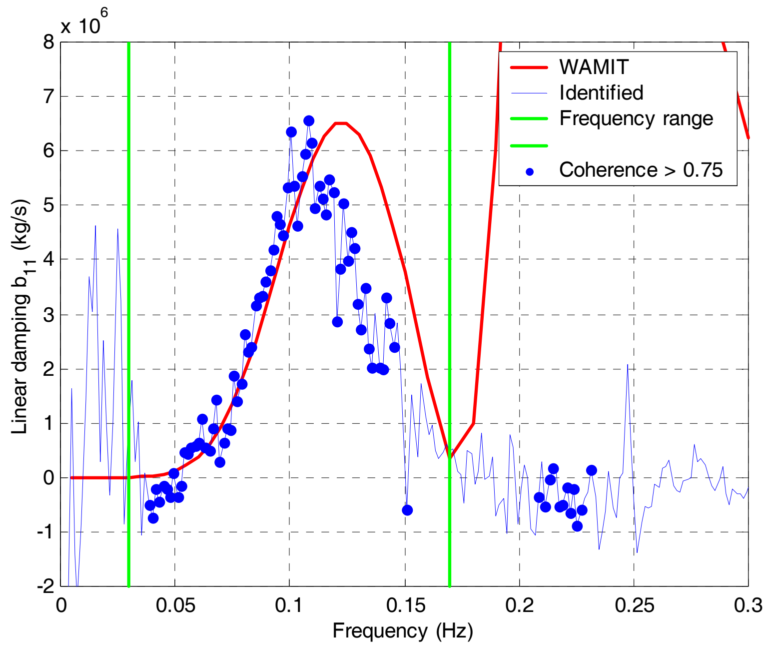


Figure 8. Comparison of the frequency-dependent surge linear damping

The identified linear stiffness, k_{11}, is presented in Figure 9. The linear stiffness identified at frequencies close to 0 Hz agrees very well with the result of the offset test performed in calm water where the linear trend line for the specified stiffness to surge offset had a slope of 4.2×10^4 N/m. In the present case, the identified parameter is not constant but displays a well-defined pattern: the linear stiffness increases in a seemingly linear manner as the frequency increases up to 0.15 Hz. Using a linear fit over that frequency range, the proportionality constant was estimated to be1.85×10^5 Ns/m. The most likely physical explanation of this phenomenon is that wave frequency motions have effects on a shorter length of tethers than slow-drift motions. At low frequencies, the tethers respond linearly from their anchors to their connections on the platform hull, and at higher frequencies the transverse drag force on the tethers keeps them relatively straight over most of the water column, while only the upper portion of the tethers moves. The decreased length of the tethers freely moving contributes to a virtual increase of the linear surge stiffness.

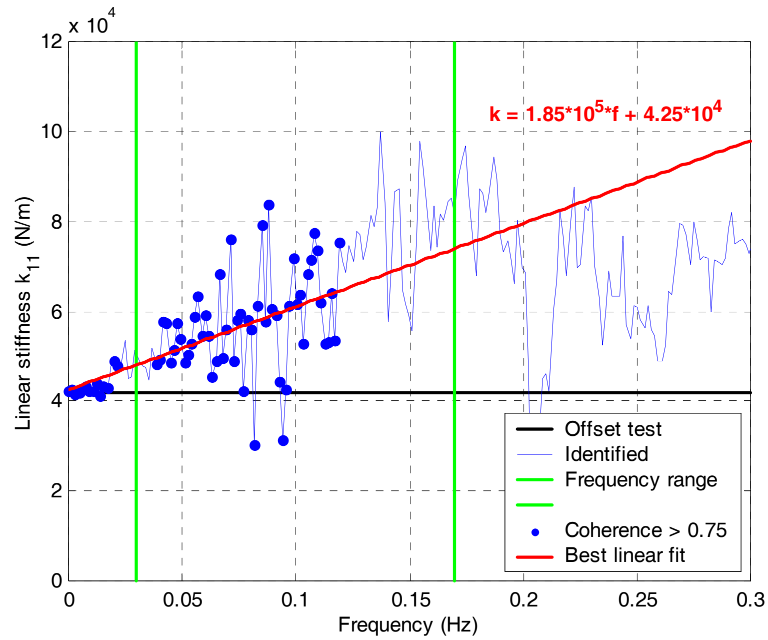


Figure 9. Comparison of the frequency-dependent surge linear stiffness.

In order to identify the pitch moment of inertia, I_{yy}, and pitch added-mass, a_{55}, the R-MI/SO model of which the output is the pitch moment excitation, $m_y(t)$, is considered. The identified moment of inertia I_{yy} and pitch added-mass, a_{55}, are presented in Figure 10. The first parameter is identified using the R-MI/SO model relative to $m_y(t)$, the pitch angular moment. The value of the identified parameter averaged over the frequency range is roughly equal to the pitch moment of inertia $I_{yy} = 3.035\times10^9\ \mathrm{kg}\times\mathrm{m}^2$, which can easily be obtained from the known pitch radius of gyration and structural mass. The pitch added-mass, a_{55}, is not recovered by this methodology. This is probably a confirmation that the platform is designed for small pitch rotations. For a severe wave environment produced in a 100-year Gulf of Mexico storm the mini-TLP exhibited pitch rotations between -2.8° and 2.9°, with a standard deviation of 0.4°. Another possible reason for the inability to correctly identify the sum of both parameters could be that the standard deviation of the measured pitch was not much greater than the accuracy of the pitch measurement. This is not totally unexpected as the platform was designed so that the pitching motion would be very small.

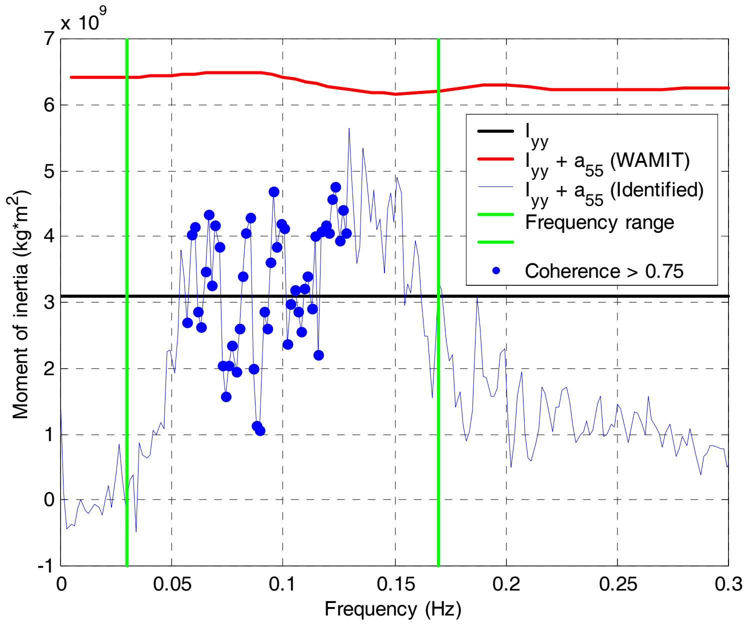


Figure 10. Comparison of the pitch moment of inertia, I_{yy}, and the frequency-dependent pitch added-mass, a_{55}.

The identified nonlinear damping, d_{11}, is presented in Figure 11. The polynomial fit of the values of high coherence indicates a clear frequency dependence of the nonlinear damping. This result at first seems to contradict the hypothesis made in a recent research study by Holappa and Falzarano (1999), which still constituted a significant advancement by proving the importance of the frequency dependence of the added-mass and damping coefficients in the simulation of nonlinear ship rolling. Indeed, in order to compare their extended state space model with full-scale experimental data, an additional nonlinear damping term had to be added to the wave damping. Since nonlinear terms cannot be included in extended state space approximations due to linearity requirement of the model, the nonlinear term was merely assumed to be constant over the frequency range. In this study, the application of the R-MI/SO method makes no restrictive assumption about the parameters and the frequency dependence of the nonlinear damping is illustrated.

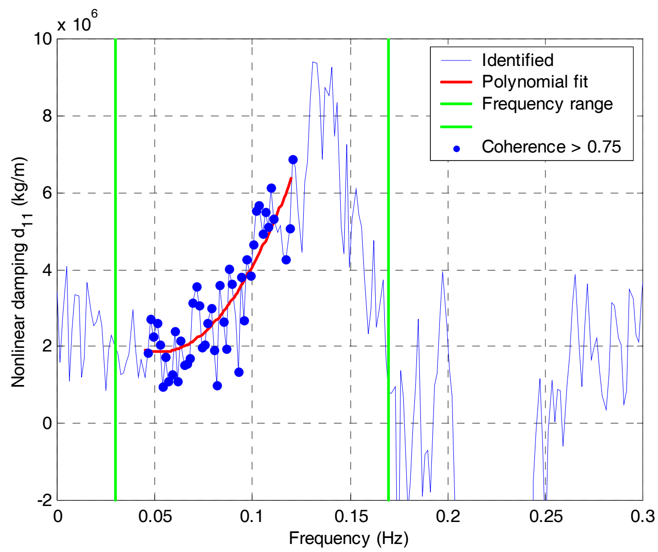


Figure 11. Frequency-dependent surge nonlinear damping, d_{11}.

4. Closing Remarks

The study, in addressing the marine riser problem, provides a sense of the difficulty in analyzing practical distributed-parameter systems. There are several interesting implications for laboratory and field measurement programs of risers. In particular, using the system identification on the measured data regardless of the source will require one to choose a finite number of the most important parameters to be investigated. Assumptions regarding the wave field, the wave force excitation model and its coefficients, and the riser system are required to resolve the parameters as was demonstrated in this study. At about mid-water depth, when the excitation is provided only by surface wave phenomena, the nature of the distribution of the response may be characterized prior to numerical analysis, and this interpretation of the data may provide an indication of the nature of the non-linearity that dominates the response behavior. The determination of the magnitude of the nonlinear coefficients requires use of the procedures presented herein.

The basic concept and objective of this system identification approach to marine risers is presented in Figure 12. To the left of that figure, there are three boxes that basically define the problem, the geometric riser specifications, the hydrodynamic force transfer coefficients and the actual measurements. There are differing levels of modeling that can be associated with the system identification method. This is important because not all models need be exceedingly complex. Once a determination is made, the governing equations are selected in a consistent manner and the system identification method can proceed with the objective to obtain the values and behavior of the design parameters of interest.

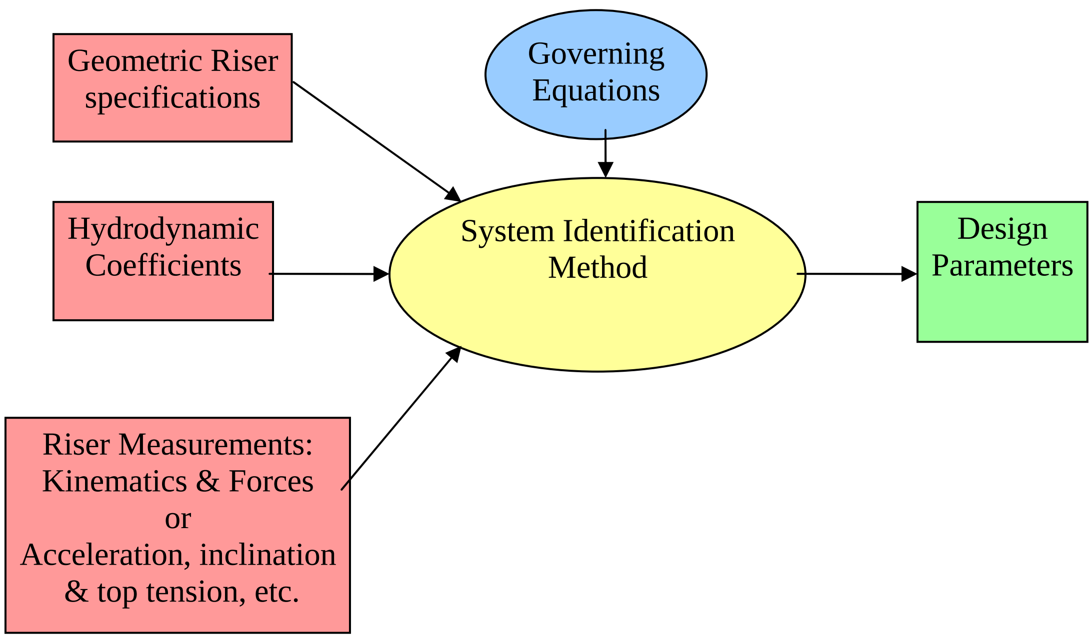


Figure 12. The basic concept and objective of using system identification methods.

In this research study, the R-MI/SO technique was used in combination with the dynamic response model for a compliant offshore structure with the objective to identify selected system parameters from measured excitation and response data. The environmental excitation was limited to only waves with the expectation that the system would behave as a weakly nonlinear system. The form of the mathematical model is initially built upon the known mechanical properties of the system. Next, by means of the computation of the original coherence functions between each potential input and the output, the most relevant inputs are selected and retained for the identification of the related parameters. The comparison of the inputs' original coherence functions within a specified frequency range is used for the sorting of the inputs before applying the conditioning procedure leading to the transfer functions, from which the system frequency dependent characteristics and model parameters are retrieved. The completeness of the model is achieved by monitoring the increase of the cumulative coherence functions and the decrease of the extraneous noise spectrum over the frequency range of interest when additional terms are included. In the case of the mini-TLP, the dynamic response model takes into account the most important mode couplings as well as Duffing type stiffness and quadratic damping nonlinearities.

The results obtained confirm the correctness of the model and demonstrate a strong consistency for the model related to surge motions, which is the predominant mode of motion of tension leg platforms in uni-directional sea states. The identified value of all the parameters included herein is satisfactory for the specific range of parameters considered and the conditions in which the input and output signals to the R-MI/SO model were recorded. The value of parameters such as the heave linear stiffness recovered via spectral averaging of the respective transfer function compares favorably to the known constant value. The identification of frequency dependent parameters, such as the surge and surge/pitch added-mass or the surge linear damping coefficient, proved to be quite accurate and in good agreement around the excitation peak frequency with the numerical predictions obtained using WAMIT. The identified value of the heave linear stiffness proved to be correct for the static case and interestingly exhibited a linear trend increasing as a function of frequency. This would not have been picked up without the implementation of a process to ensure the partition of the parameters. The recovery of the nonlinear stiffness coefficient showed that no significant error is introduced by including additional linear or nonlinear terms which contribution is negligible since the identified value of the term is approximately zero. Conversely, if one does not include sufficient terms the cumulative coherence functions will provide some guidance in choosing correctly the additional terms. It was reasoned for this study that the cause why some parameters were poorly recovered is that the particular design of the mini-TLP minimizes the motions related to these parameters, therefore making the identification process somewhat difficult. Apparently, this finding is consistent with system identification techniques in general and it is only a minor shortcoming as only the parameters associated with the largest motions are usually of real interest. In the case of the mini-TLP, the pitching motions were by design quite small as verified by the experiments. Although the pitching motions are quite small they nevertheless are quite important and contribute along with the other rigid body modes to the dynamic tendon tensions. Thus, this practical consideration suggests the inclusion of the tendon dynamics in the original formulation or as a coupled subset needs to be incorporated into the analysis procedures.

Of course, great caution must be exercised in generalizing these results beyond what has been done. Parameter evaluation is likely to be strongly influenced by the spectral characteristics of the loads used for excitation of the system. Perhaps the results could be improved if a band-limited white noise was used to persistently excite the system in the laboratory. However, using band-limited white noise excitation for the purpose of system identification is a debatable issue when dealing with real engineering applications, in view of the fact that only data due to environmental excitation can be collected in the field. Furthermore, improvements would also be significant if the kinematics and loads were measured during the same tests. Bearing this in mind and the fact that measured experimental data were used, for many of the variables a reasonable level of agreement was obtained for the identified variables over the frequency range of interest. Based upon the results obtained in this study it is apparent that the nearly perfect agreement for numerically simulated data as shown in other research investigations, for example see Spanos and Lu (1995), should not be expected when analyzing measured data.

Broadly speaking, this research study shows that no parameter can be assumed constant unless its identified value follows a horizontal line over a certain frequency range. The R-MI/SO technique is a very powerful tool for identification of frequency dependent parameters in complex offshore engineering systems. As long as the system studied can be reasonably modeled by means of a nonlinear differential equation, the R-

MI/SO technique is preferable but can be used in conjunction with Volterra methods that require large amounts of data, which are not generally available in most measurement programs (Sibetheros and Niedzwecki 2003, 2004). Although the application of this procedure to ships and other offshore platforms is straightforward, one can expect to observe unexpected results and this study provides some insight as to how one might approach the solutions.

5. Publications and Presentations Based upon this Research

Journals

Niedzwecki, J.M. and Liagre, P-Y. (2002). “System Identification of Distributed-Parameter Marine Riser Models,” *Journal of Ocean Engineering*, Vol. 30, 1387-1415.

Liagre, P-Y. and Niedzwecki, J.M. (2003). "Estimating Nonlinear Coupled Frequency Dependent Parameters in Offshore Engineering,” *Applied Ocean Research,* Vol. 25, **1**, 1-19.

Sibetheros, I. A. and Niedzwecki, J.M. (2004). “Analysis of Single and Tandem Cylinder Data Using an Orthogonal Volterra Model Approach,” *J. Ocean Engineering* in press.

Conferences

Niedzwecki, J.M. and Liagre P-Y. (2003). “Interpreting Data on Marine Riser Response Behavior Using System Identification, *OTRC Deepwater Mooring Symposium*, October 2-3, 2003, Houston, TX.

Sibetheros, I. A. and Niedzwecki, J.M. (2003). “System Analysis of the Interactive Behavior of Flexible Cylinders under Unidirectional Wave Loading,” *ISOPE 2003*, Vol. 3, May 25-30, 2003, Honolulu, Hawaii.

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