COMPUTER ANALYSIS OF THE CONSOLIDATION
OF OCEAN BOTTOM SEDIMENT

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by

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ABSTRACT

Consolidation equation and energy equations were derived in [13]. The first effort is to provide reliable, efficient codes for the solution of these two equations. Inspection of these equations shows that the consolidation equation can be solved independent of the energy equation. The solution of the energy equation requires the solution of the consolidation equation. This system is solved using a divided difference scheme for the spatial derivatives and a backward difference scheme for the time derivative. The resulting set of tridiagonal equations is easily solved.

The system adequately models laboratory tests. The energy equation can be ignored and only the consolidation equation need be solved with boundary conditions related to the applied pressure.

The model gives no consolidation for the boundary condition of field data. This has led to derivation of a more complete model and development of a different scheme for their solution.
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<td>7.</td>
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</table>
1. INTRODUCTION

1.1 Objective

We would like to solve the equations modelling the phenomena of consolidation of a soil such as the sediment at the bottom of the ocean. Further, we would like to allow additional deposition with time.

The equations are coupled sets of nonlinear partial differential equations in one spatial coordinate (depth) and time. We do not expect the availability of analytical solutions except in limited cases.

1.2 Statement to the Problem

The following consolidation and energy equations for marine sediments are taken from [13]:

\begin{equation}
Q(n) \frac{\partial^2 n}{\partial z^2} + R(n) \left( \frac{\partial n}{\partial z} \right)^2 + S(n) \frac{\partial n}{\partial z} = \frac{\partial n}{\partial t}
\end{equation}

Energy equation:

\begin{equation}
\frac{\partial \theta}{\partial t} = A_1(n, \theta) \frac{\partial^2 \theta}{\partial z^2} + A_2(n, \theta)
\end{equation}

where \( Q(n), R(n), S(n) \) are functions of the porosity, variable with depth and time; \( A_1(n, \theta) \) and \( A_2(n, \theta) \) are functions of porosity, \( n \), and temperature, \( \theta \).

We have two cases for solving equations (1.2.1) and (1.2.2):
the laboratory case and the field case. These cases have the following boundary conditions:

For the laboratory case:

\[(1.2.3) \ n = f(\text{pressure}) \text{ at } z = 0 \text{ and } z = L.\]

For the field case:

\[(1.2.4) \ n(z = 0) = 0.85 \text{ for all } t\]

\[(1.2.5) \ \frac{\partial n(z = L)}{\partial z} = 0 \text{ for all } t.\]

Note: The limits of the variable \(z\) are zero and \(L\). The lower limit is the bottom of the sample in the laboratory case and the basement in the field case. The upper limit is the thickness of the laboratory sample or the height of the sediment, the interface between the sea and the formation.
2. METHODS OF SOLUTION

2.1 INTRODUCTION

The methods of solving the equations introduced in the previous chapter include analytical techniques, numerical methods, and limiting case analysis.

2.2 ANALYTICAL TECHNIQUES

The detailed form of the equations presented in (13) immediately shows that there is a small probability of solving the equations in closed form. This leaves the value of analytical techniques to be a general understanding of the expected behavior of the solution. This will come from understanding the solution of similar linear partial differential equations which are more easily solved.

2.3 NUMERICAL METHODS

It is generally expected that the solution of problems of this difficulty will be solved or "simulated" through the use of numerical analysis. The usual procedure is to replace the spatral derivatives with "central" finite difference approximations and time derivatives with "forward" or "backward" differences to yield explicit and implicit formulae, respectively. Chapter 3 of this report is the details of a finite difference approach to the solution of this model.

2.4 LIMITING CASE ANALYSIS

A useful part of theoretical analysis of similar models is to consider the solution as time becomes arbitrarily large. In this limiting case, all derivatives with respect to time become zero as the model reaches "steady state". This simplifies the model because the partial differential equations become ordinary differential equations.
The use of the one dimensional maximum principle (see Protter & Wernberger (9)) can be used to show that the model with the field boundary conditions admits only the "unchanged" limiting case solution. This leads to the conclusion that the model is inadequate for the field case. This does not affect its adequacy for the laboratory case. The following analysis applies to the field case:

A function \( u(x) \) that is continuous on the closed interval \((a,b)\) takes on its maximum at a point on this interval. If \( u(x) \) has a continuous second derivative, and if \( u \) has a relative maximum at some point \( c \) between \( a \) and \( b \), then we know from elementary calculus that

\[
(2.4.1) \quad u'(c) = 0 \text{ and } u'(c) \leq 0
\]

Suppose that in an open interval \((a,b)\), \( u \) is known to satisfy a differential inequality of the form

\[
(2.4.2) \quad L(u) = u'' + g(x) u' > 0.
\]

where \( g(x) \) is any bounded function. Then it is clear that relations (2.4.1.) cannot be satisfied at any point \( c \) in \((a,b)\). Consequently, wherever (2.4.2.) holds, the maximum of \( u \) in the interval cannot be attained anywhere except at the endpoints \( a \) or \( b \). We have here the simplest case of a maximum principle.

An essential feature of the above argument is the requirement that the inequality (2.4.2.) be strict; that is, we assume that \( u'' + g(x) u' \) is never zero. In the study of differential equations and in many applications, such a requirement is overly restrictive, and it is important that we remove it if possible. We note, however, that for the nonstrict inequality

\[
u'' + g(x) u' \geq 0,
\]

the solution \( u = \text{constant} \) is admitted. For such a constant solution
the maximum is attained at every point. Protter [8] has proved that this exception is the only one possible.

We can repeatedly apply the maximum principle to prove that the Degenerated Consolidation Equation has a constant solution only.

2.5 Uniqueness of the Solutions of the Boundary Value Problem

Keller [6] gives complete discussions of the applicability of several numerical methods for the solution of two point boundary value problems. The equation [4.3.1.] and [4.3.2] and their associated boundary conditions give us confidence that the procedures outlined as in (9) give unique solutions of the boundary value problem.

2.6 Parameter Estimation in Quasilinear Parabolic Equations - Oil Reservoir Applications

The method utilizes Newton-Raphson method, finite differences and least squares fitting criteria to estimate the parameters appearing in systems of Quasilinear Parabolic Partial Differential Equations is described in [12]. Numerical results [12] indicate that the Newton-Raphson method yields accurate estimates for the parameters within a reasonable number of iterations. It is also found that the Newton-Raphson method is an efficient method to estimate parameters in partial differential equations.

The methods developed by StuckenBruck [12] and Childs, et.al. [3] can be used with those under development on this project to determine the "best fit" soil parameters to make the model fit the laboratory data.

3. A STATIC CONSOLIDATION EQUATION AND ENERGY EQUATION

3.1 A finite Difference Representation of the Model

The consolidation equation (1.2.1) defines the relationship of the porosity function with respect to depth and time.
This equation can be solved by a Picard iteration. This is set up by using the explicit derivatives as unknowns and the coefficients on the left hand side as known functions of the current estimate of the solution. The solution for each time step converges in two or three interactions in most cases.

3.2 Some Sample Output of the Solution of the Consolidation Equation and the Energy Equation for the Laboratory Case

As in Figure 1, we take \( N = 100 \) and divide the sample height into 99 equal parts, each part is \( \Delta z \). Porosity \( (i) \) denotes the porosity at point \( i \). The input is shown in Table 1. Then the output statistics are shown in Table 2.

By using Table 2 we get the Figure 3, which shows the process of consolidation. At time \( T = 0 \) minute, we have initial porosity 0.337 at point 1, 16, 33, 50, 66, 83 and 100; at \( T = 20 \) minutes, we have a slight consolidation; at \( T = 200 \) minutes, we have better consolidation; and at \( T = 500 \) minutes we have a reasonable approximately perfect consolidation.

By using Table 4 we get the Figure 4. It shows clearly from Figure 4 that time starts from \( T = 0 \) minute, the porosity at each point stays at initial porosity, i.e., 0.5584. As time goes on, the consolidation process is going on, and at \( T = 330 \) minutes we get a reasonable perfect consolidation.

For illite under 1520 psi, temperature at 20 degrees centigrade, we have the experiment results for the time versus sample height during the process of consolidation as shown in Table 5.
A finite difference scheme is used to solve the consolidation equation. Figure 1 shows the nodal points at which the porosities are chosen as discrete unknowns and are to be evaluated. In the Lagrangean formulation the finite differences vary with depth and time and are dependent on the local instantaneous value of the porosity.

For any point i, Equation (1.2.1) may be written as follows:

\[ Q_i(n) \left[ \frac{\partial^2 n}{\partial z^2} \right]_i + R_i(n) \left[ \frac{\partial n}{\partial z} \right]_i^2 + S_i(n) \left[ \frac{\partial n}{\partial z} \right]_i = \left[ \frac{\partial n}{\partial t} \right]_i \]

where all the coefficients and derivatives refer to point i under consideration.

Note that for the field case: the upper boundary corresponds to the bottom of the ocean. If the lower boundary is fixed, it corresponds to the basement.

Let us first express the derivatives of porosity function in terms of the dependent variable at the nodal points. For convenience we denote

\[ a = (\Delta z)_i \] and \[ b = (\Delta z)_{i+1} \]

in Figure 2.
Table 1

Input data for illite run at 20°C

<table>
<thead>
<tr>
<th>Sample</th>
<th>Sample Height HCM</th>
<th>Consolidation Constants</th>
<th>Permeability Constants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Illite</td>
<td>1.962139 cm</td>
<td>A (KG/cm²)</td>
<td>B (cm/sec)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.66232502</td>
<td>- 6.0804348</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.46E-06</td>
<td>8.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Initial Porosity 0.337</td>
<td>Initial temperature 20 degrees centigrade</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Consolidation pressure 106.4 kg/cm² (1520 psi)</td>
<td>Increment time step 20 sec</td>
</tr>
</tbody>
</table>
Table 2

Output at 1/3 points for illite run at 20°C

<table>
<thead>
<tr>
<th>Time (Min)</th>
<th>Porosity (0)</th>
<th>Porosity (33)</th>
<th>Porosity (67)</th>
<th>Porosity (100)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = 0</td>
<td>.3370</td>
<td>.3370</td>
<td>.3370</td>
<td>.3370</td>
</tr>
<tr>
<td>t = 20</td>
<td>.2970</td>
<td>.3138</td>
<td>.3294</td>
<td>.3345</td>
</tr>
<tr>
<td>t = 200</td>
<td>.2970</td>
<td>.2990</td>
<td>.3008</td>
<td>.3015</td>
</tr>
<tr>
<td>t = 500</td>
<td>.2970</td>
<td>.2971</td>
<td>.2972</td>
<td>.2473</td>
</tr>
</tbody>
</table>
It is equivalent to the system of first-order equations

\[
\begin{pmatrix}
-x^2 p \frac{\partial u}{\partial t} - u \frac{\partial x}{\partial p} \\
-x^2 p \frac{\partial x}{\partial p} - x \frac{\partial x}{\partial p}
\end{pmatrix}
\frac{\partial x}{\partial p} = \frac{\partial u}{\partial y}
\]

4.2 Theorems Related to the Existence and Uniqueness Solution of the

The differential equation of order n

\text{Equation and Energy Equation}

Approximating the partial differential equations - Consolidation

4.1 Theorems Related to the Existence and Uniqueness Solution of the

If bounded above at \( x = b \), then \( u'(b) < 0 \).

If the maximum occurs at \( x = b \) and \( g \) is bounded in every closed sub-interval of \( a \) and \( b \), and suppose \( g \) is bounded below at \( a \) and \( b \), and has one-sided derivatives of the inequality

\[ u' + g(x) < 0 \text{ in } (a, b) \]

and \( u \) satisfies the differential inequality.

Theorem 4.1.2 Suppose \( u \) is a nonconstant function which satisfies

Theorem 4.1.1 (One-dimension[al maximum principle]). Suppose

There are two theorems [9] related to degenerated consolidation

\text{Equation and the Energy Equation}

4.1 Theorems Related to the Solutions to Degenerated Consolidation

4.1 Theorems Related to the Solutions to Degenerated Consolidation

10
\[ \frac{dy}{dx} = y_1 \]
\[ \frac{dy_1}{dx} = y_2 \]

\[ \ldots \ldots \]
\[ \frac{dy_{n-2}}{dx} = y_{n-1} \]
\[ \frac{dy_{n-1}}{dx} = f(x, y, y_1, \ldots, y_{n-1}) \]

Equivalence means that any solution of the system defines a solution of the equation, and conversely. In fact any system of differential equation of order greater than 1 may be reduced to a first-order system by first solving explicitly for the highest-order derivative. This process may lead to certain difficulties, e.g., extracting roots \[10\].

If, in the equations of a first-order system, the independent variable \( t \) appears explicitly on the right side, then we write

\[ x_i = f_i(x_1, \ldots, x_n, t) \quad i = 1, \ldots, n \]

which is called a nonautonomous system. Without the explicit appearance of \( t \) we have

\[ \dot{x}_i = f_i(x_1, \ldots, x_n) \quad i = 1, \ldots, n \]

which is known as an autonomous system. Note that a nonautonomous
system can be reduced to a special autonomous system by replacing $t$ on
the right by the dependent variable $x_{n+1}$ and adjoining to the resulting
system the equation

$$\frac{dx_{n+1}}{dt} = 1.$$ 

**Theorem 4.2.1** Let the function $f_i$ be continuous in a domain $D$
(a rectangle) defined by

$$|t - t_0| < a \quad |x_i - x_i^0| < a_i \quad i = 1, \ldots, n$$

and let the following Lipschitz condition be satisfied for any two
points $\bar{x}$ and $\bar{x}$ (with the same value of $t$) in $D$:

$$|f(t, \bar{x}) - f(t, \bar{x})| < \sum_{j=1}^{n} A_j |\bar{x}_j - \bar{x}_j|$$

Then in a suitable interval $|t - t_0| < \bar{a} < a$, the system has a unique
solution $x_i(t)$ which satisfies $x_i(t_0) = x_i^0$, $i = 1, \ldots, n$ [10].

4.3 The Existence and Uniqueness Solution of the Approximating the
Partial Differential Equations - Consolidation Equation and
Energy Equation

The complete proof of existence and uniqueness of the coupled
partial differential equations (i.e., the consolidation equation and
the energy equation) is quite difficult at best.

We will refer to [9] for related results. For the consolidation
Fig. 1. Finite difference scheme
ILLITE RUN AT 20 DEGREES CENTIGRADE AND D = 8.4

Fig. 3. Porosity versus time at different points
Table 3

Input data for bentonite run at 90°C

<table>
<thead>
<tr>
<th>Sample</th>
<th>Sample Height HCM</th>
<th>Consolidation</th>
<th>Permeability</th>
</tr>
</thead>
<tbody>
<tr>
<td>bentonite</td>
<td>2.149099 cm</td>
<td>Consolidation Constants</td>
<td>Permeability Constants</td>
</tr>
<tr>
<td></td>
<td></td>
<td>A (kg/cm²)</td>
<td>B</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.4922999</td>
<td>- 5.5637999</td>
</tr>
<tr>
<td></td>
<td>Initial porosity P INIT = .5584</td>
<td>Initial temperature TMI = 90 degrees centigrade</td>
<td>Consolidation pressure P0 = 25.2 kg/cm² (360 psi)</td>
</tr>
</tbody>
</table>
### Table 4

Output at 1/3 points for bentonite run at 90°C

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Porosity (0.0)</th>
<th>Porosity (1/3)</th>
<th>Porosity (2/3)</th>
<th>Porosity (1.0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t = 0</td>
<td>.5584</td>
<td>.5584</td>
<td>.5584</td>
<td>.5584</td>
</tr>
<tr>
<td>t = 20</td>
<td>.4930</td>
<td>.5300</td>
<td>.5539</td>
<td>.5579</td>
</tr>
<tr>
<td>t = 60</td>
<td>.4930</td>
<td>.5144</td>
<td>.5361</td>
<td>.5450</td>
</tr>
<tr>
<td>t = 100</td>
<td>.4930</td>
<td>.5087</td>
<td>.5247</td>
<td>.5315</td>
</tr>
<tr>
<td>t = 160</td>
<td>.4930</td>
<td>.5037</td>
<td>.5141</td>
<td>.5184</td>
</tr>
<tr>
<td>t = 200</td>
<td>.4930</td>
<td>.5014</td>
<td>.5094</td>
<td>.5127</td>
</tr>
<tr>
<td>t = 250</td>
<td>.4930</td>
<td>.4993</td>
<td>.5052</td>
<td>.5076</td>
</tr>
<tr>
<td>t = 330</td>
<td>.4930</td>
<td>.4971</td>
<td>.5007</td>
<td>.5022</td>
</tr>
</tbody>
</table>
Fig. 4. Porosity versus time at different points.
<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Sample height (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.7725</td>
</tr>
<tr>
<td>0.2</td>
<td>0.7710</td>
</tr>
<tr>
<td>0.4</td>
<td>0.7690</td>
</tr>
<tr>
<td>0.7</td>
<td>0.7680</td>
</tr>
<tr>
<td>1</td>
<td>0.7660</td>
</tr>
<tr>
<td>2</td>
<td>0.7640</td>
</tr>
<tr>
<td>4</td>
<td>0.7600</td>
</tr>
<tr>
<td>7</td>
<td>0.7560</td>
</tr>
<tr>
<td>10</td>
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</tr>
<tr>
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</tr>
<tr>
<td>Time (minutes)</td>
<td>Sample height (inches)</td>
</tr>
<tr>
<td>---------------</td>
<td>-----------------------</td>
</tr>
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<td>0.7651</td>
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<td>0.7620</td>
</tr>
<tr>
<td>2</td>
<td>0.7576</td>
</tr>
<tr>
<td>3</td>
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</tr>
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<td>4</td>
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<td>0.7289</td>
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<td>0.7288</td>
</tr>
<tr>
<td>90</td>
<td>0.7287</td>
</tr>
<tr>
<td>100</td>
<td>0.7287</td>
</tr>
<tr>
<td>110</td>
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</tr>
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<td>120</td>
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</tr>
<tr>
<td>130</td>
<td>0.7286</td>
</tr>
</tbody>
</table>
Table 7

Output for illite run under 1520 psi and DT = 10

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>Sample height (inches)</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>0.167</td>
<td>0.7688</td>
</tr>
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<td>0.5</td>
<td>0.7656</td>
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<td>0.7624</td>
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<tr>
<td>2</td>
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</tr>
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<td>3</td>
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<td>4</td>
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<td>0.7418</td>
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<tr>
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<td>30</td>
<td>0.7319</td>
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<tr>
<td>40</td>
<td>0.7304</td>
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<td>0.7295</td>
</tr>
<tr>
<td>60</td>
<td>0.7291</td>
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<tr>
<td>70</td>
<td>0.7288</td>
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<tr>
<td>80</td>
<td>0.7287</td>
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<tr>
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<td>0.7286</td>
</tr>
<tr>
<td>100</td>
<td>0.7286</td>
</tr>
<tr>
<td>110</td>
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<tr>
<td>140</td>
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</tbody>
</table>
Fig. 5. Time versus sample height
Table 8

Output for illite run under 1520 psi and DT = 50

<table>
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<th>Time (minutes)</th>
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</tr>
</thead>
<tbody>
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</tr>
<tr>
<td>5</td>
<td>0.7503</td>
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<td>0.7425</td>
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<td>15</td>
<td>0.7381</td>
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<td>0.7353</td>
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<td>25</td>
<td>0.7334</td>
</tr>
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<td>0.7321</td>
</tr>
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<td>35</td>
<td>0.7312</td>
</tr>
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<td>40</td>
<td>0.7305</td>
</tr>
<tr>
<td>45</td>
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</tr>
<tr>
<td>50</td>
<td>0.7296</td>
</tr>
<tr>
<td>55</td>
<td>0.7293</td>
</tr>
<tr>
<td>60</td>
<td>0.7291</td>
</tr>
<tr>
<td>65</td>
<td>0.7290</td>
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<tr>
<td>70</td>
<td>0.7288</td>
</tr>
<tr>
<td>80</td>
<td>0.7287</td>
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<tr>
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<td>0.7286</td>
</tr>
<tr>
<td>95</td>
<td>0.7286</td>
</tr>
<tr>
<td>100</td>
<td>0.7285</td>
</tr>
<tr>
<td>105</td>
<td>0.7285</td>
</tr>
<tr>
<td>110</td>
<td>0.7285</td>
</tr>
<tr>
<td>115</td>
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<tr>
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<td>0.7285</td>
</tr>
</tbody>
</table>
Table 9

Output for illite run under 1520 psi and DT = 100

<table>
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<th>Time (minutes)</th>
<th>Sample height (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.7725</td>
</tr>
<tr>
<td>5</td>
<td>0.7511</td>
</tr>
<tr>
<td>10</td>
<td>0.7432</td>
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<td>15</td>
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<td>0.7295</td>
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<tr>
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<tr>
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<tr>
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<tr>
<td>105</td>
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<tr>
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<td>0.7286</td>
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<tr>
<td>115</td>
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</tr>
<tr>
<td>120</td>
<td>0.7286</td>
</tr>
<tr>
<td>Time (minutes)</td>
<td>Sample height (inches)</td>
</tr>
<tr>
<td>---------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>0.1</td>
<td>0.7725</td>
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<tr>
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<td>0.7443</td>
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<tr>
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<tr>
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<tr>
<td>110</td>
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</tr>
<tr>
<td>120</td>
<td>0.7286</td>
</tr>
</tbody>
</table>
equation, we first replace the time derivative by a backward difference formula:

\[ \frac{\Delta n}{\Delta t} = \frac{n(t) - n(t - \Delta t)}{\Delta t} \]

and approximate the square of the spatial derivative by \((\Delta n/\Delta z)^2 = (\Delta n(t - \Delta t)/\Delta z)(\Delta n(t)/\Delta z)\). This gives a linear ordinary differential equation in terms of \(n(t)\) if we use \((t - \Delta t)\) in the coefficients of the consolidation equation we then have:

(4.3.1) \[ Q(n(t - \Delta t)) \frac{\Delta^2 n(t)}{\Delta z^2} + \left\{ R(n(t - \Delta t)) \frac{\Delta n(t - \Delta t)}{\Delta z} \right\} \frac{\Delta n(t)}{\Delta z} + \]

\[ S(n(t - \Delta t)) \frac{\Delta n(t)}{\Delta z} = \frac{n(t) - n(t - \Delta t)}{\Delta t} \]

We can do likewise with the energy equation giving:

(4.3.2) \[ A_1 (n(t - \Delta t), \theta(t - \Delta t)) \frac{\Delta^2 \theta(t)}{\Delta z^2} + A_2 (n(t - \Delta t), \theta(t - \Delta t)) \]

\[ = \frac{1}{\Delta t} (\theta(t) - \theta(t - \Delta t)) \]

Further, we can write (4.3.1) and (4.3.2) as four first order equations:

\[ \frac{dn}{dz} = m \]

\[ \frac{dm}{dz} = \left( \frac{1}{Q(n(t - \Delta t))} \left( -R(n(t - \Delta t)) \frac{m(t - \Delta t)}{m(t)} \right) \right) \frac{m(t - \Delta t)}{m(t)} - \]

\[ S(n(t - \Delta t)) \frac{m(t)}{m(t)} + (n(t) - n(t - \Delta t))/\Delta t) \]
\[
\frac{d\phi}{dz} = \phi \\
\frac{d\phi}{dz} = \frac{1}{A_1(n(t-\Delta t), \theta(t-\Delta t))} \left(-A_2(n(t-\Delta t), \theta(t-\Delta t)) + \frac{1}{\Delta t} (\theta(t) - \theta(t-\Delta t)) \right)
\]

It is obvious that this set of equations meets the condition necessary for theorem 4.2.1, hence, by theorem 4.2.1, has a unique solution. This unique solution implies existence and uniqueness solution for (4.3.1) and (4.3.2).

4.4 Analytic Solutions

The consolidation and energy equations have analytic solutions reasonably available only after discarding many terms. Such solutions can be obtained through the usual separation of variables techniques if the following assumptions are made:

\[R(n) = 0\]
\[Q(n) = \text{constant}\]
\[S(n) = \text{constant}\]
\[A_1(n, \theta) = \text{constant}\]
\[A_2(n, \theta) = 0\]

This is clearly too restrictive.

4.5 A Limiting Case of the Consolidation Equation

A limiting case of the consolidation equation is obtained letting \(t = \infty\). All derivatives with respect to \(t\) then approach zero.
THE CONSOLIDATION EQUATION AS $t \to \infty$:

The consolidation equation in (1.2.1) is

\[
(1.2.1) \quad - \frac{A(B-1)C}{G_W y_0} n^{B+D-2} \frac{\partial^2 n}{\partial z^2} + CD \frac{G_s}{G_W} n^{D-2} (1-n^{-1}) \frac{\partial n}{\partial z} = \frac{\partial n}{\partial t}
\]

So as $t \to \infty$, the consolidation equation degenerates to:

\[
(4.5.1) \quad - \frac{A(B-1)C}{G_W y_0} n^{B+D-2} \frac{\partial^2 n}{\partial z^2} + CD \frac{G_s}{G_W} n^{D-2} (1-n^{-1}) \frac{\partial n}{\partial z} = 0
\]

This implies

\[
(4.5.2) \quad \frac{d^2n}{dz^2} + \gamma(\beta - n) n^{\alpha} \frac{dn}{dz} = 0
\]

The above equation is subject to the boundary conditions

\[
(4.5.3) \quad n(0) = n_0 \text{ and } n'(L) = 0,
\]

which are equivalent to (1.2.4) and (1.2.5).

**Theorem 4.5.1** If $n(z)$ is a solution to

\[
\frac{d^2n}{dz^2} + g(z) \frac{dn}{dz} = 0, \quad 0 \leq z \leq L,
\]

where $g(z) = \gamma(\beta-n) n^{\alpha}$; $n(0) = n_0$

and $n'(L) = 0$ and $n(z)$ is a bounded function on $[0, L]$. Then $n(z)$ is a constant.

Before prove theorem 4.5.1, we have to list two theorems - theorem 4.5.2 and theorem 4.5.3. By [9], we have these two theorems as follows:
Theorem 4.5.2  If \( n(z) \) has a maximum on \((0, L)\), then \( n(z) = \) constant.

Theorem 4.5.3  If the maximum of \( n(z) \) occurs at \( L \), then \( n'(L) > 0 \).

Proof of Theorem 4.5.1:
Suppose \( n(z) \) is a nonconstant solution, we are going to prove that we will get a contradiction, thus \( n(z) \) must be constant.

If \( n(z) \) is not a constant, then by theorem 4.5.2, \( n(z) \) has no maximum on \((0, L)\).

By theorem 4.5.3 and the given condition \( n'(L) = 0 \) imply that the maximum of \( n(z) \) can not occur at \( z = L \).

Conclude that if \( n(z) \) is not constant, then the maximum of \( n(z) \) occurs at \( z = 0 \).

So \( n(z) \leq n_0 \)
implies \( 0 \leq n_0 - n(z) \).

On the other hand,
let \( V(z) = n_0 - n(z) \geq 0 \)
Then \( V'(z) = -n'(z) \)
\( V''(z) = -n''(z) \)
\( = \gamma \{ \beta + \alpha - n \} n^\alpha n' \)
\( = \gamma \{ \beta + V - n_o \} (n_o - V)^\alpha (-V'). \)

Hence \( V'' + h(z) V' = 0 \) with
\( V = 0 \) and \( V'(L) = 0 \),
where \( h(z) = \gamma \{ \beta + V - n_o \} (n_o - V)^\alpha \).

Again, we cannot have the maximum of \( V \) at \( z = L \) since \( V'(L) = 0 \not> 0 \);
and if the maximum of \( V(z) \) occurs on \((0, L)\) then \( V = \text{const} = 0 \); so
unless \( V \) is constant, maximum \( V \) can only occur at \( z = 0 \).

implies \( V(z) \leq 0 \) since \( V(0) = 0 \)

implies \( n_0 - n(z) \leq 0 \)

implies \( n_0 \leq n(z) \).

But we have \( n(z) \leq n_0 \)

so \( n(z) \equiv n_0 \), which is a constant.

Hence we obtain a contradiction.

Therefore \( n(z) \) is a constant. \( \square \)

Theorem 4.5.1 tells us that there is no reason to make further research for the field case. From experiment and program output we know that the temperature keeps constant in laboratory cases.

Thus, we can see the consolidation equation (1.2.1) subject to the field type boundary conditions cannot give a "consolidated" solution. This quasistatic model is thus inadequate.
5. A DYNAMIC CONSOLIDATION MODEL

5.1 Introduction

Dr. Thompson has derived a set of equations including dynamic terms. He has assumed this is not a shock wave. That is, he has assumed that at the instant a load is applied at the surface, it is uniformly applied over the entire depth of the formation.

The equations are:

Equation of motion of water -

\[(5.1.1) \quad \frac{\partial u}{\partial z} - \tau - nG_w \gamma_0 = \frac{dV_w nG_w \gamma_0}{dt} \]

Equation of motion of solids -

\[(5.1.2) \quad \frac{\partial A^nB}{\partial z} - h(1-n) \frac{\partial u}{\partial z} + \tau - (1-n) G_s \gamma_0 \]

\[= \frac{dV_s (1-n) G_s \gamma_0}{dt} \]

Rate of loading -

\[(5.1.3) \quad \frac{\partial [A^nB + u(n+h(1-n))]_1}{\partial t} = \text{rate of loading} \]

Conservation of mass of solids

\[(5.1.4) \quad \frac{\partial ((1-n)G_s)}{\partial t} + \frac{\partial (V_s (1-n)G_s)}{\partial z} = 0 \]
Conservation of mass of water

\[
(5.1.5) \quad \frac{\partial (nG_w)}{\partial t} + \frac{\partial (V_w nG_w)}{\partial z} = 0
\]

Darcy equation -

\[
(5.1.6) \quad Cn^D (1 + \frac{1}{G_w Y_o} \frac{\partial u}{\partial z}) G_w + V_w nG_w - V_s (1-n) G_w = 0
\]

where

- \( n \) is porosity,
- \( u \) is pore pressure,
- \( V_s \) is the velocity of solids,
- \( V_w \) is the velocity of water,

and the rest are the same as before.

The equations of motion are combined by eliminating \( t \) to give:

\[
(5.1.7) \quad \frac{\gamma_0}{g} \frac{\partial}{\partial t} (G_s V_s (1-n)) + \\
\frac{\gamma_0}{g} \frac{\partial}{\partial t} (nG_w V_w) + \frac{\partial [An^B + u(n+h(1-n))] }{\partial z} \\
+ (1-n) G_s \gamma_0 + nG_w \gamma_0 = 0
\]

The approach to solving these equations is based on the assumption that the functions \( n, u, V_s \) and \( V_w \) can be represented with low order polynomials at specified values of \( t \). We can then use a finite difference approximation for derivatives with respect to \( t \).
At time $t = t$,

(5.1.8) $n = P_1(z)$

where $P_1(z) = a_0 + a_1 z + \ldots + a_k z^k$

and a's are unknown coefficients.

(5.1.9) $u = P_2(z)$

where $P_2(z) = b_0 + b_1 z + \ldots + b_k z^k$

and b's are unknown coefficients,

(5.1.10) $V_s = P_3(z)$

where $P_3(z) = c_0 + c_1 z + \ldots + c_k z^k$

and c's are unknown coefficients.

(5.1.11) $V_w = P_4(z)$

where $P_4(z) = d_0 + d_1 z + \ldots + d_k z^k$

and d's are unknown coefficients.

At time $t = t - \Delta t$,

$n = P_1(z)$

$u = P_2(z)$

$V_s = P_3(z)$
\[ V_w = P_4(z) \]

with known coefficients for each \( P_i(z) \) for \( i = 1, 2, 3, 4 \).

First, we substitute these polynomials into the boundary conditions to determine the coefficients in \( P_i(z) \). Given \( L = \) sediment depth the first set of equations are:

\[ n(o, t) = n_0 \]

where \( n_0 \) is the initial porosity i.e.,

\[
(5.1.12) \quad n(o, t) = \sum_{i=0}^{3} a_i z^i \bigg|_{z=L} = n_0
\]

implies

\[
(5.1.13) \quad a_0 + L a_1 + L^2 a_2 + L^3 a_3 = n_0
\]

\[ n(z, t) = \sum_{i=0}^{3} a_i z^i \] and since

given \( \frac{\partial n(o, t)}{\partial z} = 0 \) implies

\[
\frac{\partial n(o, t)}{\partial z} = 1a_1 + 2a_2z + 3a_3z^2 \bigg|_{z=0}
\]

implies

\[
(5.1.14) \quad 1a_1 + 0a_2 + 0a_3 = 0
\]
\[ u(L, t) = \text{depth of ocean} \times G_w \gamma_0 \]

Since
\[ u(\lambda) = \sum_{i=0}^{3} b_i \lambda^i, \]

\[ u(L) = b_o + b_1L + b_2L^2 + b_3L^3 \]

\[ = \text{depth of ocean} \times G_w \gamma_0 \]

i.e.,

(5.1.15) \[ \sum b_o + b_1L + b_2L^2 + b_3L^3 = \text{depth of ocean} \times G_w \gamma_0, \]

\[ V_s(o, t) = 0 \text{ and} \]

\[ V_s(\lambda) = \sum_{i=0}^{2} c_i \lambda^i \]

\[ = c_0 + c_1 \lambda + c_2 \lambda^2 + c_3 \lambda^3 \]

implies

(5.1.16) \[ c_0 + c_1 + c_2 \lambda^2 = 0. \]

Given \( V_w(o, t) = 0 \text{ and} \)

since \[ V_w(\lambda) = \sum_{i=0}^{2} d_i \lambda^i, \text{ it follows that} \]

(5.1.17) \[ d_0 + d_1 + d_2 \lambda^2 = 0. \]
We use differences like

\[ \frac{p_j(z) - P_j(z)}{\Delta t} \]

to approximate derivatives with respect to \( t \). The resulting equations can be explicitly differentiated with respect to \( z \). If we use polynomials through 3rd order terms in Equations (5.1.8) and (5.1.9) and through 2nd order terms in (5.1.10) and (5.1.11), then we have 14 coefficients to determine. The boundary conditions (5.1.13) through (5.1.17) give five equations. Additional equations can be written by writing "collocation" equations at different points in \( z \). These equations come from satisfying the governing Equations (5.1.3) through (5.1.7) at these collocation points. If we use four collocation points, we get 20 equations to go with the 5 boundary conditions.

A code from Childs [3] is being used to solve the system of equations such that the boundary conditions exactly and the collocation equations in a least square sense. This is incomplete at the time of this report, but, the investigators are continuing these efforts on their own time.
6. CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

A code is working for the solution of the static consolidation equations. This code works reasonable for laboratory boundary conditions. It is being adapted to work in a parameter estimation mode such that laboratory data can be used to determine the coefficients in constitutive equations.

Another code is under development for the solution of the dynamic consolidation model. When these codes are completed, an addendum to this report will be forwarded.
Transform nonlinear consolidation equation and energy equation into linear equations by backward difference and second order parabola scheme.

Declaration

Input necessary parameters associated with consolidation equation and energy equation.

Output these parameters

While input for time T is not to be cancelled

| 1. Solve porosities at time T by calling SUBROUTINE TRIDIAG. |
| 2. Set up iterations. |

| 1. Solve temperatures at time T by calling SUBROUTINE TRIDIAG.  |
| 2. Set up iteration (one iteration will be enough since energy equation is linear). |

Output the solutions for porosities and temperatures.

Fig. 7. Flowchart of a code for solution of solving consolidation equation and energy equation.
APPENDIX B
SOLVE CONSOLIDATION EQUATION
3
ENERGY EQUATION

A----COEFFICIENT FOR THE EFFECTIVE STRESS EXPRESSION. (KG/CM**2)

F = A**N*B; A,B ARE CONSTANTS

B----COEFFICIENT FOR THE EFFECTIVE STRESS EXPRESSION.
(REAL OR INTEGER).

PERMEABILITY K=C**N*D (CM/SEC)

C----COEFFICIENT FOR THE PERMEABILITY EXPRESSION.
(CM/SEC)

D----COEFFICIENT FOR THE PERMEABILITY EXPRESSION.

AREA OF WATER CAN BE EXPRESSED AS A FUNCTION OF THE
POROSITY.

A = N**E

W

E----PARAMETER OF THE AREA OF WATER
N----NUMBER OF FINITE DIFFERENCE (NOTE: IN INPUT DATA)
DT----THE INITIAL TIME STEP DT (SEC)
GS----SPECIFIC GRAVITY OF THE SOLID
GW----SPECIFIC GRAVITY OF THE WATER
GW----UNIT WEIGHT OF WATER (KG/CM**3)
TOL----ABSOLUTE ERROR BOUND FOR THE ITERATIVE SOLUTION
H----INITIAL SAMPLE HEIGHT (CM)
TIME----TIME AT WHICH THE CALCULATION SHOULD BE
TERMINATED DURING THE CONSOLIDATION PROCESS. (SEC)
PO----PARAMETER *0 FOR EXPLICIT SOLUTION AND 1 FOR IMPLICIT.
PI----THE MAXIMUM MAGNITUDE OF THE APPLIED CONSOLIDATION
PRESSURE (KG/CM**2).

S MIN, S MAX----IS THE POROSITY ON THE PREVIOUS ITERATION

TIFDM----IS THE POROSITY ON THE PREVIOUS TIME STEP

TIPH----IS THE TEMPERATURE ON THE PREVIOUS TIME STEP

P INIT----INITIAL POROSITY AT ALL POINTS (THE BOUNDARY IS
CALCULATED FROM THE KNOWN PRESSURE).
SUM DELTA-POROSITY (CHANGE THIS ITERATION)

BW----IS A FUNCTION DEPENDS ON TEMPERATURE WHERE

LOG Y = A + B LOG X

S MAX = MAX (U, U MIN

C ----IS A FUNCTION OF N WHERE G C IS A SPECIFIC

W

VW ----IS A FUNCTION OF N WHERE G C IS A SPECIFIC

W

HEAT OF THE SEA WATER ONLY IN A UNIT VOLUME OF

W

THE MINERAL-SEA WATER MIXTURE.

NOTE: G C IS A CONSTANT.

C ----IS A FUNCTION OF N WHERE G C IS A SPECIFIC

VW

HEAT OF THE MINERAL ONLY IN A UNIT VOLUME OF THE

VW

MINERAL-SEA WATER MIXTURE.

GG1, GG2, GG3----ARE THE COEFFICIENTS TO DESCRIBE THE MATERIAL.

WHERE K = GG1 + GG2 *N + GG3 *N**2
\[ H = \text{Thermal Conductivity of the Mineral-Salt Water System in Calories} \text{ C m}^{-1} \text{ sec}^{-1} \]

\[ \text{SW} \text{ is either constant or 0 (Note: is a function of temperature)} \]

\[ \text{GW} \times \text{SW} = \text{the heat supply rate in the sea water only in a unit volume of the mixture.} \]

\[ \text{SS} \text{ is a constant (Note: is a function of temperature)} \]

\[ \text{GS} \times \text{SS} = \text{the heat supply rate in the mineral only in a unit volume of the mixture.} \]

\[ \text{EM} \text{ is the same as OM.} \]

\[ \text{THI} \text{ is the initial temperature at all points.} \]

\text{Declaration}

\text{CHARACTER CHAR1*40}
\text{CHARACTER PRINTOUT*80, TEMPERATURES*80}
\text{INTEGER STEP, TMX, NUMBER OF STEPS, INCRSTEP, OUTPUT COUNT}
\text{REAL PORPH(105), POROS(105), PRX(105,3), Q(105,3), P INIT}
\text{REAL K, HO, TEMP(105), PRUPOR(105)}
\text{REAL SUM DPR, TOL DPR, THMPH(105)}
\text{REAL SUMMARY(9,0:12000)}
\text{INTEGER ISUM(0:4), OUTPUT INCREMENT}

\text{READ THE ABOVE VARIABLES OR CONSTANTS (ASSIGN VALUES FOR THEM), AND PRINT THEIR VALUES.}

\text{WRITE (6,*) 'INPUT SOIL? [ILLITE OR OTHER] AT [20 C OR OTHER]}
\text{READ (5,10) CHAR1}
\text{FORMAT (A30)}
\text{WRITE (6,15) CHAR1}
\text{FORMAT (1X, A30)}
\text{INCSTEP = 1}
\text{WRITE (6,*) 'PRINTED OUTPUT? [ALL, Partial, or NL] '}
\text{READ (5,18) END=900) PRINTOUT}
\text{WRITE (6,*) 'OUTPUT TEMPERATURES? [Y, N OR NL] '}
\text{READ (5,18) TEMPERATURES}
\text{FORMAT (A80)}
\text{OPEN (UNIT=1, FILE= *DATA*), IOINTENT= 'INPUT', STATUS= 'OLD' )
\text{AW = 0.7}
\text{READ (1,*) A*B+C*D*E*N+D,*,G*W*G0,TOL,H CM,MAX,OM,PO,NL, P INIT,NP}
\text{1 BU,CW,CVS,GG1,GG2,GG3,SW,SS,EM,TH1}
\text{IF ( PRINTOUT(1,11) .EQ. 'A' ) OR, PRINTOUT(1,11) .EQ. 'F' )}
\text{WRITE (12,20) A,B,C,D,E,N,D}
\text{WRITE (6,201) A,B,C,D,E,N,D}
\text{IF ( PRINTOUT(11) .EQ. 'A' ) OR, PRINTOUT(11) .EQ. 'F' )}
\text{WRITE (12,22) G*W*G0,TOL,H CM,MAX,OM,PO,NL, P INIT,NP}
\text{WRITE (6,22) G*W*G0,TOL,H CM,MAX,OM,PO,NL, P INIT,NP}
\text{20 FORMAT ( 'THE FOLLOWING ARE INPUT DATA ')}
\text{1 B = 'F14.9,' C = 'E14.7,' D = 'F14.9,' E = 'F9.4,'}
\text{2 N = 'I8,' DT = 'F12.1'}
\text{22 FORMAT ( 'G*W= 'G3.1,' SW= 'F7.4,' Tol= 'E10.2,' H CM= 'F9.6,' TMAX= 'I5,'}
\text{3 NL= 'I4,' P INIT= 'F8.4,' NP= 'I2,')}
\text{IF ( PRINTOUT(11) .EQ. 'A' ) OR, PRINTOUT(11) .EQ. 'F' )}
\text{WRITE (12,25) BU,CW,CVS,GG1,GG2,GG3,SW,SS,EM,TH1}
\text{WRITE (6,25) BU,CW,CVS,GG1,GG2,GG3,SW,SS,EM,TH1}
\text{25 FORMAT ( BU= 'F5.1,' CW= 'F5.1,' CVS= 'F5.1,' GG1= 'F10.6,'}
\text{1 GG2= 'F10.6,' GG3= 'F10.6,' SW= 'F5.2,'}
2. \( S = \frac{F_5.2}{x} \), \( E = \frac{F_5.2}{y} \), \( \theta = \frac{F_7.4}{z} \)

READ (1*, END=990) MAX ITR, TOL DPR, NUMBER OF STEPS
IF (NUMBER OF STEPS .LE. 0) NUMBER OF STEPS = 1000
IF (MAX ITR .LT. 2) MAX ITR = 2
MAX USE = MAX ITR * 2
IF (TOL DPR .LE. 1.E-5) TOL DPR = 1.E-5
NMS1 = N-1
NMS2 = N-2

\( P = F \) (UNIT AREA) = \( F = A \times N \times B \) BY EXPERIMENT

\( P = PD \)

ISUM(0) = 1
DO 30 I = 1, 5
ISUM(I) = (I*N+1)/6
30 CONTINUE
ISUM(6) = N
OUTPUT COUNT = 0
STEP = 1

ASSIGN THE VALUES FOR VECTOR OF POROSITIES \& TEMPERATURES
(INITIAL POROSITY \& TEMPERATURE AT ALL POINTS)
AT TIME \( T \).

DO 40 I = 1, N
PROPOR(I) = P INIT
PORPHM(I) = P INIT
POROS(I) = P INIT
TMPPTM(I) = TM1
40 CONTINUE

C
INITIAL \( T \)
\( T = 0.0 \)
DT MIN = DT
DT PREV = DT

1). DEFINE DX = H CM/(N-1)
2). INITIALIZE DXX(J,2) = DX FOR J = 1 TO N, AND DA = DX, DB = DX.
3). H INCH = H CM/2.54 (NOTE: 1 INCH = 2.54 CM)

DX = H CM / NMS1
DO 50 J = 1, N

50 DXX(J,2) = DX
DA = DX
DB = DX

CHANGE CM TO INCHES.
H INCH = H CM / 2.54
SUMMARY(1,0) = T
SUMMARY(2,0) = H INCH
DO 55 I = 0, 6
SUMMARY(I+3,0)=FOROS(ISUM(I))
CONTINUE
WRITE(6,810) T, H, INCH
WRITE(6,830) (FOROS(I),I=1,N)
IF( TEMPERATURES(1:1) .EQ., 'Y' ) WRITE(6,840) (TEMP(I),I=1,N)
IF( PRINTOUT(1:1) .EQ., 'Y' ) THEN
WRITE(12,810) T, H, INCH
WRITE(12,830) (POROS(I),I=1,N)
IF( TEMPERATURES(1:1) .EQ., 'Y' ) WRITE(12,840) (TEMP(I),I=1,N)
ENDIF

P=A**N**B IMPLIES N=EXP((ALOG(P/A))/B)

60 CONTINUE
ITER = 0
65 NO=EXP((ALOG(P/A))/B)

PH*A**RW=WEIGHT, WHERE RW IS THE UNIT WEIGHT OF WATER
IMPLIES P=WEIGHT/A=PH*RW
IMPLIES PH=P/RW=P/(GO*GW) NOTE:RW IS GO*GW
PH=P/(GO*GW)

G*N   -G*N   +G*N
2 I-1  I 1 I+1 3
IMPLIES -(G/G)*N   +(G/G)*N   -(G/G)*N
2 I-1  I 1 I+1 3
NOTE: Q(I,1)=-(G/G)
Q(I,2)=1,
Q(I,3)=-(G/G)
1
U(I)=G/G
3

SH=A*N**B/(GO*GW)
FOR EACH POROSITY N AT EACH INTERIOR POINT.
Q(I,1)=0,
Q(I,2)=1,
Q(I,3)=0,
POROS(I)=NO,
DO 125 I=2:NMS1
SH=A*EXP(B*ALOG(PRVPOR(I)))/(GO*GW)

PERMEABILITY PER UNIT AREA K=C*N**D (CM/SEC)
K=C*EXP(D*ALOG(PRVPOR(I)))

CALCULATE THE COEFFICIENTS QN,RN,SN OF THE LEFT-HAND SIDE OF THE CONSOLIDATION EQUATION.
QN=K*EXP(-E+1)*ALOG(PRVPOR(I))/(PH*E+(B-E)*SH)
RN1=PH*PRVPOR(I)*(E-1)-2*E
RN2=PH+B-2*E
RN3=PH-2*E+PRVPOR(I)*E-PRVPOR(I)
RN = K * EXP(-E2) * ALOG(PRVPOR(I)*PH*E*R1+SH*B*R2-E*R3)
QN = K * SH*B/(AW*PRVPOR(I))
FN = K * SH*B*B*(E-1)/(PRVPOR(I)*PRVPOR(I)) + B/(B*E)/AW
SH = K*D/PRVPOR(I)

REDEFINE DA = DXX(I+2), DB = DXX(I+1+2)
( DA = DB = DX * OLD VALUE IF TIME T IS NOT INCREASED BY DT )

DA = DXX(I+2)
II = I + 1
DB = DXX(I+1+2)
DAPDB = DA * DB
DAMDB = DA * DB
AMP = DA * DAPDB
BMP = DB * DAPDB
CDP = C1 * AMP
CDM = C1 * AMP
CID = (CDP - DA) / DAMDB
CIM = (CDM - DB) / DAMDB

FROM CONSOLIDATION EQUATION WE HAVE
\[ \frac{\partial N}{\partial t} = \frac{N(I+1) - 2N(I) + N(I-1)}{(2DZ)^2} + \frac{\partial \left( R^2 \frac{\partial \phi}{\partial Z} \right)}{\partial Z} \]
\[ = (N(I) - POPTM(I)/DT) \]
AND SINCE
\[ \frac{\partial \phi}{\partial Z} = 2N(I+1) + C2*I \]
\[ \frac{\partial \phi}{\partial Z} = 2N(I) + C2*I \]
\[ \frac{\partial \phi}{\partial Z} = 2N(I-1) + C2*I \]
AT Z
\[ \frac{\partial \phi}{\partial Z} = C1*I \]
AND \( \frac{\partial \phi}{\partial Z} = C1*I \)
AT Z
THE CONSOLIDATION EQUATION BECOMES
\[ Q(I+2)N(I) + Q(I+1)N(I+1) = Q(I)N(I-1) \]
\[ \text{Note:} \quad U(I) = POPTM(I)/DT \]
AND Q(I, J) = 1; 2; 3 ARE DESCRIBED AS FOLLOWS:

\[ \text{DNDDZ = C1 * PRVPOR(I+1) + C1 * PRVPOR(I) + C1 * PRVPOR(I-1) } \]
\[ \text{TERM = RH * DNDDZ + SN } \]
\[ Q(I+3) = QN * C2 + \text{TERM} * C1P \]
\[ Q(I+2) = QN * C2 + \text{TERM} * C1C - 1.0 / DT \]
\[ Q(I+1) = QN * C2 + \text{TERM} * C1M \]
\[ \text{POROS} (I) = \text{PORPTM} (I) / DT \]

CONTINUE
\[ Q(N+1) = 0. \]
\[ Q(N+2) = 1. \]
\[ Q(N+3) = 0. \]
\[ \text{POROS} (N) = 0 \]

POROSITIES AT TIME T+DT
CALL TRIAG(105, N, Q, POROS )
SUM DPR = 0.
DO 190 I = 1,N
   SUM DPR = SUM DPR + ABS( POROS(I) - PRVPR(I) )
   PRVPR(I) = POROS(I)
190 CONTINUE
ITER=ITER + 1
IF (SUM DPR .GE. TOL DPR .AND. ITER .LE. MAX USE) GO TO 65

SOLVE TEMPERATURE RIGHT HERE

IF ( TEMPERATURES(1:1) .EQ. 'Y' ) THEN
   G(1:1)=0,
   G(1:2)=1,
   G(1:3)=0.
   TEMP(1)=TM1
   BRATIO=E/RU
   F1=EXP(-(1/B)*ALOG(10*A))**(1+BRATIO)
   DO 890 I = 2, NMS1
      IA=DXX(I1,2)
      I1=I1+1
      DB=DXX(I1,2)
      DAPDB=DA+DB
      DAMDB=DA*DB
      AMP=DA*DAPDB
      BMP=DB*DAMDB
      C2P=2./BMP
      C2C=-2./DAMDB
      C2M=2./AMP
      CPA=DB/AMP
      CIP=(DB-DA)/DAMDB
      CIH=DB/AMP
      FI=A*(EXP(8*ALOG(POROS(I))))**(1-F1)*EXP(BRATIO*ALOG(POROS(I)))
      F2=POROS(I)*GW*CWS*(1-POROS(I))*GS*CVS
      F3=GG1*GG2*POROS(I)+GG3*POROS(I)*GS*CVS
      F4=POROS(I)*GW*SW+(1-POROS(I))*GS*S3
      A1=F3/F2
      DMQT=(POROS(I)-PORPTM(I))/DT
      A2=(F1/F2)*DMQT+(F4/F2)

FROM ENERGY EQUATION WE OBTAIN

(DENOTE TEMPERATURE BY V)
   A (C2P*V +C2C*V +C2M*V -)=-A *(V(I)-V0(I))/DT
   I=I+1, I=I-1
   2
 THEN WE HAVE
   G(I:3)*V +Q(I:2)*V +Q(I:1)*V =-A *(V0(I))/DT
   I=I+1, I=I-1
   2
 AND Q(I,J), J=1,2,3 ARE DESCRIBED AS FOLLOWS:

   Q(I:3)=A1*C2P
   Q(I:2)=A1*C2C-1.0/DT
   Q(I:1)=A1*C2M
   TEMP(I)=-A2-TMPPTM(I)/DT
590 CONTINUE
   Q(N+1)=0.
   Q(N+2)=1.
   Q(N+3)=0.
   TEMP(N)=TM1
CALL TRIAG( 105, N, G, TEMP )

IF( SUM DPR .LE. TOL DPR ) GO TO 700
    TTEMP = T + DT
    IF( PRINTOUT(111).EQ. 'A' ) WRITE(12,620) SUM DPR, TTEMP
    WRITE(6,620) SUM DPR, TTEMP
    FORMAT('DID NOT CONVERGE; SUM DPR = ',G12.5,' T=',G12.5)
    IF( PRINTOUT(111).EQ. 'A' ) THEN
        WRITE(12,830) (POROS(I),I=1,N)
        IF( TEMPERATURES(111).EQ. 'Y' ) WRITE(12,840)(TEMP(I),I=1,N)
    ENDIF
    WRITE(6,830) (POROS(I),I=1,N)
    IF( TEMPERATURES(111).EQ. 'Y' ) WRITE(6,840)(TEMP(I),I=1,N)
CONTINUE

CALCULATE THE HEIGHT
DO 710 J=2,N
    POR AVG=(POROS(J)+POROS(J-1))/2,
         DXX(J,1)=DXX(J,2)*(1.-P INIT)/(1.-POR AVG)
CONTINUE
H CM=0.
DO 720 I=2,N
    H CM=H CM + DXX(I,1)
H INCH=H CM/2.54
T = T + DT
MAX use = MAXTR

DEFINE IFOROS(I) = POROS(I) - PORFTH(I)

PRINT OUT 1): THE TIME T, SAMPLE HEIGHT H INCH

2): THE POROSITY DISTRIBUTION OBTAINED AT ANY TIME T,

3): TEMPERATURES

IF( STEP IS DIVISIBLE BY INCSTEP THEN PRINTOUT T, H INCH)
IMPLIED DT IS MULTIPLIED BY INCSTEP
IF( (STEP/INCSTEP)*INCSTEP .EQ. STEP ) THEN
    WRITE(6,810) T, H INCH
    FORMAT( ' THE DURATION IS , T = ',F10.1,2X,
             ' THE HEIGHT IN INCHES IS ',F12.4)
    WRITE(6,830) (POROS(I),I=1,N)
    FORMAT( ' SOLUTIONS FOR POROSITY ,/(10F8.4))
    IF( TEMPERATURES(111).EQ. 'Y' ) WRITE(12,840)(TEMP(I),I=1,N)
    FORMAT( ' SOLUTIONS FOR TEMPERATURES ,/(10F8.2))
    IF( PRINTOUT(111).EQ. 'A' ) THEN
        WRITE(12,810) T, H INCH
        WRITE(12,830) (POROS(I),I=1,N)
        IF( TEMPERATURES(111).EQ. 'Y' ) WRITE(12,840)(TEMP(I),I=1,N)
    ENDIF
ENDIF

IF( (STEP/INCSTEP)*INCSTEP .EQ. STEP ) THEN
WRITE(6,870) INCSTEP, DT
FORMAT(' NOW USING OUTPUT INCREMENT = ',I4, 'STEPS',
       $='F10.1')
WRITE(6,*)' CONTROL D TO STOP OR CHANG THE OUTPUT INCREMENT'
READ(E,*,END=900) INCSTEP
IF( INCSTEP .LT. 0 ) THEN
  WRITE(6,*)' INPUT THE NEW DT,'
  READ(E,*,END=900) DT
  IF( PRINTOUT(I11) .EQ. 'A' .OR. PRINTOUT(I11) .EQ. 'P' )
    WRITE(12,875) DT
  ENDIF
  INCSTEP = ABS(INCSTEP)
  IF( INCSTEP .LT. 1 ) INCSTEP = 100
  IF( DT .LT. DTMIN ) DT = DTREV
  STEP = 0
ENDIF
STEP = STEP + 1
RATIO = DT / DTREV
DO 880 I = 1, N
  SAVE = POROS(I)
  POROS(I) = SAVE + ( SAVE - PORPTH(I) ) * RATIO
  PVRFOR(I) = POROS(I)
  PORPTH(I) = SAVE
  SAVE = TMPH(I)
  TMPH(I) = SAVE + (SAVE-TMPPTH(I))*RATIO
  TMPPTH(I) = SAVE
ENDO
CONTINUE

OUTPUT COUNT = OUTPUT COUNT + 1
IF (OUTPUT COUNT .GT. 2000) OUTPUT COUNT = 2000
SUMMARY(1,OUPUT COUNT) = I
SUMMARY(2,OUPUT COUNT) = H INCH
DO 885 I = 0, 6
  SUMMARY(I + 3, OUTPUT COUNT) = POROS(ISUM(I))
ENDO
CONTINUE
IF( I .LT. THAX .AND. STEP .LT. NUMBER OF STEPS ) GO TO 900
WRITE(12,910)(ISUM(I),I=0,6)
OUTPUT COUNT = 1
DO 950 ISU = 0, OUTPUT COUNT
  IF (ISU .NE. (ISU/OUTPUT INCREMENT)*OUTPUT INCREMENT) GO TO 950
  WRITE(12,930)(SUMMARY(I,ISU),I=1,9)
  IF( SUMMARY(1,ISU) .GT. 60 ) OUTPUT INCREMENT = 61/DT
  IF( SUMMARY(1,ISU) .GT. 120 ) OUTPUT INCREMENT = 121/DT
  IF( SUMMARY(1,ISU) .GT. 600 ) OUTPUT INCREMENT = 601/DT
  FORMAT(1x,F14.0,F12.4,F12.4)
930 CONTINUE
950 CONTINUE
STOP
END
APPENDIX C
REFERENCES


