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Structural Reliability Fundamentals and Their Application to Offshore Structures

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U.S. DEPARTMENT OF COMMERCE, Malcolm Baldrige, *Secretary*
NATIONAL BUREAU OF STANDARDS, Ernest Ambler, *Director*

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1. INTRODUCTION

The objective of structural reliability is to develop design criteria and verification procedures aimed at ensuring that structures built according to specifications will perform acceptably from a safety and serviceability viewpoint. This objective could in principle be achieved by meeting the following requirement: failure probabilities (i.e., probabilities that structures or members will fail to satisfy certain performance criteria) must be equal to or less than some benchmark values referred to as target failure probabilities.[a] Such an approach would require [2, 13]:

1. The probabilistic description of the loads expected to act on the structure.

2. The probabilistic description of the physical properties of the structure which affect its behavior under loads.

3. The physical description of the limit states, i.e., the states in which the structure is unserviceable (serviceability limit states) or unsafe (ultimate limit states). Examples of limit states include: excessive deformations (determined from functional considerations); excessive accelerations (determined from studies of equipment performance, or from ergonomic studies on user discomfort in structures experiencing dynamic loads); specified levels of nonstructural damage; structural collapse.

4. Load-structural response relationships covering the range of responses from zero up to the limit state being considered.

5. The estimation of the probabilities of occurrence of the various limit states (i.e., of the failure probabilities), based on the elements listed in items 1 through 4 above and on the use of appropriate probabilistic and statistical tools.

6. The specification of maximum acceptable probabilities of occurrence of the various limit states (i.e., of target failure probabilities).

The performance of a structure would be judged acceptable from a safety or serviceability viewpoint if the differences between the target and the failure probabilities were either positive (in which case the structure would be overdesigned) or equal to zero.

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[a] An alternative statement of this requirement is that the reliabilities corresponding to the various limit states must be equal to or exceed the respective target reliabilities (reliability being defined as the difference between unity and the failure probability).

from the reliability analysis of exemplary designs, i.e., designs that are regarded by professional consensus as acceptably but not overly safe. Such inferences are part of the process referred to as safety calibration against accepted practice.

While there are instances where such a process can be carried out successfully, difficulties arise in many practical situations. For example, structural reliability calculations suggest that current design practice as embodied in the American National Standard A58.1 [8] and other building standards and codes is not risk-consistent. In particular, estimated reliabilities of members designed in accordance with current practice are considerably lower for members subjected to dead, live, and wind loads than for members subjected only to dead and live loads [9, 13, 17], especially when the effect of wind is large compared to the effect of dead and live loads.* Whether these differences are real or only apparent, i.e., due to shortcomings of current structural and reliability analyses, remains to be established. Thus, it is not possible in the present state of the art to determine whether it is the lower or the higher estimated reliabilities that should be adopted as target values.

In spite of both theoretical and practical difficulties, structural reliability tools can in a number of cases be used to advantage in design and for code development purposes. The objective of this report is to present a review of fundamental topics in structural reliability as applied to individual members, which are potentially applicable to ocean engineering problems. These topics include: the estimation of failure probabilities; safety indices; and safety (or load and resistance) factors.

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* The earliest justification of current design practice with respect to wind loading was traced by the authors to Fleming's 1915 monograph Wind Stresses [10], which states: "Maximum wind loading comes seldom and lasts but a short time. The working stresses used for the loading may therefore be increased by 50% above those used for ordinary live- and dead-loads."

where $X^{(1)}$ = extreme value of $\zeta(t)$ during a time interval $t_1 = T/n$, T = lifetime of structure, n = integer and $F_{X^{(n)}}$ = cumulative distribution function of $X^{(n)}$. The cumulative distribution function of $X^{(1)}$, $F_{X^{(1)}}$, is referred to as the <u>parent distribution</u> of $X^{(n)}$. Equation 2 holds if successive values of $X^{(1)}$ are identically distributed and statistically independent. An application of equation 2 is presented in the following example.

<u>Example</u>. Let $X^{(1)} \equiv U_a$ denote the largest yearly wind speed at a given location. Then $X^{(n)} \equiv U$ denotes the largest wind speed occurring at that location during an n-year period (equal to the lifetime of the structure). It is assumed that the largest yearly wind speed, U_a has an Extreme Value Type I distribution, i.e.,

$$F_{U_a}(u_a) = \exp\left[-\exp\left(-\frac{u_a-\mu}{\sigma}\right)\right] \tag{3}$$

It can be shown that

$$\mu \simeq \overline{U}_a - 0.45\,\sigma_{U_a} \tag{4}$$

$$\sigma \simeq 0.78\,\sigma_{U_a} \tag{5}$$

where $\overline{U}_a$ and σ_{U_a} = sample mean and sample standard deviation of the largest annual wind speed data, U_a. From Eqs. 2 through 5 it follows that the probability distribution of the largest lifetime wind speed, U, is:

$$F_U(u) = \exp\left[-\exp\left(-\frac{u-\mu_n}{\sigma_n}\right)\right] \tag{6}$$

where

$$\mu_n \simeq \overline{U} - 0.45\,\sigma_U \tag{7}$$

$$\sigma_n \simeq 0.78\,\sigma_U \tag{7a}$$

$$\overline{U} = \overline{U}_a + 0.78\,\sigma_{U_a}\,\ell n\; n \tag{8}$$

$$\sigma_U = \sigma_{U_a} \tag{9}$$

and n = lifetime of structure in years.

elasticity, breaking strength)[a], i.e.,

$$Q = Q(X_1, X_2, \dots X_n) \tag{14}$$

$$R = R(X_1, X_2, \dots X_n) \tag{15}$$

Substitution of equations 14 and 15 into equations 10, 11, and 12 yields the mapping of the failure region, safe region, and failure boundary onto the space of the variables $X_1, X_2, \dots, X_n$. The equation of the failure boundary thus can be written as

$$g(X_1, X_2, \dots, X_n) = 0 \tag{16}$$

The well-behaved nature of structural mechanics relations generally ensures that equation 13 is the mapping of equation 12 onto the load effect space. Equation 16 is thus the mapping onto the space $X_1, X_2, \dots, X_n$ not only of equation 12, but of equation 13 as well. Therefore, once it is made clear at the outset that the problem is formulated in the load, or in the load effect, space, it is permissible to refer generically to Q and Q_e as "loads" and to R and R_e as "resistances", and to omit the index "e" in equation 13.

The important case is noted where relations between loads and/or resistances and more fundamental variables of the problem, $X_1, X_2, \dots X_n$, can only be obtained numerically. In that case, equation 16 cannot, in general, be written in closed form.

It is useful in various applications to map the failure region, the safe region, and the failure boundary onto the space of the variables $Y_1, Y_2, \dots, Y_r$, defined by transformations

$$Y_i = Y_i(X_1, X_2, \dots, X_n) \quad (i = 1, 2, \dots, r) \tag{17}$$

For example, if in equation 16 $X_1 = \rho$, and $X_2 = U$, where ρ = air density and U = wind speed, a variable representing the dynamic pressure may be defined by the transformation $Y_1 = 1/2\ \rho U^2$, and equation 16 may be mapped onto the space of the variables $Y_1, X_3, X_4, \dots, X_n$. Another example is the frequently used set of transformations

$$Y_1 = \ell nR \tag{18}$$

$$Y_2 = \ell nQ \tag{19}$$

---

[a] These are sometimes referred to as basic variables. We will use here simply the term "variables", since what constitutes a basic variable is in many instances a matter of convention. For example, the hourly wind speed at 10 m above ground in open terrain, which is regarded in most applications as a basic variable, depends in turn upon various random storm characteristics, such as the difference between atmospheric pressures at the center and the periphery of the storm, the radius of maximum storm winds, and so forth.

We consider a failure boundary in the space of a given set of variables, and denote by S its mapping in the space of the corresponding reduced variables.

The safety index, β, is defined as the shortest distance in this space between the origin and the boundary S [14][a]. The point on the boundary S that is closest to the origin, as well as its mapping in the space of the original variables, is referred to as the <u>checking point</u>. For any given structural problem, the numerical value of the safety index depends upon the set of variables in which the problem is formulated. The examples that follow illustrate the meaning of the safety index and the dependence of its numerical value upon the set of variables being used.

<u>Example 1</u>. It is assumed that the only random variable of the problem is the load (effect) Q. The resistance--a deterministic quantity--is denoted by R. The mapping of the failure boundary

$$Q - \overline{R} = 0 \qquad (23)$$

onto the space of the reduced variable $q_r = (Q - \overline{Q})/\sigma_Q$ (i.e., onto the axis Oq_r--see figure 1) is a point, q_r^*, whose distance from the origin O is $\beta = (R - Q)/\sigma_Q$. The safety index represents in this case the difference between the values R and Q measured in terms of standard deviations, σ_Q. It is clear that the larger the safety index β (i.e., the larger the difference R - Q for any given σ_Q, or the smaller σ_Q for any given difference R - Q), the smaller the probability that $Q \geq R$.

<u>Example 2</u>. Consider the failure boundary in the load space (equation 10), and assume that both R and Q are random variables. The mapping of equation 12 onto the space of the reduced variables $q_r = (Q - Q)/\sigma_Q$ and $r_r = (R - R)/\sigma_Q$ is the line [13]

$$\sigma_Q q_r + \overline{Q} - \sigma_R r_r - R = \overline{0} \qquad (24)$$

(figure 2). The distance between the origin and this line is

$$\beta = \frac{\overline{R} - \overline{Q}}{(\sigma_R{}^2 + \sigma_Q{}^2)^{1/2}} \qquad (25)$$

---

[a] This definition is applicable to statistically independent variables. If the variables of the problem are correlated, they can be transformed by a linear operator into a set of independent variables [14]. Note that an alternative, generalized safety index was proposed in reference 15, whose performance is superior in situations where the failure boundaries are nonlinear (see also Chapter 9 of reference 16).

largest lifetime wind speed are $\overline{U}$ = 70 mph, σ_U = 8.61 mph. An expansion of equation 30 in a Taylor series about the mean, $\overline{U}$, yields

$$\overline{Q} = a\overline{U}^2(1 + V_U^2) \qquad (31)$$

and

$$V_Q \simeq 2V_U \qquad (32)$$

i.e., $\overline{Q} \simeq 13.3$ ksi and $\sigma_Q \simeq 3.27$ ksi. The equation of the failure surface in the space of the variables U, R is

$$aU^2 - R = 0 \qquad (33)$$

and its mapping in the space of the reduced variables u_r, r_r is

$$(u_r + \frac{\overline{U}}{\sigma_U})^2 = \frac{\sigma_R}{a\sigma_U^2}(r_r + \frac{\overline{R}}{\sigma_R}) \qquad (34)$$

The value of the safety index is $\beta = 4.31$ (figure 3). The coordinates of the checking point are $r_r^* = -2.51$, $u_r^* = 3.50$, to which there correspond in the U, R space the coordinates U* = 100.14 mph, R* = 26.76 ksi. It can be verified that the values of the safety index corresponding to the variables Q, R (equation 25) and ℓnQ, ℓnR (equation 29) are $\beta \simeq 4.66$ and $\beta \simeq 3.69$, respectively.

Note that the mean and standard deviation of the largest lifetime wind speed or of the largest lifetime load, which are needed for the calculation of the safety index, cannot be estimated directly from measured data, but must be obtained from the probability distribution of the lifetime extreme. This distribution is estimated from the parent distribution that best fits the measured annual extreme data. Knowledge of, or an assumption concerning, the parent probability distribution is required for the estimation of the safety index in all cases involving a random variable that represents a lifetime extreme.

Safety Indices and Failure Probabilities: The Case of Normal Variables

Consider the space of the variables Q and R, and assume that both variables are normally distributed. Note that the failure boundary (equation 12) is linear. Since the variate R − Q is normally distributed, the probability of failure can be written as

$$P_f = F(R - Q < 0)$$

$$= \frac{1}{\sqrt{2\pi}\sigma_{R-Q}}\int_{-\infty}^{0} \exp\left[-\frac{1}{2}\left(\frac{x-(\overline{R-Q})}{\sigma_{R-Q}}\right)^2\right] dx$$

$$= 1 - \Phi\left(\frac{\overline{R} - \overline{Q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}}\right) \qquad (35)$$

Table 1. Probabilities of Failure of Four Members Corresponding to Various Distributions of the Variables

Member (1)	$\overline{Q}$ (ksi) (2)	σ_Q (ksi) (3)	$\overline{R}$ (ksi) (4)	σ_R (ksi) (5)	β (Eq. 25) (6)	β (Eq. 29) (7)	P_f		
							a (Eq. 35) (8)	b (Eq. 37) (9)	c (Eq. 38) (10)
I	13.3	3.27	35.27	3.39	4.66	3.69	1.6×10^{-6}	1.1×10^{-4}	1.2×10^{-3}
II	18.0	2.79	35.27	3.39	3.93	3.69	4.2×10^{-5}	1.1×10^{-4}	1.4×10^{-3}
III	19.6	2.10	35.27	3.39	3.93	4.08	4.2×10^{-5}	2.5×10^{-5}	3.0×10^{-4}
IV	15.5	3.70	35.27	3.39	3.93	3.20	4.2×10^{-5}	6.9×10^{-4}	3.0×10^{-3}

[a] Based on the assumption that R and Q are normally distributed.

[b] Based on the assumption that R and Q are lognormally distributed.

[c] Based on the assumption that R has the distribution of Fig. 5 and that $Q^{1/2}$ has an Extreme Value Type I distribution.

is possible that two members, one subjected to gravity loads and the other to wind or wave loads, will have widely different failure probabilities even if their safety indices calculated by equation 29 are nearly equal. In this case, or in similar cases, a comparative reliability analysis would require the estimation of the failure probability by equation 22 or by alternative, approximate methods. A few such methods are briefly described in the following section.

Approximate Methods for Estimating Failure Probabilities

We first describe the method referred to as normalization at the checking point [19]. The principle of the method is to transform the variables, X_i, into a set of approximately equivalent normal variables, X_i^n, having the following property:

$$p_{X_i}(X_i^*) = f(X_i^{n}*) = \frac{1}{\sigma_{X_i^n}} \phi(x_{i_r}^{n*}) \tag{40}$$

$$P_{X_i}(X_i^*) = F(X_i^{n}*) = \Phi(x_{i_r}^{n*}) \tag{41}$$

where $i = 1, 2, \ldots, m$, the asterisk denotes the checking point, p_X and P_X are the probability density function and cumulative distribution function of X_i, respectively, f and ϕ are the normal and standardized normal probability density function, respectively, F and Φ are the normal and standardized normal cumulative distribution function, respectively, $x_{i_r}^n$ is the reduced variable corresponding to X_i^n, and σ_{X^n} is the standard deviation of X_i^n. From equations 40 and 41 it follows that

$$\sigma_{X_i^n} = \frac{\phi(\Phi^{-1}[P(X_i^*)]}{p_{X_i}(X_i^*)} \tag{42}$$

$$\overline{X_i^n} = X_i^* - \Phi^{-1}[P_{X_i}(X_i^*)]\sigma_{X_i^n} \tag{43}$$

where the bar denotes mean value. Once $\overline{X_i^n}$ and $\sigma_{X_i^n}$ are obtained from equations 42 and 43, the problem can be restated in the space of the reduced variables, $x_{i_r}^n$. The safety index, β, is the distance in this space between the origin and the failure boundary. A computer program for calculating this

Equation 47 can be written as

$$X_i^* = \gamma_{\bar{X}_i} \bar{X}_i \tag{46}$$

where

$$\gamma_{\bar{X}_i} = 1 + V_{X_i} \; x_{i_r}^* \tag{47}$$

The quantity $\gamma_{\bar{X}_i}$ is termed the partial safety factor applicable to the mean of the variable X_i.

In design applications the means, X_i, are seldom used, and nominal design values, such as the 100-year wave, the allowable steel stress, F_a, or the nominal yield stress, F_y, are employed instead. Let these nominal values be denoted by $\tilde{X}_i$. Equation 46 can be rewritten as [13]

$$X_i^* = \gamma_{\tilde{X}_i} \tilde{X}_i \qquad (i = 1, 2, \ldots, n) \tag{48}$$

where

$$\gamma_{\tilde{X}_i} = \frac{\bar{X}_i}{\tilde{X}_i} \gamma_{\bar{X}_i} \tag{49}$$

The factor $\gamma_{\tilde{X}_i}$ is the partial safety factor applicable to the nominal design value of the variable X_i.

In the particular case in which the variables of concern are the load, Q, and the resistance, R, the partial safety factors are referred to as the load and resistance factor. For the resistance factor the notation $\phi_{\bar{R}}$ or $\phi_{\tilde{R}}$ is used in lieu of $\gamma_{\bar{R}}$ or $\gamma_{\tilde{R}}$.

From the definition of the partial safety factor (equation 47) and the definition of the checking point in the space of the reduced variables corresponding to $Y_1 = \ell nR$ and $Y_2 = \ell nQ$, it follows that if higher order terms (see equation 27) are neglected

$$\phi_{\bar{R}} \simeq \exp(-\alpha_R \beta V_R) \tag{50}$$

$$\gamma_{\bar{Q}} \simeq \exp(\alpha_Q \beta V_Q) \tag{51}$$

$$\alpha_R = \cos[\tan^{-1}(V_Q/V_R)] \tag{52}$$

$$\alpha_Q = \sin[\tan^{-1}(V_Q/V_R)] \tag{53}$$

3. SUMMARY AND CONCLUSIONS

A review was presented of fundamental topics in structural reliability that are potentially applicable to ocean engineering problems. As mentioned in the report, although a number of studies concerned with structural systems have been reported (e.g., references 2 to 7), the practical usefulness of such studies remains limited, particularly as far as ocean engineering applications are concerned. The present report is concerned with applications to individual members.

Some of the potential advantages of reliability methods in the context of offshore platform analysis and design have been outlined in reference 25. In the present report such methods have been subjected to an independent critical review aimed at highlighting possible difficulties and pitfalls in their application. Principal conclusions of the review are:

1. Uncertainties with respect to structural behavior and to probabilistic characterizations of relevant parameters can render difficult, if not impossible, meaningful comparisons between estimated safety levels of members belonging to different types of structure or to structures subjected to different types of load. These difficulties are compounded by the failure of most current reliability methods to account adequately for the complexities of systems reliability behavior, particularly in cases involving time dependent loads such as wind or waves.

2. Reliability methods based on the use of safety indices cannot be applied in cases involving a random variable that represents a lifetime extreme unless an explicit assumption is made with regard to the parent probability distribution of that variable.

3. Reliability methods based on the use of safety indices or load and resistance factors can in certain instances provide useful comparisons between the safety levels of certain types of members. This is the case only if it can be determined that the relation between the safety index and the failure probability for those members is independent of, or weakly dependent upon, the relative values of the mean and coefficients of variation of the resistances and of the loads.

4. Simplified approximate expressions for partial safety factors should be used with caution, and their range of applicability should be carefully checked against the "exact" expressions from which they are derived.

The writers believe that the observations presented in this report can be helpful in ensuring that reliability analyses conducted for offshore structures can be used prudently and confidently by both practitioners and regulatory bodies.

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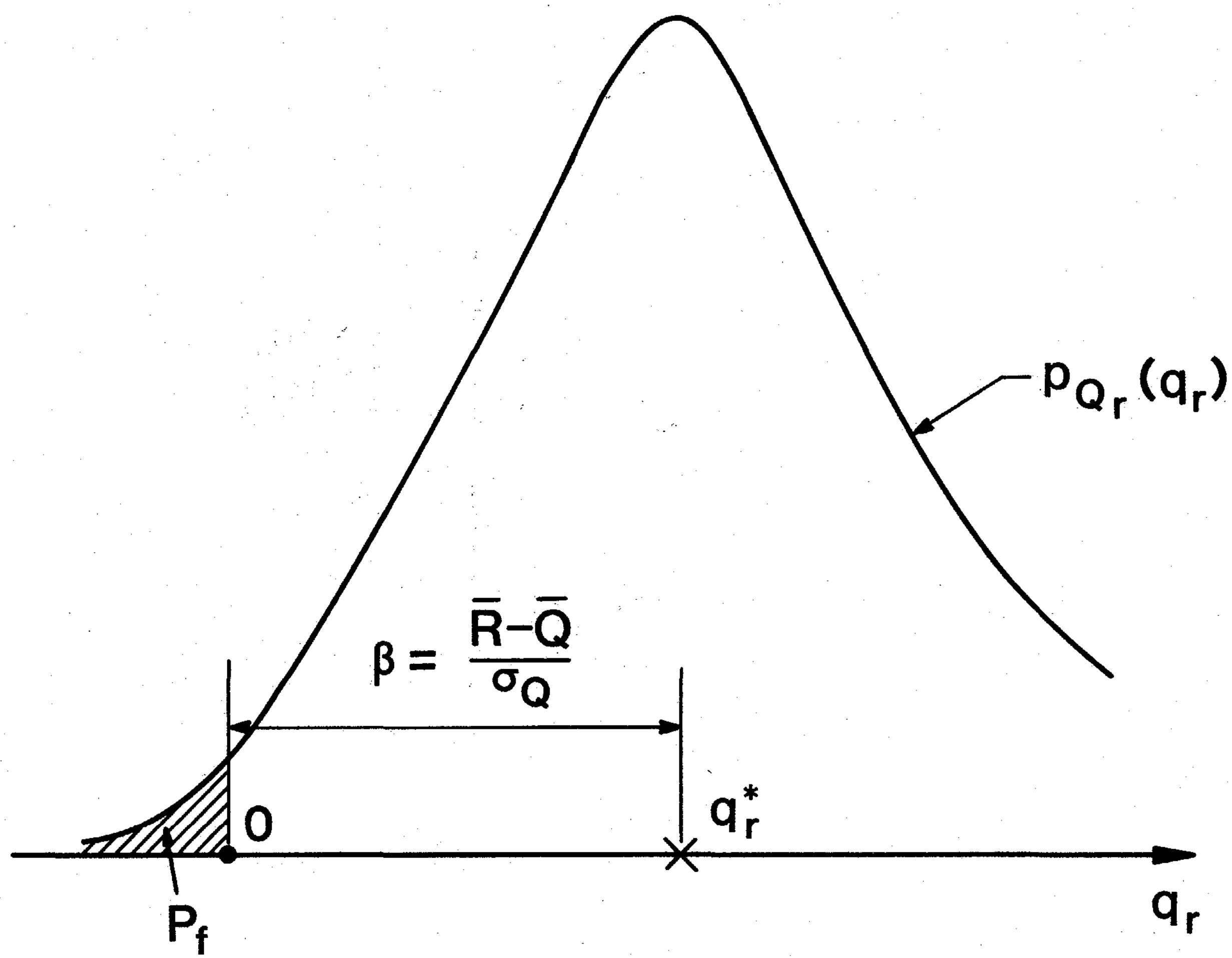

Figure 1. Safety index β in space of variable q_r. The curve $p_{Q_r}(q_r)$ represents the probability density function of q_r. The probability of failure is equal to to hatched area, P_f.

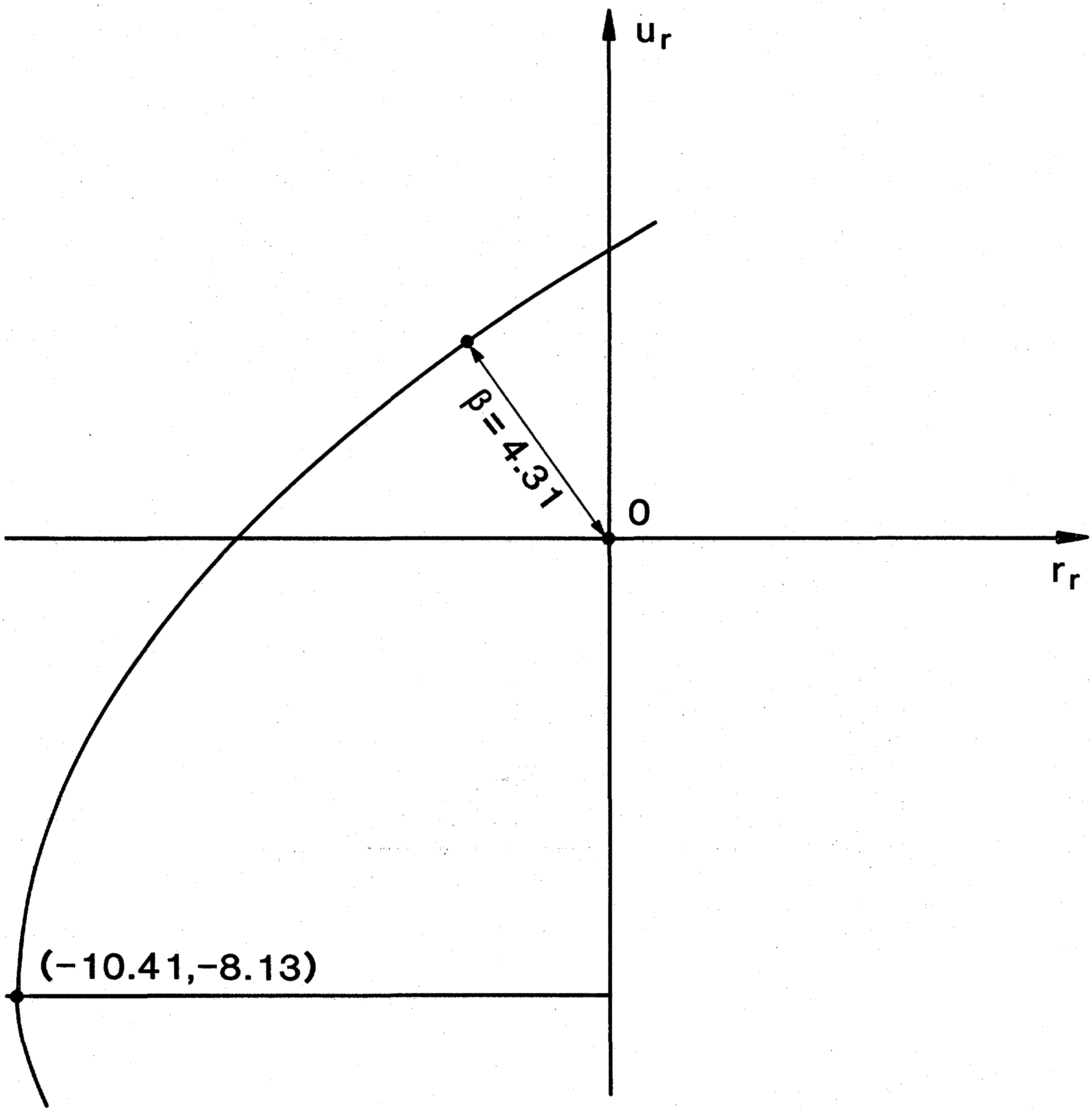

Figure 3. Failure boundary in space of coordinates u_r and r_r

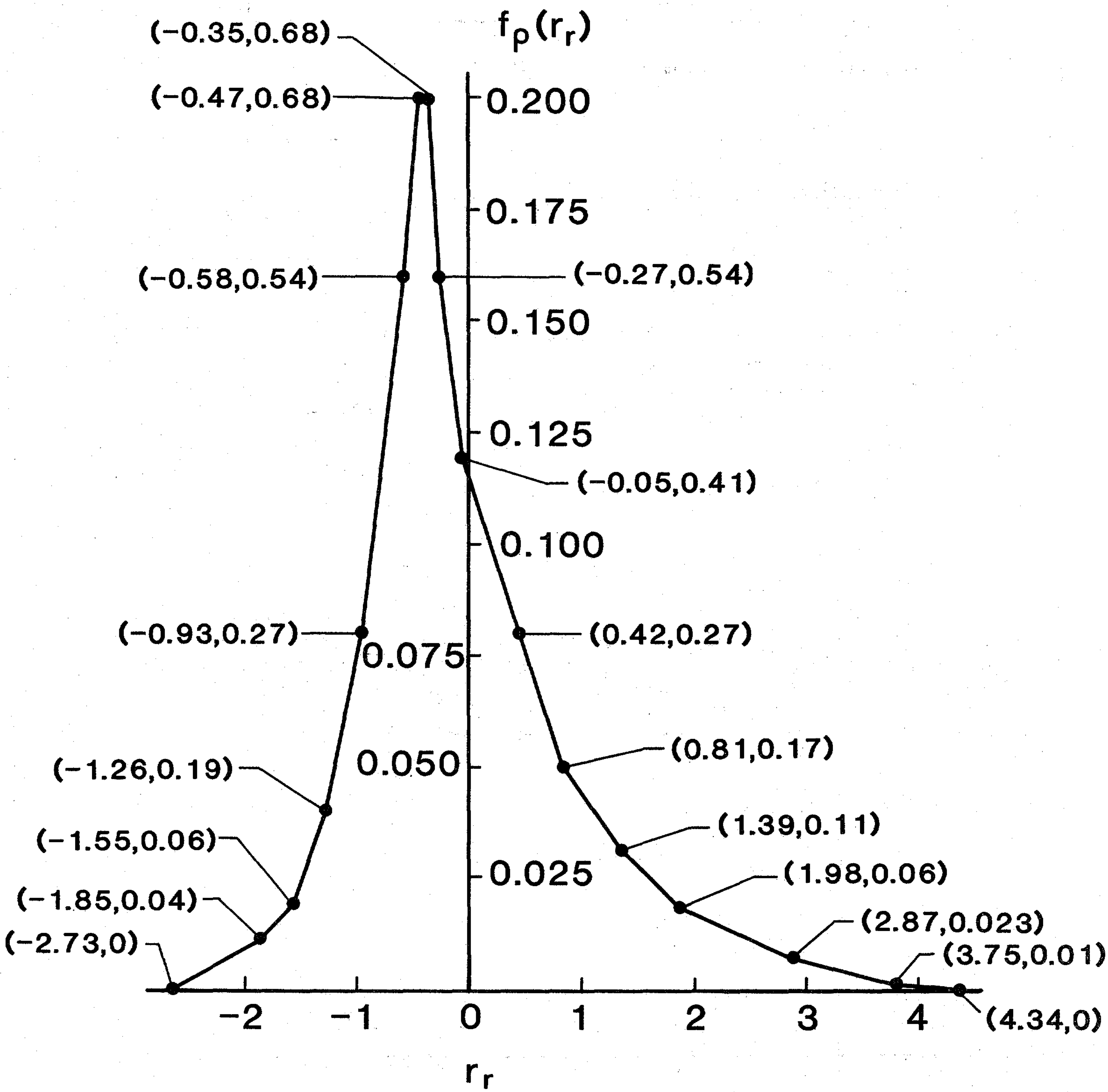

Figure 5. Probability distribution of steel yield stress