An Integrated Constitutive Theory for the Mechanical Behaviour of Sea Ice: Micromechanical Interpretation
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INTRODUCTION

Ice in general, and columnar sea ice in particular, is a very complex material which exhibits a wide range of behaviors often at the same time. As a consequence of its occurrence at high homologous temperatures, the mechanical behavior of ice is strongly influenced by temperature and rate of loading. In most applications ice behaves as a material undergoing both continuum deformation and cracking activity. Microcracking activity can be effectively idealized as continuum behavior and represented by models describing multiaxial flow with appropriate modifications to account for the damage due to micro-cracking. This type of damage manifests itself as strain-softening observed during constant strain-rate tests in compression and as tertiary creep in compressive creep tests. On the other hand, macrocracking activity implies "failure" of the material at macroscale and is a separate behavioral mechanism. A constitutive model that captures both the mechanisms of multiaxial flow as a continuum and ultimate failure by macrocracking in addition to the rate and temperature dependence is necessary to investigate the mechanics of deformation and progressive failure in ice.

Previous modeling work directly relevant to the development of such a constitutive theory is limited since most investigators have treated ice as either an elastic-plastic or a viscoelastic material under multiaxial states of stress. They include the works of Glen (1955), Palmer (1967), Shapiro (1978), Sinha (1978, 1979, 1983b), Michel (1981), Wang (1982), Morland and Spring (1981, 1982, 1983), and Ashby and Duval (1985). Studies on the damage behavior of ice have been relatively few in comparison. Sinha (1982) and Sanderson and Child (1984) have studied the occurrence of first cracks in ice under compressive creep conditions, while Schulson (1979, 1984a,b)
and Currier and Schulson (1982) have studied the brittle to ductile transition in polycrystalline ice. The use of damage mechanics theory for modeling ice is being explored by Karr (1984, 1985a,b). More recently, Szyzkowski and Glocnner (1985) have proposed a relatively comprehensive model that describes the continuum behavior of pure ice accounting for damage due to microcracking. The models cited here are highly variable in the degree to which they are related to the micromechanics of ice behavior, i.e., the physical mechanisms governing ice behavior at micorscale.

This paper presents a new rate-sensitive constitutive theory for describing the mechanical behavior of ice developed at MIT. The theory integrates (i) a potential function description for modeling deformations based on a nonlinear generalization of the Maxwell differential model and the associated flow rule with (ii) a rate-sensitive Drucker-Prager surface to describe ultimate failure by macrocracking representing either yielding of the material or fracture depending on the stress state. The constitutive model is characterized by its ability to:

(a) Decompose the various recoverable (instantaneous and delayed elastic or "primary creep") and irrecoverable (secondary creep and strain-softening or tertiary creep) components of strain.

(b) Represent continuously damaging or strain-softening material behavior during ductile-to-brittle transition in compression with a linear incremental damage accumulation model.

(c) Describe materially anisotropic mechanical behavior with a pressure-insensitive but rate-dependent potential function.

(d) Predict first crack occurrence or nucleation with a rate-dependent limiting tensile strain criterion.

(e) Distinguish the mechanisms of multiaxial flow as a continuum and ultimate failure by macrocracking leading to yielding of the material or fracture.

Further, the model shows strong dependency of the continuum behavior under creep and constant strainrate conditions. The model predictions compare very well with several independent sets of experimental data, particularly those for first-year sea ice. Data for the uniaxial "strength" of sea ice has been augmented with the extensive experimental data base available for pure crystalline ice through a normalization proposed by Weeks and Assur (1967) based on the work of Frankenstein and

Garner (1967). The effective modulus is considered below -10°C as a function of the Young's modulus of ice, the Poisson's ratio of ice, and fracture toughness integrated for the temperature range.

CONTINUUM

Uniaxial model

The nonlinear constitutive model consists of a dashpot. It is discussed in detail in Ref. 2.

Rate-sensitive constitutive equation at 1/3 power law in effective modulus approaches and the Young's modulus is integrated for the temperature range.

\[ E_{\text{eff}}(\varepsilon) = E (\varepsilon) \]

where \( E \) is the effective modulus, \( \varepsilon \) is the strain, and \( \varepsilon \) is the rate-dependent limiting tensile strain criterion.

The rate-sensitive constitutive equation at 1/3 power law in effective modulus approaches the Young's modulus as follows:

\[ 1/E_{\text{eff}} = \frac{1}{E} + \frac{1}{E_0} \]

where \( E_0 \) is the initial effective modulus, and \( E_0 \) is the initial effective modulus. The limiting tensile strain criterion is expressed as:

\[ \varepsilon = \frac{1}{E} \]

where \( \varepsilon \) is the rate-dependent limiting tensile strain criterion.
Garner (1967) to account for the presence of brine. The effect of temperature on the continuum behavior of ice is modeled in terms of an Arrhenius activation energy law that is considered by Mellar (1983) to be valid for temperatures below -10°C. The model is described in two papers by Ting and Shyam Sunder (1986a,b); the first focuses on the continuum behavior of sea ice while the second focuses on its yielding and fracture behavior. This paper presents the model in integrated form emphasizing its micromechanical interpretation.

CONTINUUM THEORY

Uniaxial model formulation
The nonlinear generalization of the two element Maxwell fluid model consists of an elastic spring in series with a viscous dashpot. The characterization of the two elements is discussed in what follows.

Rate-sensitive elastic spring As a consequence of its application at high homologous temperatures, the elastic modulus of ice is sensitive to even "slight" variations in rate of loading (Mellar, 1983) and cannot be taken as a constant. If the Young's modulus for ice, $E$, is defined to be the modulus value at very high rates of loading, then the variation of effective or apparent elastic modulus, $E_{eff}$, with rate may be expressed as:

$$E_{eff} = E \left[1 - r \exp\left(-A/E \cdot \dot{\varepsilon}^{1/N}\right)\right]$$  \hspace{1cm} (1)

where $\dot{\varepsilon}$ is the strainrate, $r$ and $A$ are constants, and $N$ is the power law index for ice. Equation (1) shows that the effective modulus tends to the Young's modulus as the strain-rate approaches infinity. When the strain-rate tends to zero, the effective modulus tends to $(1-r)E$, and for $r$ equal to one, the effective modulus tends to zero. If $r$ is zero, the effective modulus is rate-insensitive and equal to the Young's modulus. A value of $r$ less than one is necessary to model stress relaxation.

The rate-sensitive elastic spring represents recoverable strains contributed both by instantaneous elasticity and by delayed elasticity. By defining the total elastic strain to be the sum of the strains due to these components it is possible to model the rate-sensitive spring as the series combination of two springs, one with modulus equal to $E$, i.e., the Young's modulus, and the other with a modulus equal to $E_d$, the modulus of delayed elasticity. It then follows that:

$$1/E_{eff} = 1/E + 1/E_d$$  \hspace{1cm} (2)
with

$$E_d = E [1/r \exp(A/E \cdot \epsilon_1^1/N)] - 1 \quad (3)$$

Equation (3) shows that the modulus of delayed elasticity tends to infinity at infinite strain rate and to \([(1-r)/r]E\) at zero strain rate. In the latter case, the modulus tends to zero if \(r\) is one and to infinity if \(r\) is zero. Use of Eq. (2) shows that when the modulus of delayed elasticity is infinity, the effective modulus equals the Young's modulus. When the modulus of elasticity is very small with respect to the Young's modulus, the effective modulus equals the modulus of delayed elasticity. This occurs at low strain rates as \(r\) tends to one.

Nonlinear viscous dashpot The secondary creep strain rate, \(\epsilon_{sc}\), in ice is modelled in terms of the well-known Glen's power law (Glen, 1955):

$$\sigma = (A/M) \epsilon_{sc}^{1/N} \quad (4)$$

where \(A\) and \(N\) are the constants in Eq. (1), and \(M\) is a third constant. \((A/M)\) is often taken to be a single constant.

However, since \(A\) is used to describe delayed elasticity or primary creep resistance, an additional degree of freedom in the form of the constant \(M\) is used to model the secondary creep resistance. The nonlinearity is associated with the dashpot constant \(c/\epsilon_{sc}\) which is a function of the secondary creep strain rate.

The description of deformations due to delayed elasticity and secondary creep proposed here can be interpreted in terms of slip within and between ice grains along the lines suggested by previous investigators. The practical value of such an interpretation is not always clear. For example, the models of Sinha (1979, 1982) and Michel (1981) consider grain boundary sliding and basal slip in the hexagonal ice crystals as factors governing the two deformations, while the model of Duval and Ashby (1985) considers slip on the basal plane plus one other nonbasal system as being responsible for the deformations. However, in all three cases the resulting "physical" model itself is a spring-dashpot idealization of the postulated slip mechanisms.

Continuous damage model In the transition from pure ductile to pure brittle behavior under compressive loading ice behaves essentially as a continuum undergoing damage. Under tensile loading the transition region is much smaller and a continuum description of damage is of limited value. Damage or micro-

Cracking and strain rate-sensitive models Small strain rate-sensitive models are generally elastic-plastic and developed for metals. Differences between metals which approximate the model. This is not the case as the damage state that is described is 

$$A_d = D_c$$

where \(D\) is the rate of 

(3) and (4) and between zero and the material, the following strain are identified

$$\epsilon_d = c_1$$

where \(c_1\) is the strain rate that material strain tends to infinity and is the value of

The formulae appropriate for the loading history are assumed to be 

accumulated. Thus if the 

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Temperature versus 

damage is 

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strain rate.
cracking activity in ice leads to tertiary creep under constant stress loading and to strain-softening under constant strainrate loading. Damage is almost nonexistent at very small strain and strain-rate but increases as both strain and strainrate increase. Further, unloading a damaged material generally exhibits a reduction in the effective modulus of elasticity. Thus the phenomenon affects both the rate-sensitive spring and the non-linear viscous dashpot. The development here is based on the hypothesis that damage influences the constant $A$ describing the creep resistance of ice, which appears in both the elements of the generalized Maxwell model. The effect of damage on the Young's modulus is neglected. Defining $D$ as a one-dimensional damage parameter and $A_D$ as the damaged value of the constant $A$, it is possible to state that:

$$A_D = (1-D)A$$

(5)

where $D=0$ in the case of no damage, and the $A$'s in Eqs. (1), (3) and (4) are replaced with $A_D$. In general, $D$ varies between zero and one. For the case of total damage, i.e., $D=1$, the material looses all its ability to sustain stresses. The following mathematical form describes the dependence of $D$ on strain and strainrate, and satisfies the physical constraints identified above.

$$D = 1 - \left[ \exp(-c_1\varepsilon) + \exp(-c_2\varepsilon) \left(1-\exp(-c_1\varepsilon)\right) \right]$$

(6)

where $c_1$ and $c_2$ are constants. This equation shows that as strainrate approaches infinity $D$ tends to one, i.e., the material is completely damaged. Further, as strain approaches infinity $D$ tends to $\left(1-\exp(-c_2\varepsilon)\right)$, i.e., there is a limiting value of damage at any given strainrate.

The formulation of the damage parameter in Eq. (6) is appropriate for monotonic loading conditions. For a variable loading history the evolution of the damage parameter is assumed to follow the well-known Miner's linear damage accumulation rule. According to this rule, the incremental damage accumulation depends only on the current state of damage. Thus if the rate of loading (or strainrate) changes at some instant, Eq. (6) applies at the new rate with the level of damage previously accumulated. This requires the definition of an "equivalent" initial strain corresponding to the new strainrate.

Temperature effects At rates of loading where no material damage is present, the effect of temperature on the stress versus secondary creep strainrate relationship is characterized by an Arrhenius activation energy law. Mellor (1983)
states that for temperatures greater than \(-10^\circ C\) the law is not valid and that the complete empirical relation derived from experiments should be used to model the temperature dependence in such cases. Sinha (1978) has concluded that the variation of the delayed elastic or primary creep strain with temperature also follows an Arrhenius law. He found the activation energy for both the secondary creep flow and the delayed elastic deformation to be equal. The parameter \(A\), which appears in Eqs. (1), (3) and (4), describes the creep resistance of ice and is taken to follow the Arrhenius activity energy law to model temperature effects below \(-10^\circ C\), i.e.,

\[
A = A_0 \exp(Q/kT)
\]

(7)

where \(T\) is the temperature in degrees Kelvin, \(A_0\) is a temperature independent constant, \(Q\) is the activation energy, and \(k\) is the universal gas constant. As the temperature reduces, the parameter \(A\) increases in value. In consequence, the effective elastic modulus tends to the Young’s modulus and the nonlinear dashpot becomes highly viscous. Then ice displays a purely linear elastic behavior with little rate or temperature sensitivity. This model takes into account the relative temperature insensitivity of the Young’s modulus of ice and of the microcracking activity in ice.

Uniaxial model parameters. The uniaxial model has been successfully verified against several independent sets of experimental data for both pure and sea ice as described in Ting and Shyam Sunder (1985b, 1985a). The verification is based on the following values of the eight model parameters:

\[
\begin{align*}
E &= 9.5 \text{ GPa} \\
r &= 0.98 \\
A_0 &= 0.00652 \text{ MPa s}^1/\text{N} \\
N &= 3 \\
M &= 1411.2 \\
Q &= 65,000 \text{ J mol}^{-1} \\
C_1 &= 2.28 \times 10^5 \text{ s} \\
C_2 &= 1028 \text{ s}
\end{align*}
\]

The universal gas constant, \(R=8.314 \text{ J mol}^{-1} \text{ K}^{-1}\). The value of \(r\) is subject to some uncertainty since experimental data is inadequate to quantify it. Additional data may indicate that a lower value is more appropriate. All other parameters can be derived from conventional tests conducted on ice.

The models of Sinha (1978), Michel (1981), Duval and Ashby (1985), and Szyszkowski and Glockner (1985) use 7, 9, 11, and 9 parameters respectively if prediction of temperature dependence with a single parameter, for example \(Q\), is considered.
It must be recognized that (i) the models of Sinha, and Duval and Ashby do not capture material damage, (ii) verification of Michell's model with experimental data appears to be limited, and (iii) the ability of the models to describe stress relaxation has not been investigated adequately. Furthermore, the models have not been verified against sea ice data.

**Multiaxial model formulation**

Field observations have shown that sea ice, which is predominantly columnar, has two sources of anisotropy: (a) the c-axis is oriented perpendicular to the axis of crystal growth, and (b) the c-axes of different crystals or grains may show preferred azimuthal orientation in the plane on which they lie. This anisotropy strongly influences the mechanical behavior of first year sheet ice, while its influence on the behavior of multi-year floes, though less well studied, may be less. The development presented here is based on an orthotropic generalization of the uniaxial, rate-sensitive damage model for the continuum (flow) behavior of ice.

The strainrate vector due to the three components of creep is related to the stress vector through an effective stress measure for orthotropic materials with identical behavior in compression and tension, i.e.,

\[
\dot{\varepsilon}_e^2 = \frac{3}{8} \left[ \frac{a_1}{3} (\sigma_{xx} - \sigma_{yy})^2 + \frac{a_2}{3} (\sigma_{yy} - \sigma_{zz})^2 + \frac{a_3}{3} (\sigma_{zz} - \sigma_{xx})^2 + 2a_4 \sigma_{xy}^2 + 2a_5 \sigma_{yz}^2 + 2a_6 \sigma_{zx}^2 \right]
\]

with \(8 = a_1 + a_2\). This may be expressed in compact form using matrix notation as:

\[
\dot{\varepsilon}_e^2 = \frac{3}{8} \bar{\sigma}^T T \bar{\sigma}
\]

where \(T\) is a matrix containing the \(a_i\)'s and defined in Ting and Shyam Sunder (1985).

The creep strainrate vector can be related to the stress vector with the help of a scalar potential function obeying the associated flow rule. The desired relationship can be written as:

\[
\dot{\varepsilon}_{cr} = \lambda S^k
\]
where

$$\lambda = \frac{3}{8} \frac{1}{e} \left[ \frac{e}{E_d} + \left( N/A_d \right)^N \sigma^N \right]$$  \hspace{1cm} (11)$$

and

$$\sigma^* = \sigma_0$$  \hspace{1cm} (12)$$

Note that $\sigma^*$ may be thought of as a pseudo deviatoric stress vector for an anisotropic material. The evaluation of $A_d$ and $E_d$ requires knowledge of the effective creep strain rate vector which can be expressed in a manner similar to Eq. (9) as shown in Ting and Shyam Sunder (1985).

Given the stress vector, the pseudo deviatoric stresses may be obtained from Eq. (12). Then, applying Eqs. (9), (11) and (10) in succession leads to the creep strain rate vector.

**Multiaxial model parameters** Five uniaxial (compression) tests at constant strain rate may be used to derive the five orthotropic model parameters: $a_2$ to $a_6$. Note that (i) $a_1$ can be set equal to one without loss of generality, and (ii) there is experimental evidence which shows that the power-law exponent $N$ can be considered independent of the direction of loading.

The six tests (including one used to obtain the reference uniaxial parameters) can be conducted in the three directions defining the axes of orthotropy and along the $45^\circ$ axes on the three orthogonal planes contained by the axes. Ting and Shyam Sunder (1985, 1986a) provide equations to evaluate the model parameters from the experimentally obtained ratios of stress in a given direction to that in the reference direction. In first year sheet ice, the ratio of vertical stress to horizontal stress in the direction of the c-axes at constant strain rate varies from 2 to 5 (Butkovich, 1959, Peyton, 1966, Vaudrey, 1977, Sinha, 1983a, and Frederking, 1983), while the stress ratio may vary in the range 0.25-0.60 at a $45^\circ$ azimuth angle to the c-axes when they are aligned on the horizontal plane and 0.50-0.95 at a $90^\circ$ angle (Wang, 1979, Vittoratos, 1979, Richter-Menge, et al., 1985, Peyton, 1968).

This pressure insensitive material model for the continuum behavior of ice follows very well the experimental data of Frederking (1977) on the plane strain compressive strength of columnar grained and granular-snow ice. Verification of the model against data on sea ice, which is significantly less pressure sensitive than pure ice (see Richter-Menge, et al., 1985, for data on anisotropic first year sea ice), suggests that a pressure sensitive model may be adequate in many engineering applications.
YIELDING AND FRACTURE THEORY

Uniaxial model formulation
The behavior of ice in uniaxial tension and compression at constant strainrates is identical up to a transition strainrate, \( \dot{\varepsilon}_{\text{tr}} \). Above this strainrate, ice fractures in tension although it continues to flow in compression. For the range of grain sizes typically encountered in sea ice, Schulson (1979, 1984a,b) and Currier and Schulson (1982) find that the tensile strength of ice is governed by the stress to nucleate cracks (not the stress to propagate cracks) and that the tensile strength actually does represent fracture, i.e., sudden failure. The ductile to brittle transition in tension tends to occur over a negligible range of strainrates unlike that in compression where strain-softening is an important behavioral phenomenon.

In compression, the strength is governed by brittle fracture only when the strainrate exceeds a limiting value, \( \dot{\varepsilon}_{\text{tr}} \), which typically is greater than \( \dot{\varepsilon}_{\text{tr}} \) by several (5 to 6) orders of magnitude. However, microcracks can nucleate at lower strainrates as a result of material damage. The stress at which the first "important" microcrack, i.e., one that is visible to the naked eye in pristine ice, nucleates is termed the "yield" stress in compression. This yield stress is always less than (or equal to) the maximum attainable compressive stress or "strength".

A consistent model for predicting the uniaxial fracture strength in tension and yield stress in compression are developed below.

Tensile fracture strength The stress-strain-strainrate behavior in uniaxial tension prior to fracture is considered to be identical to that under uniaxial compression. Hawkes and Hellor (1972) justify this assumption from creep tests on ice. The tensile fracture strength versus strainrate relationship can be modeled as:

\[
\frac{1}{\sigma_{\text{tf}}} = \frac{1}{\sigma_{\text{tm}}} + \frac{1}{B\varepsilon^{1/N}}
\]

(13)

where \( \sigma_{\text{tm}} \) is the maximum and constant value of tensile strength at high strainrates and the second term models the
increase in tensile strength as the strainrate increases from the transition value. The value of }\textit{B}\text{ is chosen so that at }\dot{\varepsilon}_{\text{tt}}\text{ Eq. (13) predicts a value of stress, }\sigma_{\text{tt}}\text{, equal to that given by the continuum theory.}

The data of Ashby and Cooksley-Hallam for pure ice contained in Palmer et al. (1982) suggests that }\dot{\varepsilon}_{\text{tt}}=5\times10^{-8}\text{ s}^{-1}\text{ and }\sigma_{\text{tt}}=0.48\text{ MPa}.\text{ From this the material constants for the model are estimated to be:}

\begin{align*}
\text{B} & = 176\text{ MPa s}^{1/\text{N}} \\
\sigma_{\text{tm}} & = 2.0\text{ MPa}
\end{align*}

Model parameters for sea ice are estimated from the above by correcting for the presence of brine.

**Compressive yield strength** Prediction of the first "important" crack to nucleate under uniaxial compressive loading is based on the hypothesis that this occurs due to the lateral tensile strain and strainrate resulting from the Poisson effect of elasticity and the incompressibility condition of creep deformation. The first crack is postulated to occur when the lateral tensile strain equals the strain for fracture under uniaxial tensile loading at the instantaneous strainrate. The strain for tensile fracture at a given strainrate can be obtained from the tensile stress-strain-strainrate-strength model. This results in a rate-dependent limiting tensile strain criterion for first crack occurrence or nucleation. The adequacy of this criterion has been established (Ting and Shyam Sunder, 1985b, 1986b) with the help of Gold's (1972) experimental data on the time to first crack occurrence in pure polycrystalline ice during compressive creep tests.

The compressive stress at which the first crack nucleates is the desired yield stress, }\sigma_{\text{cm}}\text{. Once this stress is reached, the material continues to sustain compressive load but loses its ability to carry tensile loads in the transverse direction if applied. This is a realistic modeling assumption that is often used to describe the behavior of concrete (ASCE, 1982). Application of the cracking criterion to constant strainrate tests in uniaxial compression results in the following mathematical model for the yield stress:

\begin{align*}
\frac{1}{\sigma_{\text{cm}}} &= \frac{1}{\sigma_{\text{cm}}} + \frac{1}{\left[(\text{A/M}) \frac{\varepsilon^{1/\text{N}}}{\text{N}}\right]^2} \\
&\text{(14)}
\end{align*}

where }\sigma_{\text{cm}}\text{ is the maximum or constant value of yield stress in compression as the strainrate approaches infinity but it also represents }\dot{\varepsilon}_{\text{tt}}\text{ for a constant strainrate stress, }\sigma_{\text{tm}}\text{, which increases twice at the rate of a constant strainrate.}

Experimental }\dot{\varepsilon}_{\text{tt}}=10\text{ s}^{-1}\text{ is used to correct the theoretical value.
represents the fracture stress for strainrates greater than \( \dot{\varepsilon}_{tf} \). At these strainrates both the yield and fracture stresses are almost equal and an elastic-brittle fracture model for ice is adequate. The second term models the increase in yield stress as the strainrate increases from twice the transition value in tension. The factor of two is applied since incompressibility requires the lateral tensile strainrate to be half the compressive strainrate. Below this transition strainrate the material flows in compression without nucleating cracks to cause "yielding".

Experimental data for both pure and sea ice suggests that \( \dot{\varepsilon}_{Lc} = 10^{-2} \text{ s}^{-1} \) and \( \sigma_{ct} = 5 \text{ MPa} \). This stress value should be corrected for the presence of brine when applied to sea ice.

**Multiaxial model formulation**

A rate-sensitive and isotropic Drucker-Prager "failure" surface is used to describe the yield/fracture behavior of ice. The failure surface may be expressed as:

\[
f(\sigma) = p I_1 + \sqrt{J_2} - k
\]

where \( p \) and \( k \) are constants, and \( I_1 \) and \( J_2 \) are the first invariant of stress and the second invariant of deviatoric stress respectively. Thus, the model developed here considers the failure surface to be pressure sensitive although the flow behavior is considered to be pressure insensitive. The constants \( p \) and \( k \) may be derived from two uniaxial tests at constant strainrate, one in tension and the other in compression. The resulting equations are:

\[
p = \frac{1}{3} \left( \frac{\sigma_{cn}/\sigma_{tf}}{\sigma_{cn}/\sigma_{tf} - 1} \right)
\]

and

\[
k = \sigma_{cn} (p + 1/\sqrt{3}) \quad \text{or} \quad \sigma_{tf} (p + 1/\sqrt{3})
\]

Both \( p \) and \( k \) are functions of the effective strainrate. For strainrates less than the transition strainrate in tension, \( p \) equals zero and \( k \) is proportional to the effective stress for the continuum flow. Experimental data to develop an anisotropic model for the failure surface is not available at the present time.

For effective strainrates below the transition strainrate in uniaxial tension, ice flows as a continuum but does not "fail". The maximum stress state attainable at any given strainrate is given by Eq. (8) and termed the limiting flow
surface. When the effective strainrate exceeds the transition value in uniaxial compression, it is assumed that no flow can occur. Instantaneous elastic deformation is followed by fracture on the Drucker-Prager surface. At intermediate values of effective strainrate the limiting flow surface and the failure surface intersect. Whenever one of the principal stresses is tensile, the failure surface defines the maximum attainable stress state prior to fracture. The direction of cracking is assumed to be perpendicular to the largest principal tensile stress. When the state of stress is purely compressive, the limiting flow surface is attainable.

However, if the loading path crosses the "failure" surface, a crack is assumed to nucleate in the direction of the smallest compressive stress and no tensile stress can be sustained by the material in that direction if applied at a later time.

Haynes (1973) has compared several classical failure theories for brittle materials against his experimental data on the tensile strength of bubbly polycrystalline ice under triaxial stresses. None of the theories was able to adequately predict the data. However at approximately the same strainrate used by him, i.e., $10^{-5}$ s$^{-1}$, the model developed here indicates a ratio of $c_{on}$ to $c_{ef}$ equal to about 1.7. This appears to provide the best prediction of the measured data. These results are consistent with Schulson's (1984a, personal communication) work on the ductile to brittle transition in ice under compression if it is recognized that crack nucleation (not crack propagation) governs both the tensile fracture strength and the compressive yield stress.

CONCLUSIONS

This paper has presented a new and integrated constitutive theory for the continuum and yielding/fracture behavior of sea ice emphasizing its micromechanical interpretation. The model has been extensively verified against experimental data. A numerical solution algorithm has been developed to simulate arbitrary loading histories using the model. This has been implemented in a computer program based on the finite element method of analysis. Several boundary value problems involving ice-structure interaction have been solved using the computer code and assuming continuum behavior of ice as described in a companion paper at this conference (Shyam Sunder, 1986).

Current research at MIT is concerned with the incorporation of yielding/fracture in the numerical simulation of ice-structure interaction processes. An objective fracture energy release rate criterion is being developed to modify the "strength" based fracture criterion presented in this paper.

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Further research, both experimental and theoretical, is necessary to investigate the influence of (i) unloading/reloading and cyclic loading, and (ii) brine volume, or more generally porosity, on the multiaxial behavior of ice. The results of such research can be used to improve the model proposed here.

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