Tether Deformation and Tension Leg Platform Surge

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Estimates of tension leg platform (TLP) surge presented in the literature are generally based on the assumption that the tethers are straight at all times, i.e., that transverse deformations of the tethers due to hydrodynamic and inertial forces may be disregarded (2,3). The purpose of this note is to verify the validity of this assumption in the case of a deep water TLP.

It is assumed (2) that the equation of motion of the tethers can be written as

$$(m_o + m_a)\frac{\partial^2 y}{\partial t^2} + \lambda \left|\frac{\partial y}{\partial t}\right| \frac{\partial y}{\partial t} = T\frac{\partial^2 y}{\partial z^2} \quad (1)$$

in which m_o = mass of tether per unit length; m_a = added mass; λ = hydrodynamic drag coefficient; T = tether tension; y = horizontal displacement; z = vertical coordinate; $m_a = 0.785\rho C_a D^2$; $\lambda = 0.5\rho C_d D$; ρ = water density; D = tether diameter; C_a = added mass coefficient; C_d = drag coefficient associated with fluid viscosity effects. The boundary conditions for Eq. 1 are assumed to be

$$y(t,0) = 0 \quad (2a)$$

$$y(t,l) = y_o \cos \omega t \quad (2b)$$

at the sea bottom ($z = 0$) and at the platform elevation ($z = l$), respectively.

The following nondimensional quantities are defined: $z' = z/l$; $t' = t\omega$;

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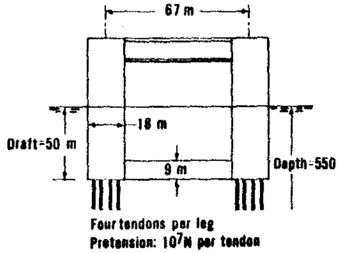

FIG. 1.—Platform Configuration and Dimensions, After (4)

$y' = y/l$; $y'_o = y_o/l$; $m' = (m_o + m_a)/m_o$; $\lambda' = \lambda l/m_o$; $T' = T/(m_o\omega^2 l^2)$. Eq. 1 can be written as

$$m' \frac{\partial^2 y'}{\partial t'^2} + \lambda' \left|\frac{\partial y'}{\partial t'}\right| \frac{\partial y'}{\partial t'} = T' \frac{\partial^2 y'}{\partial z'^2} \quad (3)$$

In general, Eq. 3 cannot be solved in closed form. For the purposes of this note it is therefore necessary to solve Eq. 3 numerically.

We consider, as an example, a TLP with the geometric characteristics shown in Fig. 1 (4,6). It is assumed that the mass of the platform is $M = 4.5 \times 10^7$ kg; that the platform is attached to the sea floor by 16 tethers with length $l = 600$ m, mass $m_o = 300$ kg/m, and diameter $D = 0.48$ m; and that the tension in each tether is $T = 10^7$ N. Changes in tether tension during the platform motion—which can be shown to be of the order of 10% or less (4,6)—are not taken into account. The platform is assumed to be loaded by waves with height $H = 25$ m and period $T_w = 15$ sec, and by a current with speed varying from 1.4 m/s at the surface to 0.9 m/s at the keel. Both the waves and the current act in a direction normal to a platform side.The hydrodynamic forces on the buoyant components of the platform are described by the Morison equation with drag coefficient $C_d = 0.6$ and inertia coefficient $C_m = 1.8$ (5,6). The calculated added mass is then $A \simeq 3.6 \times 10^7$ kg (4,6).

The equation of surge motion of the platform is

$$(M + A)\,\ddot{y}(t,l) - R(t) = F_H \quad (4)$$

in which $R(t)$ = restoring force; $F_H = F_v + F_e$; F_v = total hydrodynamic viscous force; and F_e = total wave-induced exciting force. Surge wave radiation damping and damping due to internal friction within the structure are neglected on account of their relative insignificance in the problem at hand (4). For details on the calculation of F_v and F_e for the platform of Fig. 1, see Ref. 4. The integration of Eq. 4 (4) yields a fluctuating response with amplitude $y_o \simeq 6$ m and frequency $\omega \simeq 0.42$ rad/sec. For the purposes of this investigation, the fluctuating response may, to a first approximation, be assumed to be harmonic. These values for y_o and ω are therefore used for the calculation of the coefficients in Eq. 3. The drag and added mass coefficients for the tethers, C_d and C_a, depend upon the Reynolds and Keulegan-Carpenter numbers and therefore vary along the tethers. However, their overall effect may be assumed to be equivalent, approximately, to that of the constant values $C_d \simeq 1.25$; C_a

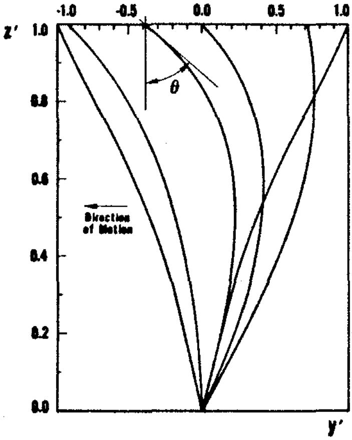

FIG. 2.—Tether Shapes for Various Platform Positions

$\simeq 0.5$ (5). Then, $m' = 1.3$; $\lambda' = 600$; $T' = 0.525$; and $y_o' = 0.01$. Using the parameter values just listed, steady state solutions for y' were obtained numerically. This was done by solving an initial boundary value problem for Eq. 3 (which is a nonlinear wave equation) and carrying the computations far enough in time that all transients vanish to single precision accuracy. Use was made of a flexible partial differential equation software package, MOL1D, (1), allowing wide choices of space and time discretizations. The package features an adaptive time step controlled by a preset error tolerance, a highly useful option for long term numerical integration. The resulting steady state solutions for y' are shown in Fig. 2 for various positions of the tether.

The restoring force in the equation of surge motion of the platform can be written as

$$R(t) = -\sum_{n} T \sin\theta \quad \text{(5)}$$

in which n = total number of tethers; and θ = angle between the vertical and the tangent to the tether at its point of articulation to the platform (Fig. 2). If the tethers are assumed to be straight at all times, then $\theta = \theta_s$, in which

$$\theta_s \approx \frac{y(t,l)}{l} \quad \text{(6)}$$

In the case of deformed tethers, $\theta = \theta_d$, in which

$$\theta_d \approx \left.\frac{\partial y(t,z)}{\partial z}\right|_{z=l} \quad \text{(7)}$$

The results obtained by integrating Eq. 3 with the parameters listed earlier were found to fit the expression

$$\theta_d \approx 2.9\,\frac{y[(t-2.8),l]}{l} \quad \text{(8)}$$

in which t is expressed in seconds. Eqs. 8, 6, and 5 show that in the case considered here the effect of the tether curvature is to cause a lag between the harmonic restoring force and the harmonic surge, and increase the amplitude of the restoring force almost threefold.

However, because in this example the restoring force is small compared to the inertia and the hydrodynamic force, it can be verified that this increase—though relatively large—is not sufficient to have a significant effect upon the amplitude of the surge response.

Similar calculations were also performed for the case $\omega \approx 0.061$ rad/sec, corresponding to a frequency of the forcing function equal to the nominal natural frequency, $n_1(\theta_s)$, and representative of fluctuations induced by an idealized fluctuating wind load. The integration of Eq. 3 yielded in this case $\theta_d \simeq \theta_s$, i.e., the deviation of the deflected shape of the tethers from a straight line was found to be negligible.

On the basis of the limited investigation presented in this note, it is therefore concluded that the lateral deformation of the tethers under the action of hydrodynamic loads does not affect significantly the surge response of deep water TLPs with characteristics similar to those assumed in the foregoing example.

This investigation was based on deliberately simplified, uncoupled models of the tether and platform motions. More elaborate models may be employed, in which the two motions are coupled, and in which the dependence on elevation of the Morison equation coefficients for the tethers is taken into account. However, in cases such as those dealt with in this note, where the effect of tether deformation on the amplitude of the surge response is negligible, it may be anticipated that integration of the coupled equations of motion will not yield results significantly different from those obtained herein.

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Appendix.—References

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