Abstract

Frictional sliding plays a fundamental role in the mechanics of ice, on scales small and large. Whether across opposing faces of microcracks within laboratory specimens, across strike-slip like features within the sea ice cover on the Arctic Ocean, or across faults within the crust of icy satellites of the Outer Solar System, sliding of ice upon itself is central to the process of brittle compressive failure. In that regard, ice is similar to rock and other brittle, crystalline materials. This review addresses that role and then focuses on the nature of friction per se. Both static friction and low-velocity (< 0.1 m s⁻¹) kinetic friction are described and then discussed in terms of the interaction of asperities that protrude from opposing surfaces and the underlying physical mechanisms. The review closes with comments on the compressive strength of ice, on the deformation of the arctic sea ice cover and on the tectonic evolution of Saturn’s Enceladus.
1.0 Introduction

Ice is an abundant and multifaceted material. It constitutes a large fraction of the crust of Jupiter’s Europa and of Saturn’s Enceladus and on Earth, in the form of ice sheets and glaciers, serves as a reservoir for more than 98% of terrestrial fresh water. Within the Arctic, the sea ice cover serves as a solar reflector and as a barrier to the transfer of heat and moisture from the ocean to the atmosphere and, as such, affects both local and global climate. From the engineering perspective, ice and icing challenge the operation of ships engaged in polar marine transportation and the safety of offshore structures employed in the exploration and harvesting of oil and gas from beneath ice-infested waters. It is a major factor as well in loads experienced by piers, docks and lighthouses situated in cold regions and in the stability of aircraft and integrity of power lines subjected to freezing rain. As a defense system, walls of ice have been used as barriers to the spreading of contaminants during the drilling of oil shale and, at the time of writing, are being built to prevent the escape of radioactive water from the site of the Fukushima nuclear power plant. On a smaller scale, micrometer-sized composite walls of ice and porous ceramics are created through the process of freeze casting of water-particulate slurries and are potentially useful as templates for biomedical devices. As an aid to transportation, roadways of ice once served as the medium for sliding heavy stones to the Forbidden City; today, they allow heavy vehicles to supply northern communities during winter. Ice serves also as a medium for recreation, through skating, skiing, curling and other activities. And then there are ice cubes, not only for cooling drinks but also for
cooling rooms. From the scientific perspective, particularly the mechanical behavior of crystalline materials to which this review relates, ice in its transparency and relatively coarse microstructure may be viewed as a model material for directly observing with the unaided eye cracks and other features that lead to brittle failure of rock and ceramics. And, apart from science and engineering, ice is occasionally the stuff poetry, music and art.

In this paper we review critically one aspect of its character; namely, the process of sliding slowly upon itself. As will become apparent, self-sliding is a basic step in brittle compressive failure, which in turn is fundamental to limiting the build-up of environmental loads on engineered structures when sheets of floating ice are pushed by wind against their sides. Sliding plays a major role as well in the break-up of the winter sea ice cover on the Arctic Ocean and in the attendant release of heat and moisture to the atmosphere. On the extraterrestrial scale, frictional sliding participates in the tectonic evolution of Enceladus and of other icy satellites, and through heat generated may account for episodic plumes of water vapour mixed with ice that are emitted through faults within Encaladus’ crust.

The paper is organized as follows: Section 2 offers a short account of the relationship between friction and fracture; Section 3 presents the salient characteristic of both static and kinetic friction; Section 4 discusses physical mechanisms; and Section 5 describes some implications re. ice mechanics. In focusing on friction, the paper expands upon a short treatment of the subject that is given in a recent book.\(^1\)

2. Relationship between friction and fracture: lessons from ice
It is perhaps not obvious that friction and fracture are related. Friction is a measure of resistance to relative movement across an interface, whilst fracture is the separation into two or more pieces of a body under load. Yet, when cognizance is taken of fracture mechanisms – specifically, of brittle compressive failure – the relationship becomes apparent.

Modern theory holds that brittle failure under compression is a multi-step process. As load is applied, cracks initiate and then propagate, interacting in the end to form a set of macroscopic faults across which post-terminal deformation becomes localized. A key step is crack sliding. Under the applied stress, opposing faces of inclined primary or parent cracks (that either pre-exist or are nucleated during loading) come into contact and then become displaced with respect to each other once the shear stress acting on the plane of the crack exceeds frictional resistance. The displacement generates tension at the crack tips and this leads to the initiation of secondary cracks that form out-of-plane extensions along the direction of loading. The extensions are termed wing cracks, Figure 1.

Displacement across the primary crack increases as the load rises, opening the wing-crack mouth and increasing the attendant mode-I stress intensity factor to the point that the wings grow, albeit in a stable manner as opposed to the unstable manner of crack growth under an applied tensile load. The energy for wing-crack growth is derived from external work on the body. Wing cracks eventually interact with each other and with free surfaces, culminating in a series of splits that lead to terminal failure, Figure 2. Under uniaxial loading, wing-crack length $l$ relative to the half-length $c$ of the primary, sliding crack is governed by the coefficient of kinetic friction $\mu_k$ and by the plane strain fracture toughness $K_{IC}$ and is given by the relationship:

$$l / c = \frac{2\mu_k}{K_{IC}}$$
\[
\frac{\ell}{c} \approx \frac{\sigma^2 \pi c (1 - \mu_k)^2}{4K_{lc}^2}
\]  

(1)

for \( \mu_k < 1 \), where \( \sigma \) denotes the applied compressive stress. Equation (1) accounts well for the growth of wing cracks in specimens of ice deformed under controlled conditions in the laboratory,\textsuperscript{10, 11} and from the geometry of wing-like cracks within the winter sea ice cover, Figure 3, it allows a reasonable estimate as well of wind-induced stresses within the cover\textsuperscript{12, 13}.

In permitting wing cracks to be seen, ice played a significant role in qualifying the frictional sliding mechanism. Earlier work revealed out-of-plane-extensions of the kind that had been predicted\textsuperscript{2-4, 6, 14, 15}, but was based on the study of crack-like slots that had been machined into brittle materials. What ice allowed for the first time was the observation via high-speed photography of natural cracks at play. There was no concern that the wings were artifacts of specimen preparation, as there was for rock\textsuperscript{16}, for owing to the optical transparency and the relatively large grain size (1-10 mm) of ice, there was no need for post-test sectioning of the material. Wing cracks were visible to the unaided eye.

Sliding also occurs under moderate confinement, but failure occurs through a somewhat different although related mechanism. Secondary cracks still form, not simply as pairs of wings but (owing to non-uniform sliding) as closely spaced sets on one side of the primary, sliding crack\textsuperscript{17}. The secondary cracks create sets of microplates, fixed on one end and free on the other, Figure 4. Frictional drag across the free end causes the plates to
bend and then break in a domino-like manner, analogous to the failure of teeth in a comb under a sliding thumb. The “comb crack” mechanism accounts for the formation of frictional or Coulombic shear faults, an example of which is shown in Figure 4a. From an analysis based upon mixed mode-I and mode-II loading that includes frictional resistance from both the free end and the sides of the microplates, Renshaw and Schulson\textsuperscript{8} derived a relationship for the failure stress $\sigma_f$, given non-dimensionally by:

$$\Sigma = \frac{\sigma_f}{K_{lc}} = \frac{\sqrt{e}}{\frac{2}{\sqrt{1 + \left(1 - \frac{1 + R}{1 - R}\right)^{2/3}} - 1}} \left(1 + 3\mu_k^2\alpha^2(1 - R)^2\right)^{1/2}$$

where $R$ denotes the ratio of the least to the greatest compressive stress ($R = \sigma_3/\sigma_1$) and $\alpha$ denotes the slenderness ratio $\alpha = \frac{h}{w}$ of the microplates (Fig. 4d). Equation (2) accounts well for the brittle compressive strength of moderately confined ice (defined below), but, owing to the dominance of axial splitting under very low confinement, is limited to around $R > 0.02$. The model accounts, too, for the strength of a variety of rock and minerals\textsuperscript{8}. The greater is the coefficient of friction, the greater is the compressive strength.

Another instance where friction and fracture are related is the transition from brittle to ductile behavior. At root, the transition marks a balance in the micromechanical competition at crack tips between two processes\textsuperscript{18}: the intensification of internal stress through crack sliding and the relaxation of internal stress through creep. A physical model has been developed\textsuperscript{8,18} that incorporates the resistance to creep, to crack
propagation and to frictional sliding and, when expressed in terms of a critical strain rate \( \dot{\varepsilon}_{kc} \), is given by the relationship (for confined, compressive loading):

\[
\dot{\varepsilon}_{kc} = \frac{25 B K^3}{d^2 \left[ (1 - R) - \mu_k (1 + R) \right]}
\]

where \( B \) denotes a temperature-dependent parameter that relates creep rate to applied stress (\( \dot{\varepsilon} = B \sigma^n \) where \( n=3 \)) and \( d \) denotes crack size (set within virgin material by grain size and within cracked bodies, by the largest stress concentrator). The greater is the coefficient of kinetic friction, the lower is the rate of stress intensification; correspondingly the greater is the applied strain rate that marks the transition.

Friction also plays a major role in another transition; namely, from frictional or Coulombic (C) faulting to non-frictional or plastic (P) faulting \(^{19-21}\). This transition is governed by adiabatic heating and sets in when ice is rapidly loaded under a relatively high degree of triaxial confinement; i.e., when the hydrostatic component of the stress state becomes sufficiently great to suppress sliding while the non-zero deviatoric component activates dislocation slip. P-faulting is not strictly a mode of brittle failure, in that crack growth and interaction are not involved, although it manifests brittle-like character in that load-bearing ability is suddenly reduced through the localization of inelastic deformation within one or more narrow bands—oriented in this case close to planes of the greatest applied shear stress. Figure 5 shows an example. The important point from the perspective of this paper is that the critical level of confinement \( R_c \).
needed to suppress sliding and to activate the C -> P transition is governed solely by the coefficient of kinetic friction $^{19,20}$:

$$R_c = \left[ \left( \frac{\mu_k^2}{1 + \mu_k} \right)^2 + \mu_k \right]^2.$$  \hspace{1cm} (4)

The greater the coefficient of friction, the lower is the critical confinement. Interestingly, there is now evidence that a similar transition may occur within Earth’s crust $^{22}$.

The importance of friction in brittle compressive failure is reflected as well in the large effect of confinement on compressive strength. For instance, when the confining stress is only 10% of the applied stress, the compressive strength of ice (taken as the most compressive applied stress at failure) increases by ~100%; and when the confining stress is 20% of the applied stress, the strength increases by 400% or more, Figure 6 (for review see Schulson and Duval$^1$). Were friction not important, the strength in the two cases just noted would have increased by about 10% and 20%, respectively. The sensitivity $q$ to confinement (i.e., the slope of the rising branch of the brittle compressive failure envelope where strength is limited by Coulombic faulting, Fig.6) is given by $q = R_c^{-1}$ and thus by the relationship $^{23}$:

$$q = \frac{d \sigma_1}{d \sigma_3} = \left[ \left( \frac{\mu_k^2}{1 + \mu_k} \right)^2 + \mu_k \right]^2.$$  \hspace{1cm} (5)

Again, the coefficient of friction is the sole factor at play.
The models described above are independent of spatial scale. Although verified through experiments in the laboratory, the question is: how well do they work on the engineering and geophysical scales? Schulson and Duval\(^1\) consider this question at length and conclude that the evidence, albeit limited, points strongly in the direction of scale-independence. From the perspective of this paper an important observation \(^{24}\) is the slope of the failure envelope that bounds in-situ measurements of the stress state within the sea ice cover on the Arctic Ocean. That slope is closely similar to the slope of the low-confinement branch of the brittle compressive failure envelope generated in the laboratory from measurements made on specimens of first-year sea ice harvested from the winter ice cover \(^{23}\). The similarity indicates that the coefficient of friction is essentially independent of spatial scale.

3. Friction of ice: observations

We turn now to friction per se of ice sliding upon itself. In the instances described above and in the examples discussed Section 5, sliding occurs relatively slowly (\( V_x \leq 0.1 \) m s\(^{-1}\)). We limit the review, therefore, to relatively slow sliding.

3.1 Static friction

Static friction is greater than kinetic friction. This difference first became clear for ice through early experiments by Bowden and Hughes \(^{25}\) who noted that when surfaces of relatively warm ice were allowed to stand in contact for a few minutes, the shear force to initiate motion was much greater than the force to sustain movement. Bowden and Hughes did not quantify the effect, although they reported that it was less significant in
colder ice. More recently, Lishman et al. ²⁶ found from a series of slide-hold-slide (SHS) experiments on salt-water ice that static friction increased with the logarithm of holding time under an applied load normal to the interface, being greater, for instance, by about a factor of two after holding for $10^3$ s at $-10^\circ$C under a normal stress of 50 kPa. Schulson and Fortt ²⁷, ²⁸ confirmed this behavior.

Figure 7 illustrates the effect. The curves shown were generated during SHS experiments ²⁷, ²⁸ that were performed in the laboratory at $-10^\circ$C using a double-shear device, on both freshwater, granular ice of no crystallographic texture and columnar-grained, first-year sea ice of salinity $\approx 5$ parts per thousand that possessed the S2 growth texture. (Appendix 1 describes the microstructure of ice.) The long axes of the columnar grains of the sea ice were oriented as they would be in a natural cover undergoing the kind of strike-slip-like deformation described by Marko and Thomson ²⁹ and by Schulson ¹³; i.e., perpendicular to both the direction of normal load and the direction of sliding.

Immediately prior to sliding, the surfaces of both materials had been freshly prepared by milling, to a roughness of $R_a < 1\ \mu m$. The SHS curves of Figure 7 are presented in terms of the ratio of the global shear stress to the global normal stress $\tau / \sigma_n$ acting on the interface (i.e., the ratio of the measured shear force to the applied normal force, where each force was divided by the apparent area of contact) versus actuator displacement, at actuator velocities from $10^{-6}$ to $10^{-4}$ m s$^{-1}$. The normal stress was constant at $\sigma_n = 60$ kPa and the holding time ranged from 1 s to $10^4$ s. The inserts show the response after holding 100 s. Upon holding under normal load, the shear stress for both kinds of ice, first increases and then reaches a maximum $\tau_{\text{max}}$, followed by a rapid descent to a “steady-state” level. The descent begins once sliding resumes (at an initial velocity greater than
that applied by the actuator of the loading system) and is complete within a sliding
distance of the order of tens of micrometers once the rate of sliding reaches the actuator
velocity. The coefficient of static friction \( \mu_s \) is defined by the ratio:

\[
\mu_s = \frac{\tau_{\text{max}}}{\sigma_n}.
\]  

Figure 7 shows that \( \mu_s \) increases with holding time \( t_h \). In comparison, the steady-state
level of shear stress and the attendant coefficient of kinetic friction \( \mu_k \) exhibits little
change upon holding.

Figure 8 quantifies this behavior. No effect is detected until holding time exceeds a
threshold period \( t_l \) that decreases with increasing velocity, from \( t_l \sim 30 \text{ s} \) at \( V_s = 10^{-6} \text{ m s}^{-1} \)
to \( t_l \sim 3 \text{ s} \) at \( V_s = 10^{-5} \text{ m s}^{-1} \) to \( t_l \sim 0.3 \text{ s} \) at \( V_s = 10^{-4} \text{ m s}^{-1} \). Once the threshold is exceeded, the
coefficient of friction \( \mu_s \) increases with the logarithm of holding time \( t_h \) for \( t_l < t_h < t_u \)
(where \( t_u \) denotes an upper limit of \( \sim 10^3 - 10^4 \text{ s} \)) and may be described by the
relationship:

\[
\mu = \mu_s - \mu_k = \beta \log_{10} \left( \frac{t_h}{t_l} \right)
\]  

where \( \beta \) is termed static strengthening; for \( t_h > t_u \) the strengthening begins to level off.
The static strengthening coefficient \( \beta \) is independent of the sliding velocity over the
range explored and has the value \( \beta = 0.3 \pm 0.03 \text{ at } -10 \text{ °C} \).
In addition to the log time description, static strengthening may be described reasonably well by the power law:

$$\Lambda \mu = A t_k^m$$  \hspace{1cm} (8)

where $A$ and $m$ are constants. Under the conditions described in the preceding two paragraphs, the exponent has the value $m=0.5\pm0.1$ for $t_i \leq t_h \leq t_u$; for $t_h > t_u$, $m < 0.2$. We return later to this simpler description to evaluate a physical model.

Temperature exerts an effect$^{27}$. Upon cooling from -10 °C to -75 °C, the strengthening coefficient falls by about a factor of three, to $\beta=0.1$; and upon cooling to -100 and to -175 °C the coefficient falls still further to below 0.03, the limit of detectability$^{24}$ in the experiments cited.

Sea ice cannot be distinguished from granular, freshwater ice$^{28}$, at least at temperatures of -10 and -30 °C. This suggests either that the difference in microstructure compensates for hidden effects of salinity and texture or, more likely based upon observations noted below, that microstructure is not a major factor in static strengthening.

The behavior just described was observed in the laboratory from a study of relatively small (cm-sized) specimens, again raising the question: is size a factor here? Consider two other SHS studies, both performed on meter-sized blocks of ice. Lishman et al.$^{26,30}$ performed tests in an ice basin on saline ice (7 ppt salinity) at -10 °C held under normal stress of $\sigma_n = 10$ kPa for periods up to $10^3$ s, sliding at $V_s = 8\times10^{-3}$ m s$^{-1}$. They obtained the value $\beta=0.48$, albeit accompanied by a larger degree of scatter. The other study was performed on the sea ice cover in Svalbard. There, Sukhoruhov and Loset$^{31}$ held blocks
of first-year sea ice under a normal stress of $\sigma_n = 4.2 \pm 0.6$ kPa at temperatures from -18 to -7 °C for periods from 5 to $\sim 10^3$ s, sliding intermittently at $V_s = 4 \times 10^{-2}$ m s$^{-1}$. They found for both dry and wet (submerged) surfaces the lower value $\beta = 0.19$, again accompanied by a relatively large degree of scatter. Given that larger specimens led to both a greater and a lesser $\beta$-value from that obtained with smaller specimens, it seems likely that the differences are related more to experimental scatter than to size per se.

Static strengthening is not limited to holding ice against itself, nor to ice. It has been observed when ice is pressed against both PMMA and steel at temperatures from -13.5 to -3.5 °C. In those cases, the value of the strengthening coefficient appears to be lower (i.e., $\beta \sim 0.1$ (our estimate) vs. $\beta = 0.30$), although this point requires further attention. The effect is found also in cold metals, in paper, in glassy polymers and in rock. In those cases, $\beta$ even is lower than in warm ice, by about an order of magnitude.

A point not examined is the degree to which statically strengthened interfaces develop cohesive strength. Cohesion is expected in warm ice and is discussed briefly in Section 4.1, although confirmatory pull-apart tests remain to be performed. Missing, too, are experiments on the role of normal load. Presumably, static strengthening increases with increasing stress, although experiments are needed to explore and then to quantify that point.

3.2 Kinetic friction

3.2.1 Stick-slip

Once static friction has been overcome sliding commences. At that point the frictional resistance decreases (except at temperatures sufficiently low to prevent static
strengthening) and tends toward a level that oscillates about a mean, in some cases by as much as ~50% but more commonly by less than 10%. The oscillation is a manifestation of a dynamic instability termed stick-slip and has been observed in a variety of scenarios: in the laboratory as either warm or cold ice slides over itself, slowly or quickly, over either a smooth or a rough interface; in an ice basin as multi-meter slabs of saline ice are pushed past a floating ice sheet; in the field as either wet or dry meter-sized blocks of sea ice are pulled over parent ice sheets; and on the Arctic Ocean as floes of sea ice either push into a developing ridge or ride over themselves to create rafts, leading to icequakes. Stick-slip is observed also when warm ice slides against steel and against PMMA. Presumably, although this point has not been explored, stick-slip is accompanied by miroseismicity.

Stick-slip in ice has not been studied systematically. Where some attention has been given, it was noted that the behavior depends upon sliding velocity and temperature, tending toward higher velocity with decreasing temperature. Specifically, from experiments in the laboratory Kennedy et al. reported stick-slip at -3 °C at velocities as low as \( V_s = 10^{-5} \) m s\(^{-1}\) and at -10 °C at \( V_s > 5 \times 10^{-5} \) m s\(^{-1}\). Similarly, Maeno et al. reported that stick-slip tended to higher sliding velocities (over the range \( 10^{-5} \) to \( 10^{-3} \) m s\(^{-1}\)) as temperature decreased from -1 to -27 °C. A quantitative relationship and physical model, however, remain to be developed.

Stick slip depends not only on the material system, but also on the compliance of the loading device. The principle is that when the decrease in frictional resistance per unit of displacement exceeds the stiffness of the loading system, the resulting imbalance in force causes the slider to accelerate, leading to unstable slip. Pritchard et al. offer a model,
developed upon pulling blocks of sea ice across the surface of a parent ice cover using a
4.8 mm diameter steel cable—similar to models presented by Persson \textsuperscript{52} for metals and by
Scholz \textsuperscript{53} for rock.

3.2.2 Definition of the coefficient of kinetic friction

Early work established that the mean shear resistance $F_s'$ is proportional to the normal
load $F_n'$ applied across the interface \textsuperscript{25}, \textsuperscript{54}. The coefficient is termed the coefficient of
kinetic friction, denoted $\mu_k$, and may be defined either by the ratio of forces and hence
stresses:

$$\mu_k = \frac{F_s'}{F_n'} = \frac{\tau_s}{\sigma_n}$$  \hspace{1cm} (9)

or by the derivative:

$$\mu_k = \frac{dF_s'}{dF_n'} = \frac{d\tau_s}{d\sigma_n}$$  \hspace{1cm} (10)

where $\tau_s$ denotes the global shear stress; $\sigma_n$ again denotes the global normal stress
acting on the interface. The direct ratio (Equation 9) includes cohesion which, if
significant, leads to an apparent reduction in the coefficient of kinetic friction with
increasing normal stress, evident, for instance, in measurements by Repetto-Llamazares
et al. \textsuperscript{55} of sliding across ruptured freeze bonds in saline ice. The derivative (Equation 10)
requires measurements over a range of normal stresses, but has the advantage of
removing cohesion from the coefficient. Under many combinations of velocity and
temperature cohesion is small \textsuperscript{27} and so the two definitions give essentially the same value. On the other hand, upon sliding very slowly at high temperature the difference in definition becomes important (more below).

Concerning the $\tau - \sigma_n$ relationship, under low normal stresses $\sigma_n < 2 \text{ MPa}$ – i.e., under normal stresses low enough (defined elsewhere \textsuperscript{27}) to avoid brittle failure – the shear resistance increases linearly with normal stress within experimental scatter. This is evident from measurements made using double-shear apparatus over relatively large ranges of velocity ($-10^{-8}$ to $10^{-2} \text{ m s}^{-1}$) and temperature (-196 to -3 °C) upon sliding across both smooth surfaces created by either milling or microtoming and rough surfaces (defined below) created by shear faulting \textsuperscript{25, 27, 39, 51}. Figure 9 shows examples from experiments at -175 °C (98 K), -140 °C (133 K), -100 °C (173 K), -50 °C (223 K) and at -10 °C (263 K). In other words, over the ranges of experimental conditions noted above, ice obeys Coulomb’s friction law:

\begin{equation}
\tau_s = \tau_o + \mu_k \sigma_n
\end{equation}

where $\tau_o$ denotes cohesive strength. Cohesion makes only a small contribution \textsuperscript{27} except, as already noted, at high temperature and very low sliding velocity (more below). In comparison, under higher normal stresses ($\sigma_n = 5$-250 MPa), there is evidence of a non-linear relationship of the form $\tau \propto \sigma_n^{2/3}$, derived from sliding experiments on shear faults \textsuperscript{42, 56-58} and on saw cuts \textsuperscript{40} performed using pressure cells. Whether the difference in functionality reflects a difference in the underlying physical process or the creation of
additional microfractures induced under the higher stresses imparted by pressure cells remains to be determined.

3.2.3 Dependence of $\mu_k$ upon velocity

Figure 10 shows the coefficient of kinetic friction for fresh-water ice sliding slowly over itself across smooth surfaces (roughness $R_a$ (O[\mu m])) at velocities from $5 \times 10^{-8}$ to $1 \times 10^{-3}$ m s$^{-1}$ at temperatures from -175 °C (98 K) to -10 °C (263 K). The data were obtained upon sliding a distance of 2 mm using a double-shear device and were obtained from the derivative (Equation 10). (Experiments in which the sliding distance ranged from almost nothing to 20 mm showed that “steady state” had been attained after sliding ~1 mm.) To discern significant effects, the experimental data from which Figure 10 was constructed had first been subjected to statistical analysis by calculating $p$-values, rejecting null hypotheses for $p < 0.05$. Such analysis showed that minor trends apparent in the data had no significance.

The friction coefficient depends significantly upon velocity both at higher temperatures ($T \geq -50$ °C) and at the lowest temperature explored (-175 °C), but not at intermediate temperatures (-100 and -140 °C). At higher temperatures, the coefficient exhibits dual behavior; namely, positive dependence at lower velocities ($V_s \leq 10^{-5}$ m s$^{-1}$), termed velocity-strengthening, where over the range noted above the coefficient increases by about a factor of two; and negative dependence at higher velocities, termed velocity-weakening, where the coefficient decreases by a factor of four or even more when the upper limit is raised to $5 \times 10^{-2}$ m s$^{-1}$. At the lowest temperature (-175 °C) the
coefficient increases with increasing velocity over the range explored, by about a factor of two. Velocity-strengthening can be described by the power-law:

\[ \mu_k = aV_s^m \]  \hspace{1cm} (12)

and velocity-weakening, by the logarithmic function:

\[ \mu_k = \mu_{k,d}(1 - \gamma \log \frac{V_s}{V_1^*}) \]  \hspace{1cm} (13)

for \( V_1^* \leq V_s \leq V_2^* \) where \( \mu_{k,d} \) denotes the coefficient of friction for dry sliding (more below) and the parameter \( \gamma = 1/\log(V_2^*/V_1^*) \) where \( V_1^* \) denotes the velocity that marks the maximum coefficient and \( V_2^* \), the velocity that marks the lowest coefficient measured in the experiments. (We relate \( V_1^* \) and \( V_2^* \) to a physical process in Section 4.2.) Table 1 lists parametric values.

Table 1: Experimentally-derived values\(^{27}\) for the constants in Equations 12 and 13 (for velocity expressed in units of m s\(^{-1}\)).

<table>
<thead>
<tr>
<th>Temperature °C</th>
<th>Parameter</th>
<th>( a ) (m(^{-m}) m(^{m}))</th>
<th>( m )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td></td>
<td>3.40</td>
<td>0.15</td>
<td>~1/3</td>
</tr>
<tr>
<td>-50</td>
<td></td>
<td>3.33</td>
<td>0.13</td>
<td>~1/3</td>
</tr>
<tr>
<td>-175 (lower velocity)</td>
<td></td>
<td>4.61</td>
<td>0.15</td>
<td>--</td>
</tr>
</tbody>
</table>
The friction coefficient, of course, cannot be lower than zero and so the logarithmic weakening just described must cease at some point. Based upon experiments at higher velocities, by Oksanen and Keinonen\textsuperscript{59}, the coefficient at -15 °C, for instance, reaches a minimum value of around 0.02 at a velocity of ~0.1 m s\textsuperscript{-1}, set by the viscosity of water produced through frictional heating. Beyond that velocity, the coefficient of friction increases. We return to this point in Section 4.2.

Dual behavior is not limited to freshwater ice sliding across a smooth \((R_a \sim 1\mu m)\) interface, nor is it limited to ice on ice or even to ice. Strengthening and weakening are evident upon sliding across smooth surfaces in saline ice\textsuperscript{41} and upon sliding across natural Coulombic shear faults, of roughness three orders of magnitude greater \((R_a \sim 1mm)\)\textsuperscript{39}, in both freshwater ice and first-year sea ice Figure 11. Dual behavior also characterizes ice/glass and ice/granite\textsuperscript{60} as well as halite\textsuperscript{61,62}, quartz\textsuperscript{63,64}, serpentine\textsuperscript{65,66} and granite\textsuperscript{67} when sliding slowly over themselves.

Returning briefly to the definition of the coefficient of kinetic friction (Equations (9) and (10)), it is important to note that when the parameter is defined in terms of the direct ratio of shear force to normal force via Equation (9), thereby incorporating significant cohesive forces, velocity strengthening is not evident when warm ice slides very slowly upon itself. Maeno et al.\textsuperscript{51} and Maeno and Arakawa\textsuperscript{43} and Kennedy et al.\textsuperscript{41} followed that practice in sliding ice at -10 °C over the range of velocity from \(10^{-7}\) to \(10^{-1}\) m s\textsuperscript{-1} and did not report strengthening at the lower velocities. Instead, they observed weakening over
the entire range. However, upon sliding ice slowly at the lower temperature of -40 °C, where less cohesion is expected, Kennedy et al.\textsuperscript{41} observed both strengthening and weakening in both freshwater ice and saline ice.

Turning to field experiments in the Arctic and to measurements made using meter-sized blocks of sea ice sliding over a natural cover, somewhat different behavior has been reported. Upon sliding blocks across natural (i.e. unprepared) surfaces at velocities within the range $5 \times 10^{-3}$ to $1 \times 10^{-1}$ m s\(^{-1}\) (i.e., at velocities over the higher end of the “low-velocity” range), Pritchard et al.\textsuperscript{46} and later Sukhorukov and Lose\textsuperscript{31} found that within experimental scatter velocity appeared to display no systematic effect on the sliding resistance. Over the range of velocity noted and for ice temperature between about -2 to -6 °C and air temperature between about -2 and -12 °C, the coefficient of kinetic friction for snow-free interfaces was measured\textsuperscript{31} to be $\mu_k = 0.45 \pm 0.05$, independent of the whether bottom ice slid across top ice or top across top. This compares with the lower values $\mu_k = 0.1$ at $5 \times 10^{-3}$ m s\(^{-1}\) and $\mu_k = 0.04$ at $5 \times 10^{-2}$ m s\(^{-1}\) for saline ice at -10 °C sliding over itself in the laboratory\textsuperscript{41}. However, when the sliding surfaces of the field tests were run in by sliding blocks multiple times across the same path, the coefficient of kinetic friction decreased with increasing velocity and tended toward the value $\mu_k = 0.05-0.1$ at $4 \times 10^{-2}$ m s\(^{-1}\), scaling as $V_s^{-1/2}$. The running in may have had two effects: it probably allowed the opposing surfaces to better fit together by reducing natural topography\textsuperscript{31} and it may have reduced the roughness of the interface. In other words, allowing for the small differences in temperature, the field and lab experiments indicate that the coefficients derived from smooth surfaces, whether on relatively small or large
specimens, are essentially the same within experimental scatter, implying that size is not a factor here.

The field tests revealed another point. Interfaces submerged in sea water offered no significant difference from dry interfaces in sliding resistance\(^31\). That point, when combined with the absence of a detectable effect on static strengthening noted above, means that the fundamental physical processes underlying static and kinetic friction are essentially insensitive to the presence of external water under insignificant fluid pressure.

A note on cohesion is worth mentioning. When Coulombic shear faults were introduced into relatively warm (at -10 °C), columnar-grained specimens of both freshwater ice and saline ice through proportional, biaxial loading, for the purpose of subsequent sliding experiments, it was found\(^39\) upon removing the ice from the loading system that the faulted specimens did not come apart. Prior to unloading, the faults had slid a few millimeters in about one second; i.e., at a velocity of \(\sim 10^{-3} - 10^{-2}\) m s\(^{-1}\). The faults, in other words, possessed cohesive strength. We return to this observation in Section 4.2.

3.2.4 Rate-state friction

To expand further upon the effect of sliding velocity, consider the results of experiments in which the sliding velocity was changed “instantaneously”\(^26,30,68\). In rock mechanics, experiments of that kind are viewed within the context of rate-state friction, described by the phenomenological relationship\(^69,70\):

\[
\mu_k = \mu_0 + A \ln \left( \frac{V_s}{V'_s} \right) + D \ln \left( \frac{V'_s \theta}{D_c} \right) \tag{14}
\]
where the first term, $\mu_0$, denotes the nominal coefficient of friction for steady-state sliding at velocity $V_s^*$, the second term describes the effect of velocity, and the third term describes the evolution of the state of the material; $A$ and $D$ are empirical constants and $D_c$ denotes a critical slip distance over which friction is assumed to decay to some fraction of the new steady-state value following the instantaneous change in rate; $\theta$ denotes a state variable that changes with time $t$ and is defined by the relationship:

$$\frac{d\theta}{dt} = 1 - \frac{V_s \theta}{D_c}.$$  \hspace{1cm} (15)

At steady state when evolution has ceased $d\theta/dt = 0$ in which case Equation (14) reduces to:

$$\mu_k = \mu_0 + (A - D) \ln \left( \frac{V_2}{V_1} \right).$$  \hspace{1cm} (16)

where the difference ($A-D$) is derived experimentally from the relationship:

$$(A - D) = \left( \mu_2 - \mu_1 \right) / \ln(V_2 / V_1)$$  \hspace{1cm} (17)

where $V_1$ and $V_2$, respectively, denote the sliding velocities before and after the velocity change, and $\mu_1$ and $\mu_2$, the steady-state coefficients of kinetic friction before and after the
velocity change. Positive values of (A-D) imply velocity-strengthening and negative values, velocity-weakening.

Table 2 lists values for (A-D) derived from instantaneous velocity change tests for sliding a few mm ice across natural Coulombic faults in freshwater ice at -10 °C \(^6\). Positive and negative values are obtained, respectively, when the velocity change is made within the velocity-strengthening and the velocity-weakening regimes, independent of the sense (up or down) of the change. When scatter in the data is taken into account, the values so derived are closely similar to values derived from data generated during constant-velocity tests. The value listed in Table 2 for the velocity-weakening regime \(((A-D) = -0.13 \pm 0.03)\) is about a factor of two more negative than the value \((A-D) = -0.07\) obtained from saline ice\(^2\), also whilst sliding at -10 °C within the velocity-weakening regime. Whether the difference reflects the difference in materials and/or in surface conditions or simply scatter in the data remains to be determined.

Interestingly, values of (A-D) for ice are greater than values derived for rock. For instance, for Westerly granite the values differ by about a factor from 4 to 80 within the velocity-strengthening regime and by a factor from 30 to 160 within the velocity-weakening regime\(^7\). This difference, we suggest, reflects a difference in the strain-rate sensitivity \(m'\) of the flow stress \((\sigma \propto \dot{\varepsilon}^{m'})\) of the two materials: for ice \(^1 m' \sim 0.3\) at -10 °C (homologous temperature of \(T_h = 0.96\)); for granite \(^2\) \(m \sim 0.01\) at room temperature (\(T_h \sim 0.3\)).

In a related study on columnar-grained saline ice of ~10 ppt salinity and 930 kg m\(^{-3}\) density, Lishman et al.\(^7\) found, upon resuming sliding at a higher rate after holding under load for a period of time, that the friction coefficient at -10 °C decreased from the static
strengthening level and reached a steady-state level not after sliding a “critical distance”
as exhibited by rock 70 but after sliding a distance that increased with increasing slip rate.
This, they suggested, indicates that saline ice conforms more to the concept of a critical
slip time than to a critical slip distance and that effects of memory persist for about 10
seconds. The meaning of this observation remain to be clarified.

Table 2: Parametric values for Equation (17) obtained by sliding freshly formed
Coulombic shear faults in freshwater ice across themselves at -10 °C 68

<table>
<thead>
<tr>
<th>$V_1$ (m s$^{-1}$)</th>
<th>$V_2$ (m s$^{-1}$)</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>(A-D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8 \times 10^{-4}$</td>
<td>$8 \times 10^{-5}$</td>
<td>0.79</td>
<td>1.02</td>
<td>-0.10</td>
</tr>
<tr>
<td>$8 \times 10^{-3}$</td>
<td>$8 \times 10^{-4}$ m s$^{-1}$</td>
<td>1.12</td>
<td>0.76</td>
<td>-0.16</td>
</tr>
<tr>
<td>$8 \times 10^{-6}$</td>
<td>$8 \times 10^{-7}$</td>
<td>1.17</td>
<td>0.91</td>
<td>0.11</td>
</tr>
<tr>
<td>$8 \times 10^{-7}$</td>
<td>$8 \times 10^{-6}$</td>
<td>1.03</td>
<td>1.26</td>
<td>0.10</td>
</tr>
</tbody>
</table>

3.2.5 Dependence of $\mu_k$ upon temperature

In addition to velocity, temperature affects the coefficient of kinetic friction. The
effect, however, is less systematic (Figure 10). Statistical analysis 27, again based upon $p$-
values, revealed significant variation at the highest velocities, but not at the lowest. For
instance, at $V_s=10^{-3}$ m s$^{-1}$ $\mu_k$ increases monotonically with decreasing temperature and
brackets the complete range of values measured from smooth surfaces ($\mu_k = 0.15$ to
0.76). At the lower velocities of $V_s=10^{-4}$ and $10^{-5}$ m s$^{-1}$, $\mu_k$ first increases as temperature
decreases from -10 to -50 °C and then decreases as temperature decreases further to -100
°C; as temperature decreases still further to -175 °C, $\mu_k$ increases again. And at the lowest
velocity of $5 \times 10^{-8}$ m s$^{-1}$, $\mu_k$ increases upon decreasing temperature from -10 to -140 °C, but then decreases upon lowering temperature to -175 °C. Temperature in other words, has a rather complex effect.

3.2.6 Dependence of $\mu_k$ upon roughness: natural vs. prepared surfaces

The other factor that affects the coefficient of kinetic friction is roughness. Resistance to sliding across natural Coulombic faults, for instance, is greater by about a factor of two or more than upon sliding across freshly prepared, smooth surfaces\textsuperscript{27,39}. The expenditure of additional energy is attributed to ploughing and to displacement of fragments of ice that become embedded in the fault\textsuperscript{20,39,74}. Snow on ice – although creating a somewhat loosely granular interface – raises sliding resistance in a similar way\textsuperscript{25,46}, increasing for instance the coefficient from $\mu_k \approx 0.5$ to $\approx 0.7$ for sea ice sliding at velocities between 10 and 40 mm s$^{-1}$ across relatively warm natural covers\textsuperscript{46}.

Concerning the functional relationship, laboratory tests\textsuperscript{27} on sub-meter sized specimens in which the roughness ranged over three orders of magnitude ($0.4 \mu m \leq R_a \leq 1300 \mu m$) showed that at -10 °C at $V_s = 10^{-4}$ m s$^{-1}$ under a global normal stress of $\leq 2$ MPa the coefficient of friction scales as $\mu_k \propto R_a^{0.08}$. In other words, large though a factor of two or more is when comparing machined versus faulted surfaces, the exponent of 0.08 indicates that the coefficient of friction is relatively insensitive to roughness.

At first glance, that result may seem surprising. Yet when cognizance is taken of the behavior of other materials sliding upon themselves, particularly ductile metals, ice fits a general picture. In such cases, the coefficient of kinetic friction is governed less by roughness per se and more by plastic deformation at the level of surface asperities.
The real contact area is essentially independent of roughness as long as the near-surface material can deform plastically. Under the experimental conditions where the roughness of ice was explored, the material did deform plastically (more below).

3.2.7 Less important factors

Microstructure: There is little evidence to suggest that the microstructure of ice is an important factor. Over the experimental ranges of velocity and temperature noted above, neither grain size (1 to 8 mm), grain shape (equiaxed vs. columnar), crystallographic texture (randomly oriented vs. aligned c-axes) nor low-level salinity (~5 ppt) exhibits a statistically measureable effect on the resistance to slow sliding across smooth interfaces. Snow, too, although more resistive to sliding than is ice, shows negligible dependence on grain size. This implies that the physical processes underlying friction are relatively insensitive to the microstructure of the parent material, unlike the processes that govern certain bulk mechanical properties.

An exception appears to be hard, second phases. Golding et al. found from experiments on the two-phase ice I-magnesium sulphate (MgSO₄·11H₂O) eutectic system, of ~0.4 to 0.5 volume fraction hydrated sulphate, that relative to single-phase ice-I the composite, when sliding slowly upon itself at the very low temperature of -175 °C, has a coefficient of kinetic friction (derived as the derivative, from Equation 10) that is greater by about a factor of 1.4; also, at -175 °C little velocity dependence is apparent. At -10 °C, the ice-sulphate composite exhibits velocity-weakening over the entire range explored (5×10⁻⁸ to 1×10⁻³ m s⁻¹). Differences like these, and possibly others – including effects yet
to be measured of embedded quartz and other particles – may play a role in the friction of extraterrestrial icy bodies.

*Slip distance*: As already noted, once the distance of sliding exceeds ~1 mm on smooth interfaces of freshwater ice, the mean sliding resistance remains constant (although stick-slip persists), at least up to a distance of 20 mm. This point was established from experiments that were performed under two relatively extreme conditions: -175 °C/1 × 10^{-6} m s^{-1} and -10 °C/1 × 10^{-4} m s^{-1}. In comparison, there is some evidence, albeit mixed, of slip weakening upon sliding up to 8 mm across Coulombic shear faults in both freshwater ice and first-year sea ice at -40 °C and -10 °C over a range at velocities from 8 × 10^{-7} to 4 × 10^{-3} m s^{-1}, Figure 11. This is probably another indication of an effect already noted; namely, that running in of rougher interfaces eases sliding resistance.

*Normal stress parallel to sliding direction*: In modeling the failure of the arctic sea-ice cover, Schreyer et al. questioned whether, in addition to traction on the plane of material failure (i.e., in addition to the normal and shear components of the stress tensor mentioned above) the normal component of stress that is oriented parallel to the sliding plane \( \sigma_{np} \) affects sliding resistance. One could imagine that under all-compressive loading, this component might act to prevent either the formation or the propagation of surface cracks (more below). Experiments, however, detected no effect, at least within columnar grained, freshwater S2 ice sliding against itself across smooth surfaces at a velocity of 8 × 10^{-4} m s^{-1} at -10 °C over the range 0.2 \( \leq \sigma_{np} \leq 1.8 \) MPa.

*Reversal of sliding direction*: The coefficient of kinetic friction is insensitive to a single change in sliding direction. This point was established from an experiment in
which, for freshwater granular ice of 1 mm grain size against itself under a normal stress of 60 kPa at either -175 °C/10^{-6} m s^{-1} or -10 °C/10^{-4} m s^{-1}, the direction was instantaneously reversed after sliding 2 mm while maintaining the sliding speed. Steady state set in immediately once the frictional force (of opposite sign) reached the pre-reversal level. Whether multiple reversals would affect sliding resistance through surface fatigue is not known.

3.2.8 Microcracking and recrystallization

Microstructural changes accompany sliding. From visual inspection and from observations using optical microscopy, Kennedy et al. 41 reported that within the regime of velocity-weakening the surface of freshwater, granular ice changed from relatively clear to rather opaque upon sliding at -10 °C at velocities from 10^{-5} to 10^{-3} m s^{-1}, accompanied by fine fragmentation (i.e., ice powdered ice) and by cracking to depths of about one grain diameter. Similarly, Montagnat and Schulson 76 reported surface cracks, oriented normal to the direction of sliding and spaced by ~0.5 mm, that developed to a depth < 0.6 mm in fresh-water, columnar ice when sliding at velocities of 10^{-5} to 10^{-3} m s^{-1}; i.e., at velocities within the upper-end and lower-end, respectively, of the velocity-strengthening and velocity-weakening regimes. Damage was not easily detected in sea ice, not because of its absence, but, as noted in a recent study on the role of damage on elastic behavior 81, because of the opacity of the material. Microcracks presumably initiate from stress concentrators and then propagate under the action of tensile stresses, generated by tangential surface tractions that develop during sliding, when the attendant mode-I stress intensity factor $K_I$ reaches the critical level $K_{IC}$. The tensile stress $\sigma_t$
reaches a maximum behind interacting asperities (defined in Section 4) and is given by the relationship \(^{82}\):

\[
\sigma_t = H \left( \frac{1 - 2\nu}{3} + \frac{4 + \nu}{8} \pi \mu_k \right)
\]

and the mode-I stress intensity factor is given by:

\[
K_I = 1.12\sigma_t \sqrt{\pi c_o}
\]

where \(H\) denotes hardness and \(\nu\), Poisson’s ratio and \(c_o\), the length of the stress concentrator. Taking the values \(\nu = 0.3\) and \(\mu_k = 0.5\) (ignoring velocity dependence), then from Equations (18) and (19):

\[
c_o \sim 0.25 \left( \frac{K_{kc}}{H} \right)^2.
\]

Taking the values \(K_{kc} = 0.1\) MPa m\(^{0.5}\) (ref. 1) and \(10 \leq H \leq 30\) MPa \(^{60}\), we obtain the estimate \(3 \leq c_o \leq 30\umu\). Doubling or reducing the coefficient of friction by a factor of two either reduces or increases the size of the stress concentrator by about a factor of three, extending the possible range to around \(1 \leq c_o \leq 100\umu\). As will become apparent from the discussion below (Sections 4.1 and 4.2), this range, perhaps fortuitously, includes the expected size of asperities.
In addition to cracking, rubbed surfaces recrystallize. Barnes et al.60 noted a zone of recrystallization extending to a depth of ~0.2 mm after slowly sliding a single crystal of warm ice across granite. Similarly, Montagnat and Schulson 76 observed boundaries of new grains of ~ 300 \( \mu m \) diameter on the surfaces of freshwater ice rubbed at -10 °C after sliding across itself at 10\(^{-5}\) m s\(^{-1}\). Some of the boundaries were intersected by cracks, implying that recrystallization had occurred dynamically. The “driving force” for the transformation is probably strain energy in the form of crystal dislocations that develop within the near-surface regions where, during sliding, the combination of surface tractions and contact forces generate sufficient deviatoric stress to activate plastic flow.

Taken together, these observations imply that inelastic deformation during sliding extends to a significant depth beneath the surface. Part of the work of friction, in other words, is expended through inelastic deformation within a narrow zone bordering both sides of the sliding interface.

4. Physical mechanisms

To understand the salient characteristics of the slow sliding of ice upon itself—namely static/ageing strengthening of the coefficient of static friction and both velocity-strengthening and velocity-weakening of the coefficient of kinetic friction—it is important to appreciate that the real area of contact is considerably smaller than the nominal or apparent area 83, 84. Contact occurs not globally across the entire interface, but locally at asperities that protrude from the opposing surfaces, as sketched in Figure 12. It is these local sites that constitute the real area of contact. The interaction of asperities is then a major contributor to friction. Others processes also contribute, including the near-surface
inelastic deformation of the kind just described, deformation of third bodies created by fracture (fragments of ice), plowing and heating. In warmer ice, cohesion (i.e., the formation and breaking of chemical bonds) and melting also play a role.

4.1 Static strengthening/ageing

To account for the increasing coefficient of static friction upon holding under load under pseudo-stationary conditions, we invoke creep of asperities in contact. The analysis below is based upon one by Schulson and Fortt\textsuperscript{28} who, for simplicity, assumed a fixed number of contacts of circular cross-section and a single initial asperity size. Although details differ, in invoking asperity creep the analysis is similar to ones developed to account for the same effect in metals\textsuperscript{85}, polymers\textsuperscript{35} and rock\textsuperscript{38}.

Consider first the average asperity size and how this parameter can be derived from experimental observations. Asperities interact for a period of time $t_i$ where that time is the sum of the average time taken for contacts to slip past each other $t_s$ plus the time of holding under stress $t_h$; i.e.

$$t_i = t_s + t_h$$  \hspace{1cm} (21)

The slip time is related to the sliding velocity:

$$t_s = \frac{2a_o}{V_s}$$  \hspace{1cm} (22)

where $2a_o$ denotes the initial average contact diameter. Although the velocity may vary from point to point (as proposed in Section 4.2), for the present discussion $V_s$ is equated
to velocity applied prior to holding. Schulson and Fortt\textsuperscript{28} suggested that holding begins to exert a detectable effect on the resistance to sliding once the threshold period $t_h$ is comparable to the slip time; i.e.,

$$t_h = t_s \approx \frac{2a_o}{V_s}.$$ (23)

Thus:

$$2a_o \approx V_s t_s.$$ (24)

From Equation (24) and from the threshold times given in Section 3.1, the average diameter of the asperities in contact across the smooth ($R_a < 1 \mu m$) interfaces in both freshwater ice and sea ice described in that section is estimated to be $2a_o \approx 30 \times 10^{-6} = 3 \times 10^{-5} = 0.3 \times 10^{-4} \approx 30 \mu m$. This dimension is smaller than the grain size by about a factor of ten and smaller than the size and spacing of brine pockets in sea ice by about a factor of three, but is of the same order of magnitude as the estimated distance of sliding that is needed to re-establish steady-state following a period of holding and it is within the range 1-100 $\mu m$ that is suggested (from different methods of analysis) for other materials\textsuperscript{52}.

We are not aware of this threshold-based analysis being developed and applied for other materials, although we expect it to have applicability beyond ice. An example is granite. In work by Marone\textsuperscript{37}, Figure 1 of that work shows $\Delta\mu - \log t_h$ plots that extrapolate to $\Delta\mu = 0$ at threshold times of $\sim 0.3$ s and $\sim 3$ s for sliding at velocities of 10
\(\mu m\ s^{-1}\) and 1 \(\mu m\ s^{-1}\), respectively. From Equation (24) the average asperity diameter is derived to be \(2a_o \sim 3\mu m\).

Returning to ice, to estimate the real area of contact and the areal density of contact points, Schulson and Fortt\(^{28}\) assumed that the normal stress supported by each contact equals the hardness \(H\) of the ice, in keeping with modern theory\(^{83, 84}\). Accordingly, the total normal load \(F_n\) supported by \(N\) contacts is given by:

\[
F_n = NH\pi a_o^2 = \sigma_n A_a
\]  \hspace{1cm} (25)

where \(A_a\) denotes the apparent area of contact. Then from Equation (25):

\[
N = \frac{\sigma_n A_a}{H\pi a_o^2}.
\]  \hspace{1cm} (26)

The real area of contact \(A_r\) is given by the relationship:

\[
A_r = \frac{\sigma_n A_a}{H}
\]  \hspace{1cm} (27)

and is expected to increase with increasing normal stress. Upon ignoring for the present purpose the rate-dependence of hardness and taking the value \(10 \leq H \leq 30\) MPa\(^{60}\) and upon using \(\sigma_n = 60\) kPa, \(A_a = 41 \times 41\) mm\(^2\) from experiments\(^{28}\), Equations (26) and (27) give for the conditions of these experiments (-10 °C, both freshwater ice and sea ice) the
ranges $5 \times 10^3 \leq N \leq 14 \times 10^3$ and $0.002 \leq A_r / A_a \leq 0.006$. In other words, the analysis indicates that the real area of contact is less than 1% of the nominal area. The areal density of contacts is given by:

$$\rho_c = \frac{N}{A_a}$$  \hspace{1cm} (28)

and has the value $\rho_c = 5.6 \pm 3 \times 10^6 \text{ m}^{-2}$; correspondingly, the mean spacing of asperities is estimated to be $s_c \approx \rho_c^{1/2} = 400 \mu\text{m}$. This value of areal density agrees extraordinarily well with the value $5.5 \times 10^6 \text{ m}^{-2}$ that was measured by Hatton et al. upon sliding columnar grained saline ice (7.3 p.p.t. salinity) at -10°C at $1 \times 10^{-4} \text{ m s}^{-1}$ under a normal stress of 10 kPa.

This mean spacing ($400 \mu\text{m}$) is interesting. It is similar to the average diameter of $d = 300 \mu\text{m}$ of the dynamically recrystallized grains that were observed following the slow sliding of ice upon itself at -10°C. It is similar, too, to the average size of the recrystallized grains that formed upon sliding a single crystal of warm ice across granite (evident from the photograph in Figure 12 of Barnes et al.). Perhaps the boundaries of new grains sublime to the extent that the grains develop a dome-like cap of radius $r$ such that the contact diameter $2a_o$ is set by Hertzian mechanics, in which case (as can be shown from the elastic deformation of two spheres in contact) $a_o \sim Hr / 4E$ where $E$ denotes Young’s modulus. Noting that $r > d$ and taking $H=20 \text{ MPa}$ and $E=10 \text{ GPa}$, this interpretation implies that $2a_o > Hd / 2E > 10^{-3} d > 10^{-3} \times 400 \mu\text{m} > 0.4 \mu\text{m}$. In other words, the minimum size of $0.4 \mu\text{m}$ is not inconsistent with the size ($2a_o \sim 30 \mu\text{m}$) derived above from the threshold time.
To obtain the coefficient of static friction, the contact area of each asperity and the shear strength of the contact must be estimated. Consider first the contact area. As mentioned already, we imagine that asperities creep under load, shortening and broadening in the process and increasing the area of contact, thereby increasing the shear resistance to sliding. From conservation of volume, the relative rate at which the contact area increases $\frac{1}{a^2} \frac{da^2}{dt}$ is equal to minus the relative rate at which asperities shorten $\frac{1}{h} \frac{dh}{dt}$. The shortening rate equals the creep rate $\dot{\varepsilon}$ which, in turn, may be described most easily by a power-law $\dot{\varepsilon} = B\bar{\sigma}^n$ (a hyperbolic law that describes both low-stress and high-stress creep would be better, but is left for future development) where $B$ and $n$ are materials parameters; the parameter $\bar{\sigma}$ denotes the effective stress which decreases as the load-bearing area increases: it may be written as $\bar{\sigma} = \bar{\sigma}_o \left( \frac{a_n}{a} \right)^2$ where $\bar{\sigma}_o$ is the effective stress at the start of holding when $t_h = 0$. Writing $\left( \frac{a_n}{a} \right)^2 = \frac{h}{h_o}$ through conservation of volume, where $h_o$ is the initial height of the asperity, the creep rate may be given as:

$$-\frac{1}{h} \frac{dh}{dt} = B\bar{\sigma}_o^n \left( \frac{h}{h_o} \right)^n. \quad (29)$$

Upon rearranging, Equation (29) becomes:
Upon integrating Equation (30) and again writing \( \left( \frac{a}{a_o} \right)^2 = \frac{h}{h_o} \), the ratio of the current to the initial contact area is given by the relationship:

\[
\left( \frac{a}{a_o} \right)^2 = \left( \frac{h}{h_o} \right) = \left( 1 + nB\bar{\sigma} \right)^{\frac{1}{n}}.
\] (31)

During holding, asperities are loaded under a multiaxial state of stress; i.e., under a normal compressive stress, taken to be the hardness of ice, and under a local shear stress \( \tau_l = \mu_k H \) that was acting on the asperity before sliding was interrupted. This shear stress component relaxes over time through creep. The fraction retained may be approximated as \( \tau_r = (1 - \alpha) \tau_l = (1 - \alpha) \mu_k H \) where \( \alpha \) is a stress-relaxation factor that ranges from 0 \( \leq \alpha \leq 1 \). Assuming for simplicity that the asperity obeys the von Mises yield criterion, anisotropic though its inelastic behavior probably is, the effective stress may be given as:

\[
\bar{\sigma} = (H^2 + 3\tau_r^2)^{\frac{1}{2}} \left( \frac{a_o}{a} \right)^2.
\] (32)

Thus, the ratio of the current to the initial asperity contact area is given as:

\[
\left( \frac{a}{a_o} \right)^2 = \left( 1 + nB\bar{\sigma} \right) \left( 1 + 3(1 - \alpha)^2 \mu_k \right)^{\frac{1}{n}} t_k.
\] (33)
Next, consider the shear strength $\tau_y$ of the contact. It is that property that governs the coefficient of static friction. As the actuator is reactivated following the period of holding and traction is once again applied to the interface, tensile stress builds up within the contact zone. The tensile stress is limited by the tensile strength $\sigma_T$ of the ice which in turn limits the shear strength. Over the range of strain rate that was likely to develop at the relatively low actuator velocities (from $10^{-6}$ to $10^{-4}$ m s$^{-1}$) explored by Schulson and Fortt$^{28}$, tensile strength exhibits essentially rate-independence$^{1,87-90}$. The shear strength is expected to display the same character. Then, under the multiaxial stress state created by surface traction and normal loads, the shear strength of the contact is given by:

$$\tau_y = \left[ \left( \sigma_t - \left( \frac{H}{2} \right) \right)^2 - \left( \frac{H}{2} \right)^2 \right]^{\frac{1}{2}}.$$  \hspace{1cm} (34)

Correspondingly, the coefficient of static friction (defined as the ratio of the measured shear force to initiate sliding $F_s$ to the applied normal force $F_n$) is given by the relationship:

$$\mu_s = \frac{F_s}{F_n} = \frac{N \pi a^2 \tau_y}{N \pi a_o^2 H} = \frac{\tau_y}{H \left( \frac{a}{a_o} \right)^2}.$$  \hspace{1cm} (35)

Thus, from Equations (34) and (35):

$$\mu_s = \left[ \left( \frac{\sigma_t}{H} - 0.5 \right)^2 - 0.25 \right]^{\frac{1}{2}} \left[ 1 + nBH^n (1 + 3(1 - \alpha)^2 \mu_s^2) t_h \right]^{\frac{1}{n}}.$$  \hspace{1cm} (36)
(Note that $H$ has a negative value in the first term of Equation (36) to reflect the stress tensor and a positive value in the second term to reflect creep stress.)

We note that in incorporating tensile strength and creep resistance, the model might appear to distinguish between saltwater ice and freshwater ice, for saline ice creeps more rapidly\cite{91,92} and is weaker in tension\cite{93}. However, that appearance is deceptive. The difference noted in mechanical properties relates to the behavior of material in bulk form. In the form of asperities, little or no difference is expected, because brine pockets – i.e., the microstructural feature that accounts for the difference in bulk behavior – are probably absent owing to their size and spacing being greater than the average diameter of asperities.

Returning to the model, Equation (36) can be evaluated at $-10^\circ C$ where parametric values are available: $n=3$ and $B=4.3\times 10^{-7}$ MPa$^{-3}$ s$^{-1}$\cite{60}, $|H|=20$ MPa, the mid-range value from Barnes et al.\cite{60} and $\mu_k=0.5$ (present work); the value of the tensile strength is taken to be the strength of a single crystal\cite{1} $\sigma_t=6$ MPa; we assume that $\alpha=0.5$. Reducing from $\alpha=0.5$ to 0 raises the calculated coefficient by about 10%. From these values, Equation (36) gives the coefficient of static friction at $-10^\circ C$ as:

$$\mu_s = 0.6(1+0.01t_h)^{\frac{1}{3}}$$

(37)

for hold time $t_h$ in seconds. This relationship is shown as the dashed curve in Figure 8, for a sliding velocity of $10^{-5}$ m s$^{-1}$. Comparison with the data in that figure shows that for hold time $t_h \leq 1000$ s the model captures the experimental measurements reasonably
well, but not completely. At the lower velocity of $10^{-6}$ m s$^{-1}$ the model overestimates the data by about 20-40% and at the higher velocity of $10^{-4}$ s$^{-1}$ it underestimates the data by the same amount. Also, the model over-estimates by about a factor of two the friction coefficient for the greater holding time of $t_h = 10^4$ s. These discrepancies could be attributed to a number of points, particularly the $n$-value and whether a single value applies over the entire period of holding. A higher value, $n=5$ and correspondingly lower value $B=5 \times 10^{-8}$ MPa$^{-5}$ s$^{-1}$, as found by Barnes et al.$^{60}$ for effective stresses as high as 20 MPa, would lead to a higher creep rate and to a greater increase in contact area and thus to greater hardening by about a factor of 1.5, bringing the model more into agreement with experiment at the highest velocity, but then over-estimating the measurement by a greater amount at the lowest velocity. Also, the model does not account for what appears from Figure 8 to be a tendency towards saturation upon holding for a prolonged period, nor does it account for the log-time dependence of strengthening (Figure 8). Log-time dependence, as noted by Brechet and Estrin$^{85}$ and discussed by Schulson and Fortt$^{28}$, requires that asperity creep be described not by a power law but by the exponential function $\dot{\varepsilon} = \dot{\varepsilon}_o \exp\left[\left(h/h_o\right)\left(\sigma_y/S\right)\right]$ where $\dot{\varepsilon}_o$ is a temperature-dependent constant, $\sigma_y$ is the yield stress and $S$ is a strain rate sensitivity parameter proportional (through Boltzmann’s constant) to absolute temperature and inversely proportional to the activation volume $v^*$ of the rate-controlling mechanism; and measures of $\dot{\varepsilon}_o$, $v^*$ and $S$ are not available to allow quantitative assessment. Despite these shortcomings, the creep-based model of static strengthening is considered to work well enough to justify further development along these lines.
Cohesion probably plays a role here. Bonding could develop during holding in several ways—through sintering via vapour transport, as discussed by Blackford; through the assistance of a liquid-like layer on the surface of warm ice (reviewed by Dash) given the observation of sub-second bonding between needles of ice when brought together under a normal load of 1 N at temperatures above ~-23 °C; or perhaps through the freezing of patches of melt-water that might form from thermal flashes at points of contact that for an instant during the sliding stage prior to holding experience velocities higher than average (more below). Whatever the mechanism, cohesion allows additional surface traction to develop during the post-holding stage of the slide-hold-slide cycle and should be included in a complete description.

4.2 Coefficient of kinetic friction

We turn now to the resistance to maintain sliding once the frictional force has fallen to a pseudo-steady-state level. As the opposing surfaces slide over each other, surface traction develops as the asperities interact, augmented perhaps by dynamic cohesion in warmer ice when sliding at lower velocities. The surface traction, in concert with the normal load and other contact forces, activate inelastic deformation within the near-surface regions, evident from the microstructural changes described in Section 3.2.8. This irreversible deformation, as already noted, is an important contributor to the kinetic friction of ice.

4.2.1 Velocity-strengthening
The rising part of the curve of the coefficient of kinetic friction vs velocity (Figures 10 and 11) for warmer ice can be understood in terms of creep deformation. Accordingly, taking the coefficient of kinetic friction to be defined by the ratio of the shear stress $\tau_c$ to maintain a given creep rate $\dot{\varepsilon}$ to the normal stress supported by the asperities, and taking again the normal stress to be limited by the hardness of the ice, which is also a rate-dependent property, we may write:

$$\mu_h = \frac{\tau_c(\dot{\varepsilon}, T)}{H(\dot{\varepsilon}, T)}$$  \hspace{1cm} (38)

Both creep strength and hardness also depend upon temperature $T$. From Barnes et al.

$$\tau_c(\dot{\varepsilon}, T) = B^{-\frac{1}{n}} \dot{\varepsilon}^{\frac{1}{n}} \exp\left(\frac{Q}{nkT}\right)$$ \hspace{1cm} (39)

and

$$H = C t^{-\frac{1}{n'}} \exp\left(\frac{Q'}{n'kT}\right)$$ \hspace{1cm} (40)

where $t$ denotes time of contact, $Q$ and $Q'$, respectively, denote apparent activation energies for the mechanisms governing creep and hardness under the imposed sliding conditions, $k$ is Boltzmann’s constant and $B$, $C$, $n$ and $n'$ are material constants. Creep (strain) rate $\dot{\varepsilon}$ is related to the average sliding velocity (i.e., the velocity imposed, $V_s$) and to thickness $h$ of the near-surface inelastic zone through the relationship $\dot{\varepsilon} = V_s / h$; time of contact is related to velocity and asperity diameter, $t = 2a / V_s$. The contribution
from near-surface creep to the coefficient of kinetic friction during dry sliding may then be expressed by the relationship:

\[
\mu_k = \frac{B^{\frac{1}{n}}(V_s / h)^{\frac{1}{n}} \exp(Q / nkT)}{C(V_s / 2a)^{\frac{1}{n'}} \exp(Q / n'kT)}.
\]  

(41)

To assess this model, we perform the following exercise: First, we limit our attention to strengthening at -10 °C. We take the value \( h = 0.1 \, \text{mm} \), based upon the extent of the deformation damage noted in Section 3.2.8 and also upon the value suggested by Barnes et al.\(^60\). Then for ice sliding at a reference velocity of \( V_s = 10^{-7} \, \text{m s}^{-1} \) the corresponding creep rate is \( \dot{\varepsilon} = 10^{-3} \, \text{s}^{-1} \). To maintain that rate of creep at -10 °C, a shear stress of \( \tau_c = 4.2 \, \text{MPa} \) is required, based upon the creep rate-stress data for freshwater ice shown in Figure 2 of Barnes et al.\(^60\), taking care to divide the applied stress given in that figure by the factor \( \sqrt{3} \) to convert normal stress to shear stress. To obtain a contact time and hence a reference hardness, we take the average contact diameter to be \( 2a = 30 \, \mu\text{m} \), derived from the analysis in Section 4.1, and again use \( V_s = 10^{-7} \, \text{m s}^{-1} \). That gives \( t = 300 \, \text{s} \) for which the corresponding hardness for the same kind of ice is \( H = 15 \, \text{MPa} \), from Figure 4 of Barnes et al.\(^60\). Thus, the model predicts \( \mu_k(10^{-7}, -10) = 0.28 \), in good agreement with the measured value (Figure 10) of \( \mu_k = 0.33 \pm 0.05 \) under the same conditions for relatively smooth (\( R_a \sim 1 \, \mu\text{m} \)) surfaces. To obtain the velocity dependence for the same kind of ice at the same temperature, we scale the shear stress and hardness noted above using the values \( n = 3.0 \) and \( n' = 4.4 \), from Barnes et al.\(^60\). Then, at the higher velocities of \( 10^{-6} \) and \( 10^{-5} \, \text{m s}^{-1} \) the model dictates that \( \mu_k(10^{-6}, -10) = 0.36 \) and \( \mu_k(10^{-5}, -10) = 0.46 \).
somewhat lower than the measured values of 0.45±0.06 and 0.63±0.06. In other words, the creep-based model indicates that for warm ice sliding over relatively smooth interfaces $\mu_k \propto V_s^{0.10}$, whilst experiment shows that $\mu_k \propto V_s^{0.13 \pm 0.01}$ (Section 3.2.3). Should this difference ($\Delta \mu_k = \mu_{k,\text{measure}} - \mu_{k,\text{model}} = 0.05$ at $10^{-7}$ m s$^{-1}$ to $\Delta \mu_k = 0.09$ at $10^{-6}$ m s$^{-1}$ to $\Delta \mu_k = 0.17$ at $10^{-5}$ m s$^{-1}$) be significant, and it is not clear that it is, it would indicate that mechanisms in addition to creep contribute to frictional resistance.

Concerning the sliding of colder ice, we imagine that asperities continue to interact and that near-surface regions continue to deform inelastically under the combined influence of surface traction and normal loads. Within the creep-based model, the influence of temperature is embodied in the apparent activation energies whose values have been reported to be $Q \sim 0.81$ ev atom$^{-1}$ (78 kJ mol$^{-1}$) and $Q' \sim 0.75$ ev atom$^{-1}$ (72 kJ mol$^{-1}$) for temperatures between about -10 and -50 °C. Then, using again the values $n = 3.0$ and $n' = 4.4$, the model predicts that upon lowering temperature from -10 °C to -50 °C, for instance, the friction coefficient is expected to increase by a factor of ~2.2.

Experiment, on the other hand, shows that over this range of temperature the coefficient increases by a smaller factor of ~ 1.3 (Section 3.2.4). Although the predicted trend is similar to the one observed, the magnitude of the effect is considerably lower than predicted. Barring the possibility that some process other than creep is at play in the colder ice, a possible explanation for this difference, albeit speculative, is that creep deformation during sliding is characterized by activation energies lower than quoted above for which some evidence is the value $Q = 0.41$ ev atom$^{-1}$ derived by Durham et al.$^{97}$ from low-temperature creep tests. In the same sense, should the shear strength and the hardness of ice become even more athermal in character and correspondingly less
dependent upon rate of loading as temperature falls still further, it would become increasingly difficult to detect experimentally an effect of velocity on the coefficient of friction, thereby accounting possibly for the apparent absence of velocity dependence in measurements made at -100 and at -140 °C (Figure 10). At those low temperatures (of -100 and at -140 °C), the primary contributor to friction might not be rate-dependent plastic deformation at all, but fracture, in keeping with the observation that both the measured coefficient of friction (Figure 10) and the expected toughness of ice (albeit, based not upon measurement but upon an extrapolation from measurements at higher temperatures, reviewed in ref. 1, Chapter 9) are about 30% higher at -140 °C than at -100 °C. The reality is that until low-temperature creep parameters are available, it is not possible to more fully access quantitatively the role of creep deformation on low-temperature sliding.

Why does velocity-strengthening reappear in still colder ice (at -175 °C), from $\mu_k = 0.3$ at $5 \times 10^{-8}$ m s$^{-1}$ to $\mu_k = 0.8$ at $1 \times 10^{-3}$ m s$^{-1}$ (Figure 10)? In addition to fracture, another mechanism probably becomes active. One possibility, discussed by Schulson and Fortt$^{27}$, is a pressure-induced phase transformation at points of contact, from ice I to ice II. The I-II transformation is accompanied by a reduction in volume of ~25% and correspondingly by a shortening of ~8%. The shortening would activate a temporary pulling away of contact, momentarily lowering sliding resistance. Once contact is lost and local pressure reduced, conditions would favour the reverse transformation, possibly at a rate lower than the forward transformation. The process would repeat in a cyclic manner, accounting as well for the unusual observation, discussed elsewhere$^{27}$, of stick-slip within the velocity-strengthening regime. It is important to note that the kinetics of
the I->II transformation are relatively sluggish \(^{98}\), at least within bodies larger than an asperity. As a result, this pull-away mechanism would be favoured at lower velocities, in keeping with the lower coefficient of friction at the lower end of the velocity range. At the higher end of the range, pull-away probably would not occur owing to the kinetic limitation and so the friction coefficient would be expected to be higher, as observed. Under those conditions friction might once again be limited by fracture, in keeping with the increase of factor of about 1.4 in both the coefficient of friction and the expected toughness (again, from extrapolation) relative to the values at -140 °C.

4.2.2 Velocity-weakening

Returning to warmer ice and to the observation of velocity-weakening, frictional heating-cum-melting is at root here. The reason is that heat created through the work of friction at higher speeds has less time to be conducted away from the interface\(^{25, 27, 41, 59, 60, 99-101}\). The potential exists, therefore, of raising the temperature of contacts to the point of melting and of providing the necessary heat of fusion to bring about the transformation. Frictional heat also softens asperities and lowers the creep strength of the near-surface deformation zone, but those effects, based upon Equation (41) and using the very warm-ice values\(^{60}\) \(Q\) and \(Q' = 1.3\) and 1.5 ev atom\(^{-1}\) (120 and 145 kJ mol), respectively, and \(n=3.0\) and \(n' = 4.2\), reduce the coefficient of friction by only \(~13\%\) and not by the factor of four or more that characterizes the velocity-weakening regime. Similarly, softening of asperities through dynamic recrystallization (Section 3.2.8) is expected to have too small an effect.
In mind is not global melting of the kind detected during high-speed sliding (> 1 m s⁻¹) of warm (-5 °C) ice that activates a second regime of velocity-strengthening⁵⁹,¹⁰² – one related to hydrodynamic effects and not to thermally activated deformation. We rule out melting of the whole interface because calculation shows that frictional work is too small both to raise the temperature of the interface to the melting point and to provide the necessary heat of fusion, even when pressure melting is taken into account (Appendix 2). Instead, we envisage localized heating-cum-melting that produces patches of water that help to lubricate the interface. Over the weakening regime (from \( V_s \sim 10^{-5} \) to \( 10^{-1} \) m s⁻¹), we imagine that the wet patches increase in both number and size to the point that the whole interface eventually becomes covered in a layer of water. Beyond that point, hydrodynamic sliding sets in and the character of friction changes⁵⁹. In other words, we attribute the reduction in the low-speed coefficient of friction from its maximum (at \(-10^0\) C of \( \mu_k \sim 0.5 \) at \( V_s \sim 10^{-5} \) m s⁻¹) to its minimum of (\( \mu_k \sim 0.05 \) at \( V_s \sim 10^{-1} \) m s⁻¹) to a transition from dry to wet sliding during which the coefficient of kinetic friction may be described by a function of the form:

\[
\mu_k = \mu_{k,d}(1 - \eta) + \mu_{k,w}
\]

or, more simply, given the very low coefficient of friction of warm ice, by:

\[
\mu_k \approx \mu_{k,d}(1 - \eta)
\]

where the subscript \( d \) on \( \mu_{k,d} \) denotes the coefficient of friction for dry sliding and may be obtained from Equation (41); \( \eta \) denotes the fraction of the interface that has transformed to water and, from Equation (13), is given by \( \eta = \gamma \log(V_s/V_1) \). The
surviving asperities counteract to some degree the lubricating effect of the water, owing to the rate dependence of creep strength and hardness, provided that they continue to flow plastically and not undergo the ductile-to-brittle transition at the higher rate. At some velocity, however, lubrication dominates frictional behavior.

What is the evidence of localized melting? Marmo et al.\textsuperscript{103}, using low-temperature scanning electron microscopy, reported refrozen water on the surface of ice that had rubbed against steel at -3.4 °C at 0.02 m s\textsuperscript{-1}. Under those conditions, the coefficients of kinetic friction of ice on steel and of ice on ice have similar values, $\mu_k = 0.1-0.05$ (for steel\textsuperscript{60,103}; for ice\textsuperscript{41}). Given that steel has greater thermal conductivity than ice and thus a greater propensity to conduct frictional heat away from a sliding interface, even more melt water is expected when ice slides upon itself. Of greater relevance to the ice-on-ice scenario is the observation (Section 3.2.3) that upon rapidly sliding (at $10^{-3}$ to $10^{-2}$ m s\textsuperscript{-1}) a short distance following their formation at -10 °C, shear faults possessed cohesion--bonding that probably developed as sliding-induced melt-water froze. Further evidence of localized melting are the tiny, globular shaped features that appeared on the surface of Coulombic shear faults upon sliding at $8 \times 10^{-4}$ m s\textsuperscript{-1} at -10 °C.\textsuperscript{44} We take those features to be frozen water.

To model this mechanism, we imagine that sliding on the scale of the asperities exhibits spatiotemporal character – analogous, perhaps, to the intermittent character of slip within crystals of ice through the action of dislocation avalanches\textsuperscript{104,105}. Accordingly, we imagine that certain asperities momentarily slip at a local velocity $V_l$ that is greater by a factor $f$ than the average applied velocity $V_a$, and that, in slipping, heat is generated sufficient to cause local melting. To remain molten, the time needed for the asperities to
slip past each other \( t_s \) must be less than the time needed \( t_c \) to conduct the heat of fusion into the surrounding ice. The slip time scales with asperity diameter and inversely with velocity and the conduction time scales inversely with the temperature difference \( \Delta T' \) between the surface and the sub-surface, and so the larger the asperity and the colder the ice, the faster must the ice slide to prevent the water layer from freezing.

To find a relationship between \( \Delta T' \), asperity diameter \( 2a \) and the applied velocity at which weakening is first detected \( V'_i \), termed the transition velocity, we imagine a thin layer of water at the melting point, of thickness \( \delta \); its heat content is \( \pi a^2 \delta L_v \) where \( L_v \) denotes the latent heat of fusion per unit volume. That heat is conducted in time \( t_c \) across an interface of area \( \pi a^2 \) and down a thermal gradient \( \Delta T / \Delta z \) on either side of the interface where \( \Delta z \sim \sqrt{h_t t_c} \) and where \( h_t \) denotes thermal diffusivity given by \( h_t = \kappa / \rho C_p \) where \( \kappa \) denotes thermal conductivity, \( \rho \) mass density and \( C_p \) specific heat. Upon equating the heat content to the heat transferred, it follows that:

\[
\frac{t_c}{2} \sim \frac{\left( \frac{t_c \delta}{2} \right)^2}{\kappa \rho C_p \Delta T^2}
\]

(44)

where the factor of 2 accounts for conduction into both sides of the interface. The slip time is given by:

\[
t_s = \frac{2a}{V'_i} = \frac{2a}{fV'_i}.
\]

(45)

Upon equating Equations (44) and (45) and rearranging, we obtain the relationship:
Thus, the model dictates that the velocity that marks the onset of velocity-weakening scales as \( V_t \propto a \Delta T^2 \).

It is not easy to assess this analysis. Neither the factor \( f \) by which the local velocity is increased over the average velocity nor the layer thickness \( \delta \) is known and only an average value of asperity size is available from the analysis of static strengthening given in Section 4.1. However, in the interests of a rough assessment, we take \( \delta \sim 1 \mu m \), based upon Colbeck’s analysis \(^{106}\) of snow friction, and we assume that \( f \sim 100 \) (as might be the case for an avalanche-like de-pinning of asperities and allowing for slip in directions inclined to the direction of macroscopic sliding). With those assumptions and upon taking the values \( 2a \sim 30 \mu m \) (from Section 4.1), \( L_v = 320 \text{ MJ m}^{-3}, \ k = 2.1 \text{ W m}^{-1} \text{ K}^{-1}, \ \rho = 917 \text{ kg m}^{-3} \) and \( C_p = 1900 \text{ J kg}^{-1} \text{ K}^{-1} \), Equation (46) yields the estimate that \( V_t \sim 4 \times 10^{-3} \text{ m s}^{-1} \) for ice initially at \(-10^\circ \text{C} \) (\( \Delta T = 10 \)). On a logarithmic scale this is about mid-way within the velocity-weakening regime observed in experiments. However, asperities almost certainly range in size. Should that range be \( 0.1 \mu m \leq 2a \leq 1 \text{ mm} \), the transition velocity would be expected to vary over the range \( \sim 10^{-5} \leq V_t \leq 10^{-1} \text{ m s}^{-1} \). In other words, the range of velocity over which velocity-weakening is observed at \(-10^\circ \text{C} \) may be a measure of the size range of the interacting asperities.

Turning to warm sea ice, the model implies that the onset on velocity-weakening in that material is expected to be about an order of magnitude higher than in freshwater ice,
owing to its higher heat capacity (i.e., \( C_p \approx 30 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1} \) at -10°C for sea ice of ~5 ppt salinity\(^{107,108}\); the values of density, thermal conductivity and latent heat of fusion are similar to those of freshwater ice). This implication agrees with the experimental results shown in Figure 11: velocity-weakening upon sliding across Coulombic shear faults at -10 °C appears to begin at \( \sim 10^{-4} \text{ m s}^{-1} \) in sea ice but at \( \sim 10^{-5} \text{ m s}^{-1} \) in freshwater ice. The heat capacity of colder sea ice is about the same as that of freshwater ice\(^{108}\), owing to the freezing of brine, implying that the effect just mentioned would not be expected at -40 °C. The data shown in Figure 11 are more scattered for sea ice, making a comparison for the colder ice less definitive.

If flash heating through localized bursts in sliding continues to generate a thin layer of water on certain asperities as temperature decreases, then the model dictates, as already noted, that the velocity at which weakening begins should increase. A drop from -10 °C to -50 °C, for instance, should lead to an increase of about a factor of \((50/10)^2 = 25\). The experimental results shown in Figure 10 for sliding across a relatively smooth interface indicate that over this same range of temperature, the velocity at which the coefficient of friction reaches a maximum does increase, but by the smaller factor of ~10. Results shown in Figure 11 for sliding across Coulombic shear faults, on the other hand, indicate that over the range from -10 °C to -40 °C temperature exerts little detectable effect on the velocity at maximum friction. These observations do not necessarily invalidate the model, because in addition to scatter in the data neither set of experiments was performed at increments of sliding velocity small enough to allow a highly accurate and precise measure of the maximum.
Returning to the coefficient of friction during partially wet/dry sliding and to Equations (42) and (43), we do not have an independent measure of the parameter $\eta$. A measure perhaps could be obtained from the electrical conductivity of the interface, following Bowden and Hughes\textsuperscript{25} who, in that way, revealed friction-induced melting beneath skis. However, based upon the foregoing discussion and upon the description of velocity-weakening given by Equation (13), we can now at least interpret the limits $V_1^* \leq V_3 \leq V_2^*$ as the velocities at which the smallest and largest asperities melt.

To some extent, the localized melting mechanism just described is conceptually similar to the asperity retraction mechanism that was proposed to account for velocity-strengthening of cold ice. Both mechanisms invoke a phase transformation accompanied by a volume reduction. The difference is that the ice I-> ice II transformation involves both nucleation and growth and these stages are thermally dependent and hence kinetically limited. That is why, as already mentioned, the I->II process is thought to be of lesser relevance at higher sliding speeds where less time is available, leading to less frictional weakening and hence to macroscopically stable slip. Melting, in comparison, does not suffer from a barrier to nucleation and growth and so is not kinetically limited (barring the kinetics of heat transfer): higher speeds enhance the transformation, leading to more frictional weakening and to macroscopically unstable slip. Although developed to account for the friction of ice, the ideas involving phase transformations of one kind or another may have relevance to rock and other materials which, as already noted, also exhibit both velocity-strengthening and velocity-weakening.

4.3 Other explanations of kinetic friction
Maeno and Arakawa\textsuperscript{43} offer another interpretation of kinetic friction. Like the one just described, their model is based upon the thermally activated deformation of asperities, but includes sintering and hence cohesion. As already suggested, when combined with their definition of friction in terms of the stress ratio (Equation 9), the presence of cohesion suppresses velocity-strengthening. In other words, the model fails to capture a key characteristic of frictional sliding.

Hatton et al.\textsuperscript{86} offer another interpretation, again based upon the deformation of asperities in contact. Their model includes both ductile and brittle behavior and incorporates a principle of maximum displacement for deformation normal to asperities and of minimum stress for failure under shear. While invoking realistic geometries – size distributions, peak height distributions and radii of curvature – the model as formulated dictates zero shear resistance at sliding velocities lower than about $4 \times 10^{-3}$ m s$^{-1}$. Although experiment shows that the resistance tends toward a very low value as velocity becomes very low, at the velocity noted the coefficient at -10 °C is relatively high and has the value $\mu_k \sim 0.5-0.6$.

Oksanen and Keinonen\textsuperscript{59} present a different interpretation of velocity weakening, albeit of weakening over higher velocities from 0.5 to 3.0 m s$^{-1}$. Over that range, the coefficient of kinetic friction scales as $\mu_k \propto V_x^{-3/2}$ and has the value, for instance, of $\sim 0.02$ at -15 °C at 0.1 m s$^{-1}$. This behavior is explained not in terms of the deformation of asperities at all, but in terms of thermal processes where the work of friction is equated to the sum of the heat conducted away from the sliding interface plus the energy expended in melting a thin layer of ice. The thin layer of water is assumed to be self-balanced in the sense that “increasing frictional heat would melt more water and if the water thickness
increases the reduction in frictional heat would cause a temperature drop at the contact below the melting point of water (sic) and the heat produced by friction is equal to the heat conducted into the two solids” 59. Whilst Oksanen and Keinonen’s model accounts well quantitatively for their observations and for those of Evans et al.109 that were obtained under similar conditions, this explanation cannot account for the velocity-weakening apparent in Figure 10, for under the lower-velocity conditions under which the data in that figure were obtained, $\nu_s^{-1/2}$ functionality would imply a friction coefficient of ~3 at a velocity of $10^{-4}$ m s$^{-1}$ at -15 °C compared with the measured value of ~ 0.5 (albeit at -10 °C). (The difference of 5 degrees could not account for this discrepancy, given the relatively small effect of temperature implied by the data in Figure 10.) Perhaps there are two regimes of velocity-weakening of warm ice, one that operates over lower velocities and results from a combination of inelastic deformation and localized melting and another that operates over higher velocities and results from thermal processes alone.

Makkonen 110 and Makkonen and Tikanmaki 111 offer a completely different interpretation of kinetic friction, one based upon thermodynamics. Their idea is that the resistance to dry sliding is governed not by the shear strength of interacting asperities, but by surface energy, specifically by the energy of surface steps or edges that are created at the nanoscale. For ice sliding upon itself under conditions where frictional heat is insufficient to melt a thin layer at the interface – i.e., dry sliding –the coefficient of friction is expressed as $\mu_k = \gamma / H\lambda$, where $\gamma$ denotes the surface energy of ice, $H$, hardness and $\lambda$, a nanoscale contact length. Although data for $\lambda$ are not available, it is assumed 110,111 that this parameter is equal to the size of the smallest cluster of H$_2$O
molecules that is stable in the solid state which, based upon calculations by Pradzynski et al., is of the order of \( \lambda = 2 \) nm. Taking this value and taking from Table 1 and Figure 2 of Makkonen and Tikanmaki the values \( \gamma = 73 \text{ mJ}^2 \) and \( H = 60 \text{ MPa} \) (where 60 MPa corresponds to the hardness for an indentation time of \( \sim 10^{-4} \text{ s} \) (which is about the amount of contact time \( t = \lambda / V_c \) to be expected for sliding at \( \sim 10^{-5} \text{ m s}^{-1} \) across an asperity \( \sim 2 \) nm in diameter), the thermodynamic model dictates \( \mu_k = 0.6 \) for dry sliding at -10 °C. This value agrees closely with the value measured at -10 °C for smooth surfaces sliding at \( \sim 10^{-6} \) to \( 10^{-5} \text{ m s}^{-1} \). The problem, however, is that the thermodynamic model does not account for velocity-strengthening. The only parameter in the model that exhibits rate dependence is hardness and that dependence leads to a reduction in the coefficient of kinetic friction during dry sliding and not to an increase with increasing velocity. An increase of two orders of magnitude in sliding velocity, for instance, leads (from Equation (40)) to an increase of about a factor of two in hardness and, thus, to an expected reduction in the friction coefficient of a dry interface by about the same factor.

The thermodynamic model, incidentally, appears not to be compatible with one based upon the thermally activated deformation of asperities. This may be seen as follows: Consider a block of ice of length \( L \), width \( W \) and height \( H \). Allow the upper half to be displaced over the lower half by a distance \( b \) through the action of a shear force \( F_s \). The shear force does work on the system, given by \( F_s b \). The displacement creates two steps, each of area \( Wb \), and thus new surface of energy \( 2\gamma Wb \) where \( \gamma \) is surface energy per unit area. If the shear force arises owing to a process that operates within the contact area, as assumed in the shear strength model, then the shear force during sliding is the product of the shear strength \( \tau_s \) of the interface and the area \( WL \) of the sliding surface,
\( F_s = \tau_s WL \). Upon equating the work done on the system, to the increase in surface energy (\( \tau_s WLb = 2\gamma Wh \)) the shear strength is given by \( \tau_s = 2\gamma / L \). This implies that the strength of the interface, and hence the coefficient of kinetic friction, decreases as the length of the block increases. There is no evidence that ice behaves in that manner.

5. Implications for ice mechanics

5.1 Brittle compressive strength

Returning to the role of friction in the fracture of ice, it is interesting to note that, just as the coefficient of friction falls from a maximum to a minimum value over about four orders of magnitude of applied velocity, the brittle compressive strength of ice falls from a maximum to a minimum over about four orders of magnitude of strain rate (for review of strain-rate softening of ice, see ref 1, Chapter 11). We think this correlation is not fortuitous. As discussed in Section 2, frictional sliding underlies brittle failure and in the mechanistic-based relationships described there the unconfined compressive strength scales as \( \sigma_c \propto 1 / (1 - \mu_k) \) for \( \mu_k < 1 \). This means that a reduction in the coefficient, from a maximum of \( \mu_k = 0.6 \) to a minimum of \( \mu_k \sim 0.02 \) at -10 °C, is expected to lead to a reduction in strength by a factor of ~2.5. This expectation is in good agreement with experiment \(^{18}\) which shows that the compressive strength, although scattered, falls by about the same factor within the strain-rate weakening regime from \( \sim 10^{-4} \text{ s}^{-1} \) to \( \sim 1 \text{ s}^{-1} \) at -10 °C.

The other correlation between friction and fracture pertains to high-rate sliding and deformation. As noted above, at sliding velocities above about 1 m s\(^{-1}\) the coefficient of kinetic friction of warm ice increases with velocity, from \( \mu_k = 0.020 \) at 0.1 m s\(^{-1}\) to 0.03 at
3 m s^{-1} and scaling roughly with $\mu_k \propto V_s^{1/2}$; correspondingly, at strain rates above ~ 1 s^{-1} the brittle compressive strength of warm ice increases with strain rate^{113,114}, through the relationship $\sigma_c = 9.88 \varepsilon^{0.14}$ MPa (for strain rate in units of s^{-1}). These are small effects and the scatter in the compressive strength is too large to allow meaningful quantitative comparison with the $1/(1 - \mu_k)$ function. Nevertheless, the qualitative similarity suggests once again a fundamental link between friction and fracture. Should frictional sliding continue to be a significant factor in the compressive strength of ice under conditions of dynamic loading, fresh fracture surfaces would be expected to be wet.

On the strength of cold ice, Arakawa and Maeno^{115} found that the unconfined brittle compressive strength of granular ice of ~ 1 mm grain size upon loading at a strain rate of $4 \times 10^{-5}$ s^{-1} increased by a factor of ~2.2 upon lowering the temperature from -50 to -173 °C, from ~35 to ~76 MPa. Some of that increase can be attributed to an increase in resistance to crack propagation, although that part, based upon discussion elsewhere\(^1\), is expected to be rather small and to lead to an increase in strength by only a factor of ~1.4. The other part can be rationalized in terms of friction and the discussion in the preceding paragraph. Accordingly, if it is assumed that each grain within the aggregate contained one crack of grain dimensions and that sliding across the faces of the cracks accounts for inelastic strain, and assumed further that the sliding velocity is roughly equivalent to the twice product of crack length and applied strain rate, or ~8 × 10^{-8} m s^{-1}, then from Figure 10 the appropriate values of the coefficient of kinetic friction are $\mu_{k_{-50}} \approx 0.3$ and $\mu_{k_{-173}} \approx 0.4$. Correspondingly, the friction-based increase in the compressive strength is expected to ~$((1/(1-0.4))/(1-0.3))$~1.2. The remaining factor of ~2.2/(1.4×1.2)~1.3 comes from confinement and from the influence of friction on the sensitivity of strength to
confinement. Although the experiments by Arakawa and Maeno\textsuperscript{115} were performed under uniaxial loading, the ice was actually constrained, for the specimens were bonded to copper platens whose ratio $\nu/E$ (where $\nu$ denotes Poisson’s ratio and $E$, Young’s modulus) differs by a factor of ten from that for ice; i.e., $\nu/E_{\text{ice}} \sim 0.03 \text{ GPa}^{-1}$; $\nu/E_{\text{Cu}} \sim 0.003 \text{ GPa}^{-1}$). As mentioned in Section 2 (Equation 5), confinement strengthens ice by an amount that depends not only on the degree of confinement, but also on the friction-dependent sensitivity to confinement. This sensitivity increases by a factor of $\sim 1.3$ upon decreasing temperature from -50 to -173 °C owing to the increase in the friction coefficient. In other words, by incorporating the effect of temperature on the coefficient of kinetic friction, essentially of the difference in strength of ice at -50 and -173 °C can be explained.

5.2 Sliding of the arctic sea ice cover

Satellite imagery has revealed strike-slip-like sliding across fault-like features that can extend thousands of kilometers\textsuperscript{13, 29, 116-120}. Should sliding on the geophysical scale exhibit similar character to sliding on the laboratory scale, then, once initiated, slip would be expected to be stable at lower velocities, but unstable at higher velocities. In other words, episodes of sudden slip would be expected as wind-driven driving forces build-up to the point where sliding velocity becomes high enough to impart velocity-weakening. Once the driving force is relaxed, stable deformation would be expected to resume.

Strike-slip-like sliding faults (also termed linear kinematic features by the ice-ocean community) often form as intersecting sets whose angle of intersection is around $2\psi = 35\pm15^\circ$\textsuperscript{13, 29}. If the faults are truly geophysical-scale Coulombic shear faults, as
proposed\textsuperscript{13} and if the intersecting features are conjugate sets oriented on either side of the direction of maximum compressive stress, then theory holds that\textsuperscript{121}:

\[ \tan 2\psi = \frac{1}{\mu_k}. \] (47)

Equation (47) implies that for strike-slip-like sliding \( \mu_k = 1.4 \pm 0.9 \). This value includes the value \( (\mu_k \leq 1.8) \) measured across Coulombic faults in the laboratory, suggesting, as noted earlier, that the coefficient of kinetic friction, is largely independent of spatial scale.

The foregoing comments imply that the sea ice cover deforms intermittently and heterogeneously. Statistical analyses of ice buoy drift and examination of satellite-derived images\textsuperscript{24,122-124} show that intermittency and heterogeneity are indeed characteristics of the deforming cover. Realization of this character has led to the development of new sea ice models that directly incorporate friction within the context of a failure criterion\textsuperscript{79,125-131}.

As well as sliding within the plane of the cover, sea ice slide can slide over itself through a process termed “rafting”. In that scenario, sliding probably will be affected by the presence at the interface of slush and/or snow. The magnitude of effect, however, is not clear, for systematic studies on the role of interfacial matter have not been performed.

5.3 Tectonic evolution of icy satellites

Finally, consider the “tiger stripe” rifts within the south polar region of Saturn’s satellite Enceladus\textsuperscript{132-135}. It has been suggested\textsuperscript{136} that the water-rich and ice-rich plumes
that are periodically emitted through the fractures\textsuperscript{137-139} could be energized through frictional sliding of the icy crust and attendant heating, driven by diurnal, tidal-based cyclical shear along the faults. Two parameters would then be important: the coefficient of static friction, which sets the level of the driving shear stress to initiate sliding, and the coefficient of kinetic friction, which governs the shear stress needed to maintain sliding and to generate heat. If the coefficients are too high, then the mechanism cannot operate.

Smith-Konter and Pappalardo\textsuperscript{140} and Olgin et al.\textsuperscript{141} modeled the process. They invoked Coulomb’s law, validated through the work reviewed here, assumed zero cohesion and assumed that the faults extend to a depth of 2 km within a spherically symmetric icy shell 24 km thick. When loaded under a diurnal shear stress of \(\sim 45\) kPa, a diurnal normal stress of \(\sim 70\) kPa and an overburden pressure of \(\sim 200\) kPa – a combination obtained from a numerical code\textsuperscript{142} that gives tidal-induced stresses at any point on the surface of a satellite for both diurnal and non-synchronous rotation – the faults are expected to exhibit “slip windows” during two parts of the \(360^\circ\) diurnal cycle, from \(\sim 55^\circ\) to \(\sim 105^\circ\) (left-lateral displacement) and from \(\sim 195^\circ\) to \(\sim 255^\circ\) (right-lateral displacement). This result was obtained using a static coefficient of friction of \(\mu_s = 0.2\). The windows narrow for a higher value, closing at \(\mu_s \geq 0.3\).

Since the above analyses were performed, more is known about the friction of cold ice. Assuming (i) that water ice Ih is the primary constituent of Enceladus’ icy shell, even though spectroscopic data indicate the presence as well of hydrated salts, frozen gases, clathrates and possibly other minerals\textsuperscript{143, 144}, (ii) that small amounts of impurities and second phases do not greatly affect frictional resistance, (iii) that the sliding speed is \(\sim 10^{-6}\) - \(10^{-5}\) m s\textsuperscript{-1}, based upon a displacement of \(\sim 0.5\) m over a tidal period of 1.37 days\textsuperscript{135}. 

59
and taking the surface temperature at the south pole of Enceladus to be -188 to -158 °C or even higher, based upon results from NASA’s Cassini mission (http://www.nasa.gov/mission_pages/cassini/multimedia/pia06432.html), then the new knowledge indicates that the coefficient of kinetic friction of cold (-175 °C) ice is expected to be $\mu_k = 0.37 \pm 0.04$ at $5 \times 10^{-7}$ m s$^{-1}$, rising to $\mu_k = 0.53 \pm 0.04$ at $10^{-5}$ m s$^{-1}$. At -140 °C $\mu_k = 0.48 \pm 0.04$ and at -100 °C, $\mu_k = 0.41 \pm 0.06$, independent of velocity. Assuming further that holding under stress during the period of several hours outside the “slip windows” when the rifts are closed does not begin to heal the faults, in the way that holding for up to ~3 hours at -175 °C and at -100 °C under 60 kPa does not increase sliding resistance$^{27}$, then under the assumed conditions the static and kinetic coefficients are expected to have about the same value. The implication is that frictional sliding across Enceladus’ “tiger stripes” may be more difficult than previously modeled.

**Appendix 1: The microstructure of ice**

Ice possesses 13 different crystal structures, depending upon temperature and pressure, as well as two or more amorphous states. Under terrestrial conditions, the stable crystalline form is hexagonal, denoted Ih. Unlike hexagonal metals which possess a closely-packed structure, ice Ih possesses an open crystal structure, evident from the fact that terrestrial ice is less dense than water (917 vs 1000 kg m$^{-3}$ at the melting point). Most natural features are polycrystalline aggregates, comprised in some cases (glaciers and icebergs) of equiaxed and randomly oriented grains 1-10 mm in diameter, termed granular ice; others features (floating ice sheets) are comprised of columnar-shaped grains in which the crystallographic c-axes exhibit preferred orientation. In lake ice, for
instance, the c-axes are often vertically aligned (so-called S1 crystallographic growth
texture\textsuperscript{145}), whilst in sea ice they are confined to the horizontal plane and either randomly
oriented within that plane (S2 growth texture) or aligned (S3 texture). Hail stones are also
polycrystalline aggregates. The exceptions are icicles that can form as single crystals.
Natural features generally contain air bubbles. Sea ice also contains sub-millimeter sized
pockets of brine that become entrapped during solidification and constitute, in
combination with entrapped air, about 5\% by volume of the microstructure at -10 °C. The
actual volume fraction of brine at equilibrium depends on the temperature of the ice and
is governed by the water-sea salts phase diagram\textsuperscript{146}. In addition, sea ice contains a system
of brine-drainage channels on the order of a centimeter in diameter and spaced on the
order of ten centimeters. The microstructure of icy satellites is not known, but shows
evidence from spectroscopic data of multiple phases in the form hydrated salts, frozen
gases, clathrates and possibly other minerals\textsuperscript{143,144}.

A more complete account of the structure of ice is given elsewhere\textsuperscript{1,146}.

**Appendix 2: Frictional heating and pressure melting**

The frictional heat generated per unit area per unit time $q_r$ is the product of the friction
force and the sliding velocity divided by the real contact area (given by Equation (27)):

$$q_r = \frac{F_f V_s}{A_r} = \frac{\mu_k \sigma_n A_n V_s H}{A_r \sigma_n} = \mu_k V_s H . \quad (A1)$$

In turn, upon assuming that all frictional work is dissipated in heat (an upper limit, given
that some energy is also dissipated through mechanical processes when sliding across
rough interfaces) the steady-state increase in temperature within the contact zone is given by\textsuperscript{147}:

\[ \Delta T_{\text{total}} = \frac{4q_0a_0}{\kappa \sqrt{\pi}} \]  \hspace{1cm} (A2)

where \( \kappa \) denotes thermal conductivity and \( 2a_0 \) denotes the average diameter of the asperities. The temperature rise on each surface is one-half of this value and is given by:

\[ \Delta T = \frac{2\mu k V_s H a_0}{\kappa \sqrt{\pi}} \]  \hspace{1cm} (A3)

Taking for an upper limit the values \( \mu_k = 1.4 \) (for sliding on Coulombic shear faults), \( V_s = 10^{-3}-10^{-2} \text{ m s}^{-1}, H = 30 \text{ MPa}, 2a = 30 \times 10^{-6} \text{ m} \) (from Section 4.1) and taking \( \kappa = 2.2 \text{ W m}^{-1} \text{ C}^{-1} \), Equation A3 predicts a maximum increase in surface temperature of \( \Delta T = 0.3-3.0 \degree C \). A smaller increase is expected for smoother surfaces where ride-up and particle/fragment displacement contribute less to friction. The conclusion is that for slowly-sliding ice whose temperature is initially more than a few degrees below the melting point frictional work is probably too small to both raise the temperature of the whole interface and to provide the necessary heat of fusion to cause global melting.

That conclusion appears not to change when pressure melting is taken into account. Under a pressure of \( p = H / 3 = 10 \text{ MPa} \) the Clausius-Clapeyron relationship for ice dictates a reduction in the equilibrium melting point of \( \Delta T \sim 0.8 \degree C \).
Acknowledgements

The author acknowledges valuable discussions with Profs. Harold Frost, Knut Hoyland, Francis Kennedy, Lasse Makkonen, Aleksey Marchenko, Carl Renshaw and Jerome Weiss. The work was supported in part by the U.S.-Norway Fulbright Foundation through the award of the Arctic Chair 2013-2014 and by U.S. Dept. of the Interior-Bureau of Safety and Environmental Enforcement, contract nos. E12PC00033 and E12PC00064.
References


**Figure Captions**

**Figure 1:**
(a) Photograph of a wing crack in columnar-grained, freshwater ice. The wing crack is centered in the image and is shown in 3D; parts of other cracks are also shown. The wing crack was introduced by loading ice uniaxially across the columns (i.e., along the vertical direction of the image) at -10 °C at a strain rate of 10⁻³ s⁻¹. The long axes of the columnar grains run more or less in and out of the field of view, but are inclined slightly with respect to the normal to show the 3D character of the crack. The wings are oriented sub-parallel to the direction of loading and stem from a through-thickness primary crack that is inclined by about 45 ° to the loading direction. The length of the primary crack is governed by the ~10 mm diameter of the columnar-shaped grains. The ice possessed the S2 growth texture (Appendix 1), which had the effect of constraining inelastic deformation the plane perpendicular to the long axis of the grains.

(b) Schematic sketch in 2D of the formation of a wing crack. Wings initiate within the tensile T field (i) at the tips of an inclined, parent crack and then lengthen (ii) owing to sliding across the parent. Under the far-field applied compressive stress σ sliding is effected by the difference between the shear stress τ resolved onto the plane of the primary crack and the frictional resistance across that plane which is given by the product of the normal stress across the primary crack σₙ and the coefficient of kinetic friction μₖ.
Figure 2:
Photograph of a series of wing cracks in 2D (arrowed) in ice of the kind described in the caption to Figure 1 and loaded as described there. (The long axis of the columnar-shaped grains is perpendicular to the image). Upon lengthening the wings interacted to form axial splits along the direction of loading.

Figure 3:
Photograph of cracks within the first-year ice cover on the Beaufort Sea. Banks Island is shown near the right-hand side under the scale mark. Note the wing-like character of the triangular-shaped features stemming up and down from the opposite tips of what appear to be inclined, primary cracks. Arrows indicate the shear stress on one possible primary crack. The photograph was obtained on 11 February 1983 from a satellite using infrared sensors of 0.6 km resolution. The image was obtained from file transparencies provided through the U.S. Air Force Defense Meteorological Satellite Program. The transparencies are archived at NOAA and at the University of Colorado, CIRES/National Snow and Ice Data Center. (From ref. 12).

Figure 4:
(a) Photograph of conjugate sets of Coulombic shear faults that were generated within columnar-grained freshwater ice of the kind described in the caption to Figure 1. The ice was proportionally loaded biaxially across the columns along the loading path $R = \sigma_2 / \sigma_1 = 0.1$ and where $\sigma_1$ (vertical) and $\sigma_2$ (horizontal) denote the least and the most compressive principal stresses, respectively. The ice was deformed at $-10 ^\circ$C at a strain rate of $\dot{\varepsilon}_1 = 5 \times 10^{-3} \text{s}^{-1}$. The columnar grains run in and out of the image. The faults are inclined by about $30^\circ$ to the direction of greater stress; their orientation is governed by the coefficient of friction, Equation 47. (From ref. 17.)

(b) Photograph of fault BC of (a) viewed in thin section. The inclined, whitish band shows the fault; it is comprised of microcracks. (From ref. 17.)

(c) Photograph of region A of fault BC shown in (b), in still thinner section. The damage within the region has taken the form of a comb-crack. The secondary cracks that comprise the comb stem from one side of the inclined, primary crack and tend to lengthen along the direction of greater compressive stress. (From ref. 17.)

(d) Schematic sketch of a comb-crack. Secondary cracks initiate from one side of a primary crack that is inclined by an acute angle $\psi$ to the most compressive principal stress. The secondary cracks create sets of slender micro-plates of length $h$ and width $w$, fixed on one end and free on the other. Axial loading produces a load/unit depth $P$ on the plates and frictional drag their free ends produces a moment/unit depth $M$. The moment is of greater importance than the axial load and eventually becomes large enough to cause the micro-plates to break via mixed mode-I and mode-II crack propagation. (From ref. 1.)

Figure 5:
Photograph showing a plastic fault within columnar-grained freshwater ice of the kind described in the caption to Figure 1. The ice was proportionally loaded triaxially along the loading path $\sigma_1 : \sigma_2 : \sigma_3 = 1.0 : 0.5 : 0.2$ at -10 °C at a strain rate of $\dot{\varepsilon}_1 = 6 \times 10^{-3}$ where the two more compressive stresses $\sigma_1$ and $\sigma_2$ were applied across the columns and the least compressive stress $\sigma_3$ was applied along the columns. The fault is inclined by about 45 ° to the long axis of the grains. (a) thin-section (~1 mm) viewed under natural light where, owing to absence of significant damage, the fault is only faintly visible; (b) same feature as (a), but viewed through crossed-polarizing filters. The recrystallized grains within the fault, shown in (b), formed dynamically and are a consequence as opposed to a cause of faulting. (From ref. 20.).

Figure 6:
Brittle compressive failure envelope for columnar-grained, freshwater ice of the kind described in the caption to Figure 1. The column diameter is 2-3 mm. The ice was proportionally loaded biaxially across the columns along the loading paths $0 \leq \sigma_2 / \sigma_1 \leq 1$ at -10 °C at a strain rate of $\dot{\varepsilon}_1 = 4 \times 10^{-3}$. Failure along the rising branch of the envelope occurs via Coulombic faulting; along the descending branch failure occurs via across-column spalling out of the loading plane. (From ref. 1.)

Figure 7:
Plots of frictional shear resistance $\tau$ divided by normal stress $\sigma_n$ vs. displacement from slide-hold-slide tests at -10 °C on first-year sea ice (a-c) and on freshwater ice (d-f) at pre-hold and post-hold sliding velocities from $10^{-6}$ to $10^{-4}$ m s⁻¹, upon holding for a period ranging from 1 s to $10^3$ s. The inserts show expanded segments of reloading after holding for 100 s. (From ref. 28.)

Figure 8:
Plots of the coefficient of static friction $\mu_s = \tau_p / \sigma_n$ vs. log₁₀ hold time, constructed from the data shown in Figure 7 plus more of the same kind, where $\tau_p$ denotes the peak shear stress upon re-initiation of sliding. Averages and standard deviations are shown. The dashed curve on the middle plot was calculated from Equations 36 and 37. (From ref. 28.)

Figure 9:
Shear stress $\tau$ (kPa) vs. normal stress $\sigma_n$ (kPa) vs. temperature (-175 °C(98 K) to -10 °C (263 K)) and sliding velocity (5x10⁻⁸ m s⁻¹ to 1x10⁻³ m s⁻¹) for granular, freshwater ice of 1 mm grain size (solid squares) and of 8 mm grain size (open squares). Grain size exhibits no systematic effect. Note the overall linear character of the $\tau - \sigma_n$ relationship. (From ref. 27.)

Figure 10:
Graphs of the coefficient of kinetic friction vs. sliding velocity at temperatures from -175 °C to -10 °C, derived from the kind of data shown in Figure 9 and after the data were subjected to statistical analysis to isolate significant effects. The bar through each point denotes the standard deviation. (From ref. 27.)
Figure 11:
Graphs illustrating the effect of sliding velocity and displacement on the coefficient of kinetic friction across Coulombic shear faults within (a,b) columnar-grained, freshwater ice and (c,d) first-year sea ice harvested from the Arctic Ocean. Sliding across opposing faces of the faults at -40 °C (a,c) and at -10 °C (b,d). The coefficient exhibits both velocity-strengthening (at lower velocities) and velocity-weakening. (From ref. 39.)

Figure 12:
Schematic sketch of the mechanical interaction of asperities of average diameter $2\alpha_n$ that protrude from opposing faces loaded under a shear stress $\tau$ and a normal stress $\sigma_n$. The interface is displaced at a relative velocity $v$. 