Measured and Predicted Dynamic Response of a Single Pile Platform to Random Wave Excitation

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ABSTRACT

The dynamic response characteristics of an operational single pile platform are investigated in detail. Wind, wave, and response time histories recorded on the platform in March 1980, form the basis for comparison of predicted and measured dynamic response. In the predictive analysis, the components of the total modal damping are separately computed. These damping components include the steel hysteretic, the wave radiation, the viscous hydrodynamic, and the soils damping. Response in the two fundamental bending modes of the structure are predicted using a technique based on the principle of reciprocity for ocean waves. Good agreement between predicted and measured response levels is attained. Combination of the results of the response prediction method with the results of a dynamic finite element model of the platform leads to a versatile expression for the mean rate of accumulation of fatigue damage. This expression, wave spreading factors, and climatological data are used to estimate a fatigue life for the structure.

INTRODUCTION

The understanding of the dynamic response of deepwater structures is a key element in the prediction of fatigue life. This is particularly true of structures with lightly damped natural vibration modes whose natural periods exceed three seconds. There are very few structures that respond linearly in most sea conditions and have sufficiently simple configurations to allow accurate theoretical predictions of dynamic response in directionally spread random seas. The vertical cylindrical caisson satisfies these conditions.

A complete dynamic analysis of a single caisson structure is described in this paper. The modal damping ratio is predicted by analytic means. A finite element model is used to predict the natural mode shapes and soil behavior. The mean square response of the structure is predicted and compared to field measurements with very good agreement. A fatigue life computation is presented which accounts for the variation in directional spreading of the seas.

The caisson has many similarities to other more complex fixed deepwater structures. It has similar mode shapes, natural periods, and damping ratios. The caisson has the same range of Reynolds numbers and Keulegan-Carpenter numbers as large tubular members on jacket structures. It has similar soil behavior. As a consequence the insights gained from an in-depth analysis of this simple structure provide a better understanding of the behavior of much larger and complex but dynamically similar deepwater platforms.

PLATFORM DESCRIPTION

The single pile platform which is the focus of this report is a triple-decked single well gas production platform located in South Marsh Island Block 33 in the Gulf of Mexico. A three dimensional drawing of the platform, which is operated by AMOCO Oil Company, is shown in Figure 1. The caisson stands in 89 feet of water and the pile diameter is four feet at the water line and seven at the mudline. The annulus between the main pile and the drive pile is grouted. The natural periods of the two orthogonal fundamental flexural modes of the structure are almost identical, each with a value of approximately 3.1 seconds. With these natural periods, the platform exhibits significant first mode dynamic response.

In March, 1980, the authors recorded wind, wave and response time histories on the structure for later analysis and comparison with predicted response values. A complete description of the data analysis is contained in Ref. [1]. A total of six reels of data were recorded on three separate days. A summary of the pertinent information derived from the five reels which contained horizontal biaxial accelerometer data is shown in Table 1. The other reel of data, reel 2, had time histories from four accelerometers distributed vertically over the structure but pointing in the same direction. This data was used to estimate mode shapes and is reported in OTC Paper No. 4286 [2]. One of the five remaining data sets, reel 5, included tests of a dynamic absorber. These results are described in OTC Paper No. 4283 [3].
SINGLE-DEGREE-OF-FREEDOM EQUIVALENT MODEL

This single pile platform responds significantly in only its fundamental bending modes. To predict the response in these natural modes, an equivalent, linear, single degree of freedom (SDOF) system was defined for both modes. This technique is applicable for structures which behave linearly and have small damping. The total response is obtained by superposition of the modal responses.

The equation of motion that represents first mode response of the caisson when excited by ocean waves contains terms which depend on the relative acceleration and velocity between the water particles and the generalized coordinates which represent platform motions. For this structure in commonly occurring low to moderate sea states, non-linear drag force or velocity dependent excitation is negligible and can be dropped. However, this does not necessarily imply that viscous damping losses can be ignored. These must be evaluated separately.

When the total damping is small, damping can be modelled using an equivalent linear dashpot which equates platform energy losses with the SDOF model energy losses. The SDOF equivalent equation for a mode is of the form

\[ M\ddot{q} + (R_{ST} + R_{RAD} + R_{VH} + R_{SOIL})q + Kq = F(t) \]  

where \( M \) = modal virtual mass of the structure (includes added mass)  
\( R_{ST} \) = modal steel hysteretic damping  
\( R_{RAD} \) = modal radiation or wave making damping  
\( R_{VH} \) = modal viscous hydrodynamic damping  
\( R_{SOIL} \) = modal soils damping  
\( K \) = modal stiffness of the structure  
\( q \) = generalized coordinate obtained from modal analysis for the particular mode  
\( F(t) \) = total linear modal force

The undamped modal natural frequency, \( \omega_n \), and damping ratio, \( \xi_n \), are

\[ \omega_n = \sqrt{K/M} \]  
\[ \xi_n = \frac{R_{ST} + R_{RAD} + R_{VH} + R_{SOIL}}{2\omega_n M} \]  

Using these, the SDOF equivalent system reduces to

\[ \ddot{q} + 2\omega_n \xi_n \dot{q} + \omega_n^2 q = \frac{F(t)}{M} \]  

DAMPING RATIO PREDICTION

Steel Hysteretic Damping

Steel hysteretic damping refers to the energy lost due to internal dissipation within a steel member under cyclic loading. As discussed in Ref. [1], the fraction of the total strain energy dissipated per cycle due to steel hysteretic damping within the outer pile of the caisson may be approximated by the following:

\[ \frac{\Delta V_{ST}}{V_{ST}} = 2\xi_{ST} = 2\xi_{0.0047} \]  

where \( \Delta V_{ST} \) = strain energy dissipated in the outer pile per cycle  
\( V_{ST} \) = peak strain energy stored in the outer pile  
\( J = \text{constant} = 500 \times 10^{-12} / \text{psi} \) for SAH 1020 steel  
\( E = \text{Young's modulus} = 29.5 \times 10^6 \text{ psi} \) for SAH 1020 steel.

The energy loss per cycle can be simply expressed in terms of a linear equivalent damping ratio for a single degree of freedom system. The correct relationship is given by

\[ 4\pi\xi_{ST} = \frac{\Delta V_{ST}}{V_{ST}} \]  

Therefore,

\[ \xi_{ST} = \frac{0.0047}{2} = 0.24\% \]

This damping ratio is assumed to apply to the other materials in the structure as well; particularly the grout. The grout would probably have somewhat higher losses than this and therefore the value of 0.24% as an overall material hysteretic damping is likely a lower bound. In the remainder of the report this damping will continue to be referred to as \( \xi_{ST} \) in reference to the losses in the steel. However, it is applied to the entire mechanical energy of the structure.

Radiation (Wave Making) Damping

A platform oscillating in the ocean creates waves which radiate outward, dissipating energy away from the structure. For a vertical cylinder of uniform diameter, the modal radiation damping can be estimated using linear potential flow theory. A derivation is contained in a 1976 report by Petrauskas [4].

Based on these results, an approximate expression for the modal radiation damping coefficient for a non-uniform vertical cylinder which creates deepwater radiated waves is...
Equation 11 is valid for deep water waves which decay exponentially. Using a Bretschneider (ITTC) [6] two parameter wave spectrum,

\[ G_n(f) = \frac{1}{4} H_s^2 \frac{f_p^4}{\hat{f}^2} e^{-1.25(f_p/\hat{f})^4} \]  

where \( H_s \) is significant wave height

\( f_p \) is peak wave frequency (Hz).

estimates of the modal viscous hydrodynamic damping were obtained for the sea states corresponding to experimentally derived values of \( H_s \) and \( f_p \). Typical values of \( \xi_{\text{RAD}} \) in the experiments were in the vicinity of 0.15%. The detailed results will be summarized later. As computed above, the modal viscous hydrodynamic damping is probably an upper bound since a unidirectional wave spectrum has been used to compute \( \xi_{\text{RAD}} \). Spreading would decrease the effective \( \xi_{\text{RAD}} \).

**Soils Damping**

Compared with the other components of the total damping, the characterization and modeling of soils damping is more complex and less well established. Two types of soil damping exist: material (internal) soil damping, which is a hysteretic form of damping, and geometric (radiation) damping, which is analogous to wave making damping. Material damping is usually specified as the fraction of soil strain energy dissipated per cycle and is expressed as \( 4\pi \times \text{times a constant soils damping ratio} \), \( \xi_{\text{SMD}} \), which is believed to have a value between 3 and 10%. \( \xi_{\text{SMD}} \) is also known as the specific damping ratio. Geometric damping is present only if the frequency of oscillation exceeds a threshold value which depends on the soil stratum. In general, geometric damping is not significant for small structures at the frequencies associated with wave loading, in Gulf of Mexico sediments.

As developed in detail in Ref. [1], an equivalent modal soils damping ratio \( \xi_{\text{SOIL}} \) can be estimated from the soil material damping ratio \( \xi_{\text{SMD}} \) by computing the ratio of the energy loss per cycle in the soil to the total caisson energy. The method utilizes the lumped soil spring foundation of the finite element idealization of the caisson to compute the soil strain energy. The final expression for \( \xi_{\text{SOIL}} \) is

\[ \xi_{\text{SOIL}} = \frac{AV_{\text{SOIL}}}{4\pi V_T} = \frac{\xi_{\text{SMD}}}{\text{W} \sum \frac{K_s(z_1)\psi^2(z_1)}{2}} \]  

where \( AV_{\text{SOIL}} = 4\pi \xi_{\text{SMD}} V \text{SOIL} \)

\[ V_T = \frac{1}{2} K_o \frac{Z_o^2}{2} \text{W}_n \]  

is total caisson energy

\[ K_s(z_1) \]  

lumped soil spring stiffness at \( z = z_1 \)

\[ \psi^2(z_1) \]  

value of the mode shape at \( z = z_1 \)

\( n \) is number of lumped soil springs used in this finite element model.
Equation 13 was evaluated using the four soil springs of the finite element model of the caisson and the results are shown in Table 2. This technique is approximate and research leading to the development of new techniques is warranted.

**TOTAL DAMPING RATIO MEASUREMENTS**

The total modal damping was estimated for each of the two fundamental bending modes from biaxial accelerometer data recorded in March 1980. To isolate the fundamental modal directions, a correlation function rotation scheme based on a Mohr’s circle algorithm was used. In this analysis, the modal orientation is defined as that orientation for which the time histories from a biaxial pair of accelerometers would have a minimum coherence at the natural frequencies of the two fundamental bending modes. This procedure is described in Ref. [11]. Once the modal orientation is determined, single-channel MEM (Maximum Entropy Method) spectral analysis is used to estimate natural frequencies and damping ratios. The damping ratio estimate is based on the half-power bandwidth method [7].

The results of the damping estimation are shown in Table 2 for three different days of testing. In this table, items 2, 3, 4, 6, and 7 were theoretically obtained, as previously described; item 1 is an experimental value. The error bounds on the measured total damping ratio are 95% confidence bounds. The viscous damping ratios. The damping ratio estimate is based on the half-power bandwidth method [7].

**RESPONSE PREDICTION**

After the modal equivalent damping components have been estimated, the next step is to predict modal response. The reciprocity method of response prediction, proposed by Vandiver [8], will be used for this purpose. This approach utilizes the principle of reciprocity for ocean waves which relates the radiation damping of a structure oscillating in a calm sea to the linear wave force exerted on the structure if it were held fixed in incident waves. This technique yields a simple result for the mean square modal response and its use is valid only for lightly damped modes excited by linear wave forces. This method directly incorporates wave spreading effects.

The caisson is an ideal structure to test this modal response prediction technique because the structure is axi-symmetric and the two fundamental bending natural frequencies are almost identical.

To demonstrate the performance of this response prediction method, the coordinate system shown in Figure 2 was used. In this figure, the x and y axes define the orientation of the two fundamental bending modes of the caisson. The angle 0 defines the angle between the x-modal direction and the basic wave direction in a directionally spread sea. The terms $\alpha_x^2$ and $\alpha_y^2$ represent the mean square modal displacement response on the helideck along the x and y modal axes, respectively.

The prediction of the mean square modal response $\sigma_x^2$ of the x-mode within two half power bandwidths of the resonant frequency $f_x$ is shown in Equation 14.

$$\frac{\sigma_x^2}{\sigma_{SOIL}^2} = \frac{C_x}{80 \pi^3 f_x^5} \frac{G_x(f_x)}{f_x^{RAD}(f_x)} \frac{G_x(f_x)}{f_x^{T}(f_x)}$$

The expression for the y-mode is identical with the x subscripts replaced by y subscripts. The term $C_x$ depends on both structural geometry and the directional wave amplitude spectrum. In this paper, the directional wave amplitude spectrum is modelled using frequency independent spreading functions as defined in the following equation.

$$G_y(f, \theta) = G_x(f) D(\theta)$$

Where $G_y(f, \theta)$ = directional wave amplitude spectrum

$D(\theta) = $ spreading function

$\theta = $ angle of incidence of various wave components

When integrated over all possible wave incidence angles, the directional wave amplitude spectrum satisfies

$$C_x(f) = \int_0^{2\pi} C_x(f, \theta) d\theta$$

For a single pile platform, the expressions from Ref. [8] for $C_x$ and $C_y$ simplify to

$$C_x = 2\int_0^{2\pi} D(\theta) \cos^2 \theta d\theta$$

and

$$C_y = 2\int_0^{2\pi} D(\theta) \sin^2 \theta d\theta$$

The sum of Equations 17 and 18 is trivial to evaluate, because $\sin^2 \theta + \cos^2 \theta = 1$. Therefore

$$\int_0^{2\pi} D(\theta) d\theta = 1.0$$

and therefore

$$C_x + C_y = 2.0$$

Therefore, even though the actual spreading function is unknown, the total mean square response may be predicted by summing the individual mean square responses $\sigma_x^2$ and $\sigma_y^2$.

$$\sigma_x^2 + \sigma_y^2 = \frac{\beta_y^2}{40 \pi^3 f_x^5} \frac{G_x(f_x)}{f_x^{RAD}(f_x)} \frac{G_x(f_x)}{f_x^{T}(f_x)}$$

where it has been assumed that the natural frequencies, modal masses and damping ratios of both modes are approximately equal. The x subscript has been retained.
The measured data is in terms of acceleration. For purposes of comparing predicted and measured data it is convenient to express Equation 21 in terms of mean square acceleration. The mean square response within a half power bandwidth of the natural frequency is a narrow band random process and the following simple relation may be used for either the x or y directed modes.

\[ a_x^2 = 164 f_0^4 s_x^2 \]  \hspace{1cm} (22)

Therefore, the total mean square acceleration may be expressed as,

\[ a^2 = a_x^2 + a_y^2 = \frac{2 \rho g 3}{\pi a_x^2} + \frac{G_x(f)}{\pi a_x^2} \]  \hspace{1cm} (23)

The natural frequency \( f \) of this platform was measured as well as the total modal damping \( \zeta \). The modal mass was determined from the finite element model and the radiation damping was computed theoretically. Therefore using measured values of the wave amplitude spectrum evaluated at the natural frequency it is possible to predict the total mean square acceleration response of the structure and compare it to that observed.

The results are summarized in Table 3 for data taken on two separate days. The predicted and measured values are in very close agreement. The actual error in the predictions is substantially less than might have been expected considering the uncertainty in the estimates of the total damping ratio. This is the first known experimental confirmation of the response prediction technique embodied in Equation 14.

**DESCRIPTION OF THE FINITE ELEMENT MODEL**

In order to estimate both the fundamental flexural mode shape and the relationship between platform dynamics and stresses within the pile, a 21-mode two-dimensional dynamic finite element (F.E.) model of the single pile platform was developed. In this model, the platform was represented using 14 beam elements and the soil was replaced by four linear soil springs. A schematic of the F.E. model is shown in Figure 3 and a detailed description of its formulation is contained in Ref. [1]. To achieve model natural periods which closely matched the measured values required iteration of the soil spring stiffnesses.

Both the fundamental flexural mode shape and the soil spring stiffnesses required in the damping estimation were obtained from the optimized F.E. model. In addition, the maximum stress in the pile, when subjected to sinusoidal wave excitation, at the natural frequency was found to be at a level approximately 26 feet below the mudline. For vibration in the fundamental mode, the helicopter deck displacement to maximum stress transfer function was determined to be 4.95 KSI/foot. This value is required later in the prediction of fatigue life.

**DYNAMIC RESPONSE FATIGUE LIFE ESTIMATION**

For offshore structures experiencing significant dynamic response in low and moderate sea states, the governing design criteria is often the prevention of failure caused by low stress, high cycle fatigue. To estimate the dynamic response fatigue life of the operational single pile platform, the reciprocity method of response prediction and the dynamic finite element model were combined with a fatigue accumulation model which assumes the stress process is a narrow band Gaussian process with Rayleigh distributed peaks. The result is a versatile expression for the mean rate of accumulation of fatigue damage. The details of this procedure are outlined below.

The expression for the mean rate of accumulation of fatigue damage, based on a stress range S-N curve, is from Ref. [3]

\[ \dot{F}_1 = \frac{\nu_0^+}{b/2} \frac{f_0^2}{c} \]  \hspace{1cm} (24)

where \( \dot{F}_1 \) = mean rate of accumulation of fatigue damage at a location in the structure which experiences a mean square stress \( a_s^2 \)

\[ \nu_0^+ = \text{average zero crossing rate of the stress process} \]

\[ b, c = \text{constants of the stress range S-N curve} \]

The dynamic finite element model was used to determine the relationship between the maximum stress in the pile and helideck displacements in mode x. Mathematically, this relationship can be expressed as

\[ a_x^2 = B a_s^2 \]  \hspace{1cm} (25)

where \( B = \) maximum stress/helideck displacement transfer function

\[ B = 4.95 \text{ KSI/ft.} \]

A value of \( C_x^2 \) was estimated using the reciprocity method, as defined in Equation 14, in conjunction with the Bretschneider (ITTC) wave spectrum of Equation 12. This technique requires that the wave environment at the location be modeled with a set of significant wave height (\( H_s \)) and peak wave period (\( T_p \)) pairs, each with an assigned annual probability of occurrence (\( P_i \)). The resulting expression for the mean rate of accumulation of dynamic response fatigue damage in mode x due to sea state i is shown in Equation 26.

\[ \dot{F}_1 = \frac{f_x}{c} \left[ \frac{f_x^2}{f_0^2} \left( \begin{array}{ccc} B P_i^3 & C & \frac{2 f_x^4}{T_p} e^{-1.25 \left( f_x^2 / f_0^2 \right)} \\ \frac{2 f_x^4}{T_p} e^{-1.25 \left( f_x^2 / f_0^2 \right)} & b/2 \end{array} \right) \right] \]  \hspace{1cm} (26)

where \( \nu_0^+ = \frac{f_x}{T_p} \)

in this equation, all the parameters which affect the dynamic response fatigue life of a structure are included in a readily usable form. Finally, a fatigue
life estimate (FLE) can be obtained using

\[ \text{FLE} = \sum_{i=1}^{J} F_i P_i \]  

where \( J \) = total number of sea states.

### Fatigue Life Calculation

The fatigue life of the operational single pile platform was estimated using the twelve Gulf of Mexico sea states given in Ref. [10]. For each sea state, Equation 26 was used to compute the rate of accumulation of fatigue damage associated with first mode response of the caisson. The AWS-X modified stress range S-N curve was used to determine values for \( b \) and \( c \) \((b = 4.38, c = 2.64 \times 10^{11})\). In addition, it was assumed that the modal frequency and modal mass remain constant over the lifetime of the structure. For the initial estimate, the ratio \( \frac{f_{\text{RAD}}}{f_{\text{p}}} \) was fixed for all sea states at 0.2. The selection of a representative value of \( C \) is not straightforward. However, two useful limiting cases may be easily evaluated.

The least damaging case is one in which the directional spreading is uniform over all angles. The result of this is that both modes must respond equally and therefore, \( C_x = C_y = 1.0 \). These two limiting cases will yield upper and lower bound fatigue life estimates for this caisson when substituted into Equations 26 and 27. The result for the uniform spreading case is a fatigue life of 161 years. Unidirectional spreading reduces this estimate to a worst case fatigue life of 35 years. These calculations were computed using a conservative estimate of \( \frac{f_{\text{RAD}}}{f_{\text{p}}} = 0.2 \). In fact, the field measurements indicate it is approximately half that value. Accounting for this increases the estimated fatigue life by a factor of approximately four. These results indicate that the fatigue life of this structure is between 140 and 644 years. These predictions do not account for fatigue damage caused by quasi-static response of the structure to large low frequency waves. Though many fewer in number, these waves will reduce the fatigue life somewhat. Non-linear drag exciting forces at the natural frequency of the structure have also been neglected.

### CONCLUSIONS

In this paper a complete dynamic analysis of a single caisson platform has been presented. A finite element model provided mode shapes, stress transfer functions and some insight into the behavior of the soil. Predictions of all components of damping were made. Field measurements were used to obtain estimates of natural frequencies, damping ratios and mode shapes (see OTC 4286). The measured mean square response was compared to predictions based on the reciprocity method and found to be quite accurate. Finally, an analysis of the fatigue life of the structure was performed, in such a way that the sensitivity of fatigue calculations to directional spreading of the waves was clearly indicated.

### NOMENCLATURE

- \( \alpha \) \( \beta \) \( \gamma \) modal amplitude
- \( b, c \) constants of the stress range S-N curve
- \( B \) maximum stress in the pile per foot of helideck displacement
- \( C_D \) drag coefficient
- \( C_x, C_y \) constants dependent on directional spreading
- \( d \) pile diameter
- \( D(\theta) \) wave spreading function
- \( E \) Young's modulus of elasticity
- \( f \) cyclic frequency (Hz)
- \( f_p \) peak wave frequency
- \( f(t) \) modal wave force on fixed structure
- \( G_{\eta_x} \) wave amplitude spectrum
- \( G_{\eta_y}(f, \theta) \) directional wave amplitude spectrum
- \( h \) water depth
- \( H_s \) significant wave height
- \( j \) constant in steel hysteretic damping expression
- \( k \) wave number
- \( K \) total modal stiffness
- \( K_s(z_i) \) soil spring stiffness at \( z = z_i \)
- \( M \) total modal virtual mass
- \( P \) annual probability of occurrence for sea state \( i \)
- \( P_{\text{RAD}} \) modal radiation damping coefficient
- \( P_{\text{ST}} \) total modal damping coefficient
- \( p \) modal displacement coordinate
| $R_{\text{VH}}$ | modal viscous hydrodynamic damping coefficient |
| $T_p$ | peak wave period |
| $V_{\text{SOIL}}$ | peak strain energy stored in the soil during a cycle |
| $\Delta V_{\text{SOIL}}$ | strain energy dissipated in the soil per cycle |
| $V_{\text{ST}}$ | peak strain energy stored in the steel members during a cycle |
| $\Delta V_{\text{ST}}$ | strain energy dissipated in the steel members per cycle |
| $V_T$ | maximum energy stored in the total structure during a cycle |
| $\Gamma()$ | Gamma function |
| $\theta$ | wave incidence angle |
| $\theta_0$ | mean wave direction of directionally spread sea |
| $V_o$ | zero upcrossing frequency of stress process (Hz) |
| $\xi_{\text{RAD}}$ | modal radiation damping ratio |
| $\xi_{\text{smd}}$ | specific damping ratio of soils |
| $\xi_{\text{SOIL}}$ | modal soils damping ratio |
| $\xi_{\text{ST}}$ | modal steel hysteretic damping ratio |
| $\xi_T$ | total modal damping ratio |
| $\xi_{\text{VH}}$ | modal viscous hydrodynamic damping ratio |
| $\rho_w$ | density of water |
| $\psi(z)$ | mode shape ordinate at elevation $z$ |
| $\omega_n$ | undamped modal natural frequency |
| $\sigma_a$ | total mean square helideck acceleration due to $x$ and $y$ first mode responses |
| $u_r$ | root mean square relative velocity |
| $\sigma_{x,y}^{2}$ | mean square helideck displacement in $x$ and $y$ modes |
| $\sigma_{x,y}^{2}$ | mean square helideck accelerations in $x$ and $y$ modes |

**REFERENCES**


## TABLE 1
SUMMARY OF WAVE, WIND, AND RESPONSE DATA

<table>
<thead>
<tr>
<th>Date</th>
<th>REEL #1</th>
<th>REEL #2</th>
<th>REEL #3</th>
<th>REEL #4</th>
<th>REEL #5</th>
<th>REEL #6</th>
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<tbody>
<tr>
<td>Observed Wave Height (ft)</td>
<td>NW 1-3</td>
<td>ENE 5-8</td>
<td>ENE 2-4</td>
<td>ESE 3-5</td>
<td>ESE 3-5</td>
<td>ESE 3-5</td>
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<tr>
<td>Observed Wind Speed (knots)</td>
<td>NW @ 20</td>
<td>ENE @ 30</td>
<td>ENE @ 10</td>
<td>ESE @ 20</td>
<td>ESE @ 20</td>
<td>ESE @ 20</td>
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<td>Peak Wave Period (sec)</td>
<td>7.28</td>
<td>6.79</td>
<td>7.10</td>
<td>7.10</td>
<td>7.10</td>
<td>7.10</td>
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<td>Orientation of Mode x</td>
<td>25°S of E</td>
<td>13°N of E</td>
<td>35°N of E</td>
<td>7°S of E</td>
<td>15°S of E</td>
<td>7°S of E</td>
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<td>Mode x Natural Frequency Est. (Hz)</td>
<td>.325</td>
<td>.323</td>
<td>.323</td>
<td>.323</td>
<td>.324</td>
<td>.324</td>
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<tr>
<td>Mode Total Damping Ratio Est.</td>
<td>1.1+/-.3</td>
<td>1.0+/-.4</td>
<td>.9+/-.3</td>
<td>1.9+/-.6</td>
<td>.9+/-.4</td>
<td>.9+/-.4</td>
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<tr>
<td>Orientation of Mode y</td>
<td>20°N of E</td>
<td>32°S of E</td>
<td>10°S of E</td>
<td>38°N of E</td>
<td>30°S of E</td>
<td>30°S of E</td>
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<td>Mode y Natural Frequency Est. (Hz)</td>
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<td>.328</td>
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<td>.327</td>
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<td>1.9+/-.7</td>
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*Dynamic Absorber was operating during this reel. See OTC 4283*

## TABLE 2
DAMPING SUMMARY

<table>
<thead>
<tr>
<th>MODAL DAMPING RATIO ESTIMATES (%)</th>
<th>Reel 1, 3/24/80</th>
<th>Reel 2, 3/25/80</th>
<th>Reel 3, 3/28/80</th>
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</thead>
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<td>1.1+/-.3</td>
<td>1.3+/-.3</td>
<td>1.0+/-.4</td>
</tr>
<tr>
<td>Mode y</td>
<td>1.4+/-.4</td>
<td>.9+/-.2</td>
<td>1.1+/-.3</td>
</tr>
<tr>
<td>1. $\xi_m$ - Total Measured</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2. $\xi_{ST}$ - Steel Hysteretic</td>
<td>0.24</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>3. $\xi_{RAD}$ - Radiation</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>4. $\xi_{VH}$ - Viscous Hydrodynamic</td>
<td>0.11</td>
<td>0.11</td>
<td>0.17</td>
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<tr>
<td>5. $\xi_{T} = (\xi_{RAD} + \xi_{VH})$ + $\xi_{ER}$</td>
<td>.64</td>
<td>.64</td>
<td>.48</td>
</tr>
<tr>
<td>6. $\xi_{SOIL}$ for $\xi_{SMD} = 0.03$</td>
<td>0.53</td>
<td>0.53</td>
<td>0.53</td>
</tr>
<tr>
<td>7. $\xi_{SOIL}$ for $\xi_{SMD} = 0.05$</td>
<td>0.88</td>
<td>0.88</td>
<td>0.88</td>
</tr>
</tbody>
</table>
### TABLE 3

**COMPARISON OF PREDICTED AND MEASURED HELIDEOCK ACCELERATION RESPONSES**

<table>
<thead>
<tr>
<th>Date Recorded</th>
<th>REEL 3</th>
<th>REEL 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date Recorded</td>
<td>3/25/80</td>
<td>3/28/80</td>
</tr>
<tr>
<td>Natural frequency $f_x$ (Hz)</td>
<td>0.323</td>
<td>0.323</td>
</tr>
<tr>
<td>$G_n(f_x)$ (ft$^2$/Hz)</td>
<td>1.2</td>
<td>0.7</td>
</tr>
<tr>
<td>$f_{RAD}(/n_x)$ (%)</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>$f_{T}(/n_x)$ (%)</td>
<td>1.0±/-.4</td>
<td>0.9±/-.2</td>
</tr>
<tr>
<td>$f_{RAD}/f_{T}$</td>
<td>1.10</td>
<td>1.22</td>
</tr>
<tr>
<td>$\sigma_p^2$ (ft$^2$/sec$^4$)</td>
<td>1.09</td>
<td>0.71</td>
</tr>
<tr>
<td>(95% confidence bounds)</td>
<td>(.78 to 1.82)</td>
<td>(.50 to .92)</td>
</tr>
<tr>
<td>$\sigma_m^2$ (ft$^2$/sec$^4$)</td>
<td>.91</td>
<td>.72</td>
</tr>
</tbody>
</table>

- $M = 3162$ lb-sec$^2$/ft, (slugs), modal mass
- $p_w = 1.988$ lb-sec$^2$/ft$^3$(slugs/ft$^3$)
- $g = 32.11$ ft/sec$^2$
- $\sigma_p^2 = $ total predicted mean square helideck displacement
- $\sigma_m^2 = $ total measured mean square helideck displacement

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Fig. 1 — Caisson production platform
Fig. 2 — Coordinate system definition

Fig. 3 — Finite element model