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The Significance of Dynamic Response in the Estimation of Fatigue Life

The fatigue life of offshore structures is investigated under the conditions that dynamic response to waves is assumed to play a significant role. Under these conditions, the study emphasizes the significance of the placement of natural frequencies and the amount of modal damping. The results may be used to assess the confidence bounds on fatigue life estimates that result from uncertainties in design stage estimates of structural natural frequencies, and damping.

Introduction

The purpose of this analysis is to investigate the sensitivity of fatigue life calculations to variations in natural frequencies and modal damping ratios. The results of such an analysis may be used to reveal the extent to which uncertainties in the estimates of such parameters will affect the estimated fatigue life of offshore structures excited by waves.

This analysis does not consider the uncertainties in material properties or the fatigue damage accumulation models themselves. This area is left to the materials specialists. This study also leaves to others the analysis of the uncertainties associated with the description of the sea states to be encountered by the structure. A structural model and its idealization are selected and one method of wave force estimation is used. The wave force model assumes that drag exciting forces are negligible and that finite wave amplitude effects are not significant. In any specific application these two assumptions can and should be checked. However, for the computation of high cycle-low stress fatigue damage on large deepwater structures these assumptions are usually valid.

For the case that drag excitation cannot be neglected, the results of some recent research at MIT are mentioned. With these results the second-order statistics of response may be estimated including nonlinear drag exciting forces.

The exclusion of finite wave amplitude effects is probably valid for large deepwater structures in low to moderate seas, which contribute the most to high-cycle, low-stress fatigue damage. The governing nondimensional parameter is likely the ratio of wave amplitude to water depth for slender bottom mounted structures. However, this is an area in which some additional research is justified.

The Fatigue Accumulation Model

For the purpose of this study the assumed form of the fatigue damage accumulation model is that used by Crandall and Mark [1] when the stress history is assumed to be described by a narrow band random process. This formulation implicitly assumes a Palgren-Miner rule for damage accumulation. Equation (1) describes the mean rate of accumulation of the fatigue damage index for a location β in the structure due to a directionally spread random sea with mean direction θ_o.

$$F(\beta,\theta_o) = \frac{\nu_o^+}{c}(2^3\sigma_s^2)^{b/2}\Gamma(1+b/2) \qquad (1)$$

$F(\beta,\theta_o)$ = the mean rate of accumulation of the fatigue damage index at position β, due to a wave field with nominal direction of propagation θ_o

σ_s^2 = the mean square stress at position β

ν_o^+ = the average zero upcrossing rate of the stress process in Hz

$\Gamma(\ )$ = the Gamma function

b,c = constants of the S-N fatigue curve of the material as defined by equation (2), where N is the number of cycles to failure with a stress range S

$$NS^b = c \qquad (2)$$

This model, and the material constants b and c are assumed fixed. This leaves ν_o^+ and σ_s^2 as variables to be considered.

ν_o^+ depends on the frequency content of the wave spectrum as well as the wave amplitude to stress transfer function for the structure. If the structure has no natural frequencies in the region of significant wave force, then the response is generally quasi-static in nature and ν_o^+ is governed primarily by the frequency content of the wave spectrum. When the stress is primarily due to the response at a natural frequency, then ν_o^+ is strongly dependent on the natural frequency.

In both of the cases the response is approximately narrow band and the use of equation (1) is appropriate. In the case that the response spectrum is composed of significant quasi-static and dynamic response peaks then it may be necessary to modify the foregoing equation. One such modification is the use of a final correction factor, such as proposed by Wirsching [2], in which rain flow cycle counting procedures are used to obtain a correction factor to account for broad band stress spectra. The use of such a correction factor is assumed to be valid here.

The task is then to investigate the sensitivity of the mean square stress σ_s^2 and the average zero upcrossing frequency,

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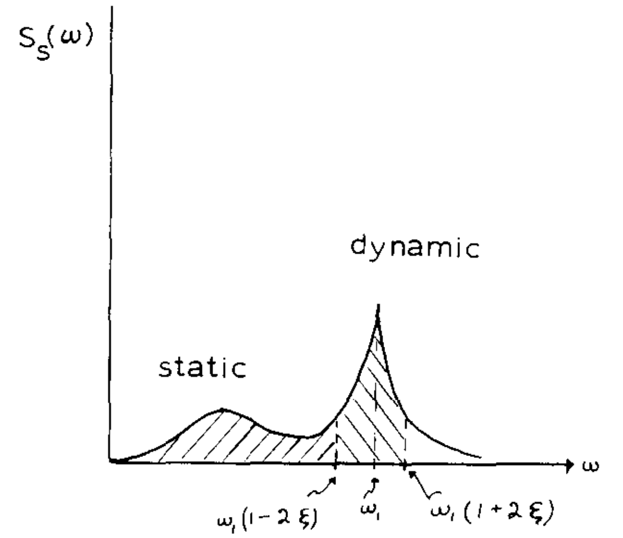

Fig. 1 The partitioning of stress into static and dynamic components

ν_o^+, to variations in structural natural frequency and modal damping.

Quasi-Static and Dynamic Contributions to Mean Square Stress

In this study, it is assumed that mean square stress at a point in a structure may be approximated by the sum of a quasi-static component due to low frequency waves and a dynamic component due to the damping controlled response of natural modes of the structure excited by the higher frequency components of the wave spectrum. This is comparable to the procedure of supplementing a full static finite element solution with the dynamic contributions of the significantly responding natural modes.

In this analysis the response is assumed to be quasi-static up to within one-half power bandwidth of the lowest natural frequency of the structure. Furthermore, the lowest natural frequency is not allowed to be less than the peak frequency of the wave spectrum. The computation of the mean square stress is then accomplished by summing the mean square static component with the dynamic contributions.

The quasi-static component of stress at a specific location is assumed uncorrelated with the dynamic components. However, for closely spaced natural frequencies, correlation between the stress components of two or more natural modes may have to be considered.The partitioning of static and dynamic contributions to the total stress is illustrated in Fig. 1, a stress spectrum with a quasi-static stiffness controlled peak and one damping controlled resonant peak.

The quasi-static mean square stress, σ_q^2, is obtained by integrating the stress spectrum up to $\omega_c = \omega_1\ (1-2\xi)$ where ω_1 is the lowest natural frequency and ξ is the modal damping ratio of that mode.

$$\sigma_q^2 = \int_o^{\omega_c} S_s(\omega)d\omega \tag{3}$$

where $S_s(\omega)$ is the stress spectrum.

For a complex structure σ_q^2 could be computed from a static finite element model. The calculation of the static mean square stress may include the influence of drag forces, in which an equivalent linearization procedure has been used or a more accurate nonlinear wave force spectrum has been computed using the results of Dunwoody [3]. Drag forces are neglected in the examples of this report.

This static approximation does neglect any dynamic amplification at frequencies below the cut off.

The average zero upcrossing frequency of the static component of stress is computed from the zero and second-order moments of the truncated spectrum

$$\omega_q^2 = \frac{\int_o^{\omega_c} \omega^2 S_s(\omega)\ d\omega}{\int_o^{\omega_c} S_s(\omega)\ d\omega} = \frac{1}{\sigma_q^2}\int_o^{\omega_c} \omega^2 S_s(\omega)\ d\omega \tag{4}$$

σ_q^2 and ω_q^2 for the example calculations are assumed to be provided for the purposes of the remaining discussions.

The dynamic or damping controlled contributions to the mean square stress are computed separately. The area under the stress spectrum as shown in Fig. 1 for $\omega_1(1-2\xi) \le \omega \le \omega_1(1+2\xi)$ is defined as the mean square dynamic response for mode 1.

There may be more than one mode which has significant dynamic response. The dynamic contribution of each must be separately evaluated. In this report the mean square dynamic response of all significant modes will be computed using techniques described by Vandiver [4]. In this refrence it is shown that the mean square dynamic response of an individual mode x is given by

$$\sigma_x^2 = \frac{2.5 C_x \rho_w g^3}{m_x \omega_x^5} S_\eta(\omega_x) \frac{R_r(\omega_x)}{R_T(\omega_x)} \tag{5}$$

where

σ_x^2 = mean square dynamic deflection of xth normal mode
m_x = modal mass
ω_x = natural frequency
$S_\eta(\omega_x)$ = wave amplitude spectrum evaluated at ω_x
ρ_w = density of water
g = acceleration of gravity
$\frac{R_r(\omega_x)}{R_T(\omega_x)}$ = ratio of the radiation (wave making) to total modal damping evaluated at ω_x

Nomenclature

A = constant of proportionality
b,c = constants of SN fatigue life curve
C_x = factor which accounts for spreading of waves
E = Young's modulus
$F(\ )$ = rate of fatigue damage accumulation
g = acceleration of gravity
K_x = modal stiffness
m_x, M_x = modal mass
N = number of cycles
$R_r(\omega)$ = radiation damping
$R_T(\omega)$ = total damping
$S_{max}(\omega)$ = Krogstad upper-bound wave spectrum
$S_\eta(\omega)$ = wave amplitude spectrum
$S_s(\omega)$ = stress spectrum
γ = Wirsching correction factor
$\Gamma(\ )$ = Gamma function
ν_o^+ = average zero upcrossing frequency in Hz
ω = frequency in radians per second
ω_p = frequency of the peak of the wave spectrum
ω_x = natural frequency of mode x
ξ = modal damping ratio
ρ_w = density of water
σ_d^2 = mean square dynamic response
σ_q^2 = mean square static response
σ_s^2 = mean square stress
σ_x^2 = mean square deflection for mode x
θ = angle of wave incidence

This result is valid for lightly damped modes excited by linear wave forces. The constant C_x depends on structural geometry and wave spreading and is assumed to have been evaluated as described in reference [4]. Through knowledge of the mode shape and structural details, the mean square stress at a specific location can be related to σ_x^2.

If there is more than one mode contributing in a significant way to the dynamic response then the stress at any specific location in the structure will depend upon the superposition of stresses from each mode. If the natural frequencies of each responding mode are different, (at least so that their damping controlled peaks as defined in Fig. 1 do not overlap), then the stresses contributed by each may be assumed to be uncorrelated and the total mean square stress will be the sum of the mean square stresses due to each individual mode. This is a consequence of the fact that waves, and hence wave forces, of different frequencies are uncorrelated. If two peaks overlap then the correlation between stress components must be included.

The mean zero upcrossing frequency for mode x is simply $\omega_x/2\pi$. The mean upcrossing frequency for the combined static and dynamic stress history may be computed as a weighted sum of the individual contributions as shown in equation (6) for a system with a single dynamic component.

$$\nu_o^+ (H_z) = \frac{1}{2\pi} \left[\frac{\omega_q^2 \sigma_q^2 + \omega_1^2 \sigma_{d1}^2}{\sigma_q^2 + \sigma_{d1}^2} \right]^{1/2} \tag{6}$$

where ω_q^2 and σ_q^2 reflect the static response and ω_1^2 and σ_{d1}^2 are the natural frequency and mean square dynamic stress contributed by mode 1.

The Effect of Natural Frequency on Fatigue

If the fundamental flexural natural period of a steel jacket structure was taken to be 3.5 s for the purpose of fatigue life computation, and the as-installed natural period turned out to be 4.0 s, how much would the estimated fatigue life be reduced? Recalling equation (1) and adding γ, a Wirsching-type correction factor to account for broad-banded spectral effects, yield

$$F = \gamma \frac{\nu_o^+}{c} (2^3 \sigma_s^2)^{b/2} \Gamma(1 + b/2) \tag{7}$$

Assuming that wave-spreading effects have been taken into consideration, then a variation in the estimated natural period of a mode will influence three parameters in equation (7): γ, ν_o^+ and σ_s^2. σ_s^2 will change because its dynamic component will change. This is because the wave spectrum is a rapidly changing function of frequency, and as can be seen in equation (4), the mean square dynamic response is proportional to the wave spectrum divided by the natural frequency raised to the fifth power. ν_o^+ will change, as can be seen in equation (6), because it depends on the natural frequency as well as on the mean square dynamic stress; γ may change because the broadbandedness of the stress spectrum may change. If an asterisk is used to denote the result with a shifted natural frequency, then the ratio of fatigue damage between two cases may be expressed as

$$\frac{F^*}{F} = \left(\frac{\gamma^*}{\gamma} \right) \left(\frac{\nu_o^{+*}}{\nu_o^+} \right) \left(\frac{\sigma_s^{2*}}{\sigma_s^2} \right)^{b/2} \tag{8}$$

The two extreme cases are simple to evaluate. The first is when the estimated and actual natural periods are so short that the dynamic component of σ_s^2 is negligible. This is true for most structures when the lowest natural frequency corresponds to a period of 2.4 s or less. In this case $F^*/F = 1.0$.

The more interesting extreme is when σ_{d1}^2, the dynamic component of stress of a single natural mode is assumed to be much larger than the static component. This may not always be the case, but provides a useful upper bound on the variation of fatigue with natural frequency. One way to estimate this case is through the ratio of fatigue damage at two different natural frequencies.

$$\frac{F^*}{F} = \left(\frac{\nu_o^+}{\nu_o^+} \right) \left(\frac{\sigma^*_{d1}}{\sigma_{d1}} \right)^b = \left(\frac{\omega_1^*}{\omega_1} \right) \left(\frac{\sigma_{d1}^*}{\sigma_{d1}} \right)^b \tag{9}$$

Because the process is narrow banded, the Wirsching correction factor reduces to 1.0 for both cases, and the upcrossing frequency reduces to the natural frequency divided by 2π.

$$\nu_o^+ = \frac{\omega_1}{2\pi} \tag{10}$$

The only remaining step is to evaluate the frequency dependence of σ_{d1}^2, the mean square stress from dynamic response of the mode. This is quite easy and may be estimated directly from equation (5), with one minor modification. In normal mode formulations, the product of the modal mass and the natural frequency squared is simply the modal stiffness.

$$M_1 \omega_1^2 = K_1 \tag{11}$$

If the natural frequency varies because the modal stiffness is different than expected, then the effect on mean square stress should be evaluated using equation (5). However, if the modal mass varies, then the effect on mean square stress should be evaluated after substituting equation (11) into equation (5), as follows.

$$\sigma_1^2 = \frac{2.5 C_1 \rho_w g^3}{K_1 \omega_1^3} S_\eta(\omega_1) \frac{R_r(\omega_1)}{R_T(\omega_1)} \tag{12}$$

If it is assumed for small variations in natural frequency that the ratio between mean square modal deflection and mean square stress at a location of concern remains constant, then the frequency dependence of the mean square stress is the same as that for mean square deflection as given in equations (5) or (12). This is essentially an assumption that the mode shape does not change, which is not true, but is adequate here for the purpose of a simple check on sensitivity to changes in natural frequency. Therefore, stress and deflection may be related as shown.

$$\sigma_{d1}^2 = A^2 \sigma_1^2 \tag{13}$$

If there is any substantial wave spreading, such as cosine squared, then C_1 is only weakly dependent on frequency and is assumed not to vary. Similarly the ratio of radiation to total damping is assumed constant in comparison to other sources of variation. Lumping all constant quantities into A^2 in equation (13), two expressions for $\sigma_{d1}2$ result, depending on whether the source of change was mass or stiffness.

$$\sigma_{d1}^2 = \frac{A^2}{M_1} \frac{s_\eta(\omega_1)}{\omega_1^5} \quad \begin{pmatrix}\text{stiffness} \\ \text{changes}\end{pmatrix} \tag{14}$$

$$\sigma_{d1}^2 = \frac{A^2}{K_1} \frac{S_\eta(\omega_1)}{\omega_1^3} \quad \begin{pmatrix}\text{mass} \\ \text{changes}\end{pmatrix} \tag{15}$$

It remains only to evaluate the frequency dependence of the wave spectrum.

Krogstad [5] has presented evidence that wind-driven wave spectra may be modeled at frequencies higher then the frequency of the peak in the wave spectrum as given in the following:

$$S_{\max}(f) = 1.62 \times 10^{-3} f^{-4.6} \text{m}^2 - \text{s} \tag{16}$$

This is the upper-bound curve for spectral values, but possesses the frequency dependence characteristic of the high frequency side of wind-driven wave spectra.

Expressed as a function of ω, equation (16) takes the form

$$S_{\max}(\omega) = \frac{1}{2\pi} \times 1.62 \times 10^{-3} \left(\frac{\omega}{2\pi}\right)^{-4.6} \quad (17)$$

Assuming all of the constants in this spectrum are absorbed into the constant A^2 in equations (14) or (15) yields

$$\sigma_{d1}^2 = \frac{A^2}{M_1} \frac{1}{\omega_1^{9.6}} \quad \begin{pmatrix}\text{stiffness}\\ \text{changes}\end{pmatrix} \quad (18)$$

$$\sigma_{d1}^2 = \frac{A^2}{K_1} \frac{1}{\omega_1^{7.6}} \quad \begin{pmatrix}\text{mass}\\ \text{changes}\end{pmatrix} \quad (19)$$

Substituting each of these expressions into equation (9) and setting the slope, b, of the S – N curve equal to 4.1 for welded tubular joints yields

$$\frac{F^*}{F} = \left(\frac{\omega_1{}^*}{\omega_1}\right)^{-14.6} \quad \text{(mass changes)} \quad (20)$$

$$\frac{F^*}{F} = \left(\frac{\omega_1{}^*}{\omega_1}\right)^{-18.7} \quad \text{(stiffness changes)} \quad (21)$$

Therefore, if the natural frequency is 10 percent greater than predicted, then the fatigue life will be increased by a factor of 4.02 or 5.94, depending on the source of the error.

These examples were upper bound situations in which the quasi-static contributions to mean square stress were assumed small. In most cases of practical interest both contributions will be of importance and the sensitivity to natural frequency variation will not be so extreme.

The Effect of Damping on Fatigue

A variation in the estimated damping of a normal mode influences the mean square dynamic contribution to the total stress directly, and the average up-crossing frequency indirectly, because of its dependence on the mean square dynamic stress.

To place an upper bound on the significance of an error in the prediction of modal damping an analysis similar to the previous section may be performed. If only the dynamic component of a single mode is presumed to contribute to the total mean square stress, then proceeding as before leads immediately to the following conclusion:

$$\frac{F^*}{F} = \left\{\left(\frac{R_r(\omega_1)}{R_T(\omega_1)}\right)^* \Big/ \left(\frac{R_r(\omega_1)}{R_T(\omega_1)}\right)\right\}^{b/2} \quad (22)$$

All terms involving frequency directly cancel out because the natural frequency does not change in the example.

The method of computing mean square dynamic stress used in this analysis is somewhat unconventional and not widely used in the industry. Therefore, to reflect conventional practice the same upper bound on the sensitivity of fatigue damage calculations to variations in estimated total damping may be expressed as follows:

$$\frac{F^*}{F} = \left(\frac{\xi_T}{\xi_T{}^*}\right)^{b/2} \quad (23)$$

when ξ_T and $\xi_T{}^*$ are the estimated and actual total modal damping ratios, which are commonly estimated in the range from 1 percent to 5 percent.

It is the position of the author that the uncertainty in estimating the ratio of the radiation to total damping is much less than the uncertainty in estimating the total modal damping itself. Furthermore, the use of equation (12) leads to estimates of mean square dynamic stress which are bounded because the ratio of radiation to total damping is at most 1.0. No such upper bound exists when conventional methods of computing dynamic response are used.

Furthermore, conventional methods of estimating response requrie independent estimates of the modal wave force spectrum and the total modal damping. This ignores the fact that the modal radiation damping and the linear modal wave force spectrum are proportional to one another [4]. Thus, two sources of uncertainty enter the calculations where only one exists.

For the sake of example, suppose in either formulation the damping is overestimated by a factor of 2.0. This will lead to an overestimate of the fatigue life by a factor of

$$(2)^{b/2} = 4.14 \text{ for } b = 4.1 \quad (24)$$

for the extreme case of no static contribution to the stress.

Conclusions

By means of general formulations and a specific example, the dependence of fatigue on the uncertainties related to natural frequencies and damping ratios have been demonstrated.

Uncertainties related to the prediction of structural natural frequencies are primarily related to the structural idealizations or models used in the design process. The greatest weakness is probably in the area of foundation modeling. The behavior of soil under cyclic loading conditions remains a rather uncertain field. Assumptions regarding soils stiffness have dramatic impact on the estimation of structrual natural frequencies.

The uncertainties related to damping estimates have several sources. One of the greatest is a general lack of accurate estimates of damping on existing structures. This issue and a method for obtaining improved measurements of damping on existing structures are addressed by Campbell [6]. The second reason for uncertainty is that direct estimation of individual components of damping are rarely made, and the knowledge required for making such estimates is not widely available in the industry. To understand the complete damping problem one must understand the fluid mechanics, the soil mechanics, the structural mechanics, and their interaction. A final source of misuse of damping is that the relationships between exciting forces and damping mechanisms are too frequently ignored.

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References

1 Crandall, S. H., and Mark, W.D., *Random Vibration in Mechanical Systems*, Academic Press, 1973.

2 Wirsching, P. H., and Light, M. C., "Probability-Based Fatigue Design Criteria for Offshore Structures," Report to the American Petroleum Institute, API-PRAC Project H 15, Nov. 1979.

3 Dunwoody, A. B., and Vandiver, J. K., "The Influence of Separated Flow Drag on the Dynamic Response of Offshore Structures to Random Waves," presented at the International Symposium on Hydrodynamics in Ocean Engineering, Trondheim, Norway, Aug. 1981.

4 Vandiver, J. K., "Prediction of the Damping Controlled Response of Offshore Structures to Random Wave Excitation," *Society of Petroleum Engineers Journal*, Feb. 1980.

5 Krogstad, H. E., et al., "Analysis of Wave Spectra from the Norwegian Continental Shelf," *Proceedings of the POAC*, Trondheim, 1979.

6 Campbell, R. B., and Vandiver, J. K., "The Estimation of Natural Frequencies and Damping Ratios of Offshore Structures," *Proceedings of the 1980 Offshore Technology Conference*, Paper No. OTC 3861, Houston, 1980, pp. 53–61.