Optimization of Horizontal Well Completion

Final Report

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Objective

The productivity of a horizontal well depends on the reservoir flow characteristics, while the reservoir flow characteristics are functions of reservoir parameters and wellbore geometry as well as the hydraulics of the wellbore. To obtain a comprehensive model for horizontal well performance, the influence of well completion on both wellbore hydraulics and reservoir flow performance should be taken into account. Therefore, the objectives of this study are as follows.

1. Experimentally investigate the flow behavior in horizontal wells with both perforation completion and slotted-liner completion. The experimental work is conducted to investigate the effects of the different completion geometry, densities and phasing on the flow behavior in the horizontal well;
2. Based on the experimental study, a wellbore flow model is developed which can be used for various completion scenarios;
3. Develop a reservoir performance model which considers the effect of flow convergence toward slots and perforations on the surface of the well;
4. Couple the wellbore hydraulics and reservoir models to build a comprehensive model that considers the interaction between the horizontal wellbore and the reservoir through small openings on the surface of the well;
5. Develop efficient algorithms to numerically evaluate the complex analytical expressions;
6. Develop a user-friendly software for horizontal well completion design that can be used to perform sensitivity analysis and optimal well completion design;
7. Investigate various completion scenarios to develop completion guidelines for optimzing the well performance.

Over the last two years, we have satisfied these objectives. These achievements are provided to our member companies in the form of this final report and software of Horizontal Well Completion Optimization (HORCOM).

Literature Survey

In a horizontal well, depending upon the completion method, fluid may enter the wellbore at various locations along the well length. The pressure distribution in a horizontal well can influence the well completion and well profile design, as well as having an impact on the production behavior of the well. Therefore, both the pressure drop versus flow behavior along the well and the relationship between the pressure drop along the well and the influx from the reservoir need to be understood.

The petroleum industry started investigating horizontal wellbore hydraulics in the late 1980's. A new friction factor correlation for horizontal wellbore was proposed by Asheim *et al*[1] which includes accelerational pressure losses due to continuous fluid influx along the wellbore. They assumed that the injected fluid enters the main flow with no momentum in the axial direction. Kloster[2] performed experimental work and concluded that the friction factor versus Reynolds number relationship for perforated pipes with no injection from the perforation does not show the characteristics of regular pipe flow. The friction factor values were 25-70% higher than those of regular commercial pipes. He also observed that small injections through perforations reduced the friction factor.

Yuan[3] and Yuan *et al*[4] studied the flow behavior in perforated horizontal wells and slotted-liner completed horizontal wells. By using the principles of mass and momentum conservation, a general horizontal well friction factor expression was developed. Horizontal well friction factor correlations for limited cases of completion geometry were developed by applying experimental data to the general friction factor expression. It was observed that the friction factor of a perforated pipe with fluid injection could be either smaller or greater than that of a smooth pipe, depending on influx to main flow rate ratios. Because the available data consider either single opening or limited multiple opening cases, the influence of the shape of the area of the opening was not thoroughly investigated. Yuan's work, however, forms the basis of this extension on wellbore hydraulics

In 1990, Dikken[5] emphasized the importance of wellbore pressure losses for an openhole horizontal well for the first time. He, however, used the assumption of uniform specific productivity to couple the wellbore and reservoir flows. This assumption, in fact, neglects the influence of wellbore hydraulics on the reservoir performance. Therefore, it cannot predict the correct flux and pressure drawdown along the well length.

In 1993 and 1995, Ozkan & Sarica *et al*[6,7] used the physical coupling conditions (pressure and flux continuity at the well surface) to obtain a solution to compute the open-hole horizontal well performance.

In 1994, Yildiz & Ozkan[8] studied the performance of selectively completed horizontal wells (i.e., only some segments of the well are open to flow with arbitrary distribution of the open interval and skin). They derived a general Laplace space solution describing the transient pressure response. The flow rate distribution is obtained as a result of a matrix solution. They also derived the asymptotic solution for different periods of time. In their model, the wellbore pressure losses are neglected (assumption of infinite conductivity).

In 1990, Ahmed, Horne, and Brigham[9] presented an analytical solution for flow into a vertical well via perforations using Green's functions. This solution contains products and series of Bessel functions and their derivatives. An array of eigenvalues is computed from an implicit equation and are used in the computation of the solution. They failed to calculate the explicit equation for the eigenvalues. Although they considered the perforation as a surface source and performed coordinate transforms to express the integration for the complicated perforation geometry, they still treated the perforations as line sinks. For our project, the integration along perforation surface on horizontal well is also difficult. So this method of coordinate transform might be of special meaning as a reference.

Spivak and Horne[10] studied the transient pressure response due to production with a slotted-liner completed vertical wellbore using source function method in 1982(?). They modeled the slots as line sources of finite length. However, the simplification they used is not applicable for general slot distributions.

Hazenberg and Panu[11] investigated flow into perforated drain tubes. The problem considered in their work bears similarities to the horizontal well problem and has potential of yielding a simplified solution.

In 1991, Landman[12] studied the optimization of perforation distribution for horizontal wells. The model couples Darcy's flow for each perforation in an infinite reservoir with the pipe flow (1-D momentum equations). Thus the perforated well is treated like a pipe manifold with T-junctions representing the perforations along the wellbore. The authors claimed that their model takes into account the wellbore pressure drop and the effect of perforation distribution. They, however, used a simple approximation for the reservoir flow and their wellbore pressure model does not consider the effects of perforation distribution on the flow pattern and mechanism. This paper still provides useful discussion on the perforating optimization strategies.

In 1998, Yildiz and Ozkan[13] presented a 3-D analytical model for the analysis of transient flow toward perforated vertical wells. This work based on their previous 2-D partially opening model. In their model, the perforations are presented as line sources. They applied Laplace transform to time and Fourier transform to x, y and z coordinates. A pseudo-skin expression is derived from the long-time solution to estimate the inflow performance. The treatment for perforations gives us a good reference to solve for the perforated horizontal wells.

In 1999[14], Ozkan, Yildiz and Raghavan investigated the transient pressure behavior of perforated slant and horizontal wells and discussed the implications of perforations on the analysis of pressure and derivative responses. The results presented are derived from a 3D analytical model. It is shown that convergent flow into perforations significantly influences the early time flow characteristics. Their model and 3-D geometrical treatment provides the basis for our long-time asymptotic solution of perforated horizontal wells.

Experimental Study on Hydraulics of Completed Horizontal Wellbore*

1. Test Facility

An existing small scale Tulsa University Fluid Flow Projects (TUFFP) test facility (Fig. 1) was used to acquire data for different horizontal well completion geometries. The test facility is composed of three parts: a flow loop, test sections (Fig. 2) and an instrumentation console. The flow loop consists of the liquid handling system (water tank, and screw and centrifugal pumps) and metering and flow control sections (turbine meters, temperature transducers, a pressure transducer and control valves). The test section consists of a perforated or slotted test pipe, 50 layers of cloth to ensure uniform influx from the openings, a 6-in. diameter casing housing and instruments to measure the pressures and differential pressures. Water is used as the testing fluid.

2. Tests

Ten new test sections were designed for the investigation of the effects of slot/perforation density and phasing. Each test section is made up of a 10-ft long, 1-in. diameter horizontal pipe with a 4-ft long test section. Experiments were conducted under steady state flow conditions with Reynolds number ranging between 5,000 and 60,000. The following parameters are considered:

- Perforation density and phasing.

* This part of work was finished by Weipeng Jang

- Slot density and distribution.

Table 1 and Table 2 list the different combinations of the above parameters for perforated pipes and slotted liners, respectively. In total, 17 different combinations were available for the analysis of the effects of the completion geometry on the horizontal well fluid behavior. 7 of the 17 combinations, which are denoted by "X" in Table 1 and Table 2, were investigated by Yuan (1997). The remaining 10 combinations, which are denoted by "•" in tables, were investigated in this study.

3. Model Development for Apparent-Friction-Factor

In this study, the general model developed by Yuan et al. (1996) was adopted to analyze the acquired data.

Consider an incompressible fluid flowing isothermally along a uniformly perforated pipe of a cross-section A. The area of each perforation is A_p. Fluid is injected through the perforations into the main flow stream uniformly as illustrated by Fig. 3. The momentum balance for the control volume in the axial direction is

$$p_1 A - p_2 A - \tau_w \cdot \pi \cdot d \cdot \Delta x = \beta_2 \rho \bar{u}_2^2 A - \beta_1 \rho \bar{u}_1^2 A - \rho V_x V_r \beta_p A_p n \tag{1}$$

where p_1 and $\bar{u}_1$ are the pressure and average velocity at the inlet of the control volume, and p_2 and $\bar{u}_2$ are the pressure and average velocity at the exit. n is the number of perforations along the distance Δx.

For the three terms on the left-hand side of the above equation, we assume that average properties completely define the flow field. The first two terms on the right hand side of the equation use the average velocities by introducing momentum correction factors, β_1 and β_2, which are defined by the following equation:

$$\beta = \frac{1}{AV^2}\int_A u^2 dA \tag{2}$$

where u and V are the velocity distribution and the average velocity in cross-section A, respectively.

The last term of Eq. (1) represents the acceleration of flow resulting from fluid injection. When the injected fluid enters the main flow stream through the perforations, the streamlines change directions. Each local mean velocity is tangent to the streamlines and can be divided into two components, V_r and V_x, as shown in Fig. 3. Fluid is transported into the main flow with a radial velocity component V_r, while retaining some axial momentum from velocity component V_x. V_r is equal to V_p due to continuity. β_p is the momentum correction factor for the influx stream .

For multiple injection points, it is convenient to use average properties. The average velocity over Δx is $\bar{u}$ and is defined as follows:

$$\bar{u}=(\bar{u}_1+\bar{u}_2)/2 \tag{3}$$

A mass balance for the control volume is given by

$$\bar{u}_1 A+nV_pA_p=\bar{u}_2 A \tag{4}$$

The influx rate through each perforation is

$$q_{in}=V_pA_p \tag{5}$$

The total volumetric influx rate is

$$Q_{in}=nV_pA_p \tag{6}$$

Velocities $\bar{u}_1$ and $\bar{u}_2$ may be eliminated by employing Eqs. (3) and (4). An apparent friction factor, defined as the ratio of the net imposed external forces to the inertial forces, can be given by:

$$f_T = -\left(\frac{p_2 - p_1}{\Delta x}\right) / \frac{\rho \bar{u}^2}{2d} \tag{7}$$

which is an average friction factor over a length Δx.

The wall friction factor f_w is defined as

$$f_w = (8\tau_w)/(\rho \bar{u}^2) \tag{8}$$

Let

$$\varphi = n/\Delta x \tag{9}$$

$$\bar{Q} = (\pi d^2 \bar{u})/4 \tag{10}$$

$$\phi = V_x/\bar{u} \tag{11}$$

where φ is perforation density.

An expression for the apparent friction factor can then be found by substituting Eqs. (6) through (11) into Eq. (1). Rearranging and simplifying:

$$f_T = f_w + 2d\cdot\left(\frac{\beta_2-\beta_1}{\Delta x}\right) + 2d\varphi\cdot\frac{q_{in}}{\bar{Q}}\cdot \left[\beta_1+\beta_2-\phi\beta_p+\left(\frac{n}{4}\cdot(\beta_1-\beta_2)\cdot\frac{q_{in}}{\bar{Q}}\right)\right] \tag{12}$$

Let

$$C_n = \left[\beta_1+\beta_2-\phi\beta_p+\left(\frac{n}{4}\cdot(\beta_1-\beta_2)\cdot\frac{q_{in}}{\bar{Q}}\right)\right] \tag{13}$$

Equation (12) then becomes

$$f_T = f_w + 2d\cdot\left(\frac{\beta_2-\beta_1}{\Delta x}\right) + C_n 2d\varphi\cdot\frac{q_{in}}{\bar{Q}} \tag{14}$$

The second term on the right-hand side of Eq. (14), $2d(\beta_2-\beta_1)/\Delta x$ is caused by a change in the velocity profile in the x direction. No attempt has been made to evaluate this term in this study. However, this term is negligible in this project since the small rate injection will not affect the velocity field significantly, except in the near wall region. Using Blasius formula $f_w=a(N_{Re})^b$, we get

$$f_T = aN_{Re}^b + C_n 2d\varphi\cdot\frac{q_{in}}{\bar{Q}} \tag{15}$$

where C_n, a, and b are determined experimentally for the different completion scenarios.

4. Results and Discussions

4.1 Multiple Slots Cases

A total number of 360 tests were conducted using the four multiple slots test sections. Figures 4 to 7 show the variations in apparent friction factor with influx to main flow rate ratios and Reynolds numbers for the four test sections with multiple slots completions. Each figure is plotted as apparent friction factor f_T vs. Reynolds number N_{Re}, and the different data series represent experimental results at different influx to main flow rate ratios. As we can see from the figures, in most test sections the f_T is greater than the smooth pipe friction factor calculated from the Blasius formula for all influx to main flow rate ratios. When the influx to main flow rate ratio approaches zero, the f_T vs. N_{Re} curve will move closer to the curve predicted by the Blasius formula.

Lubrication effects were found for the first test section when the flow rate ratio is 1/1000. In all the cases, the f_T decreases considerably with the decreasing of influx to main flow rate ratio at high flow rate ratio cases. However the decrease of the friction factor is negligible at very low influx/main flow rate ratios. We can predict that the friction factor will approach a constant at very small influx over main flow ratios. For a given flow rate ratio, f_T decreases with the increasing of Reynolds number. For a given completion density, f_T is always the smallest when the phasing is 90^o.

Applying regression analysis we get the following correlations for a, b and C_n in Eq. (15):

$$a = -0.611656 + 0.0651749\varphi + \frac{84.2771}{\alpha} \quad (16)$$

$$b = -0.198817 + \frac{1.41439}{(\varphi - 4.5)^2 + 16.0191} - \frac{12.7926}{\alpha} + 0.000133823\varphi \quad (17)$$

$$C_n = 2.25 - 0.0161188\varphi + \frac{12.9954}{\alpha} \quad (18)$$

Next we briefly discuss the effects of the completion phasing on the liquid behavior in horizontal wells. As we can see from Figure 8 and Figure 9, other parameters being equal, the decreasing of the completion phasing from 360^o to 180^o and then to 90^o decreases the total friction factor. The friction factor is smallest when the phasing is 90^o. The possible reasons why the completion phasing has such significant effect on flow behavior in horizontal wells can be: 1). When the phasing is smaller, say 90^o, the influx can be considered entering from all sides, thus there is smaller twist (distortion) against the main stream velocity profile and therefore there's smaller pressure loss due to momentum change. 2). When the influx enters the main flow from more than one direction, a larger area of the boundary layer is lubricated than if the influx is entering from one direction (360^o). The lubrication of the influx can lessen the extent of surface roughness introduced due to completion.

The effect of slots density upon the pressure drop behavior in horizontal wells in this study is quite straightforward: other completion parameters being equal, the apparent friction factor in general increases with the completion density mainly due to increased influx introduced by the extra openings (Figure 10). However this may not necessarily be true when the influx over main flow rate ratio is very small, as we will discuss in the multiple perforation section.

4.2 Multiple Perforation Cases

A total of 490 experimental tests are conducted on the six multiple perforation test sections. The data acquisition and data analysis procedure for multiple perforation cases are the same as those of multiple slots cases. And the general trends of the pressure drop behavior are the same except in this section no lubrication effects were observed.

The following three equations are obtained through regression analysis to estimate a, b and C_n:

$$a = 0.24163 + 0.00275189\varphi^{1.9} + \frac{808.542e^{-\varphi+2.33}}{\alpha} \quad (19)$$

$$b = -0.25 + 0.0334316\,\varphi - 000179399\,\varphi^2 - 0.447302\,\varphi^{0.5} + \frac{0.0513603\,e^{-\varphi}}{\varphi \cdot \alpha} \quad (20)$$

$$C_n = 0.00467386\alpha + \frac{0.556613 \cdot \varphi^{0.235} \cdot e^{\frac{\alpha}{90}+4.9081}}{\varphi \cdot \alpha} + 1.24243 \quad (21)$$

As we mentioned early in the multiple-slots section, the completion phasing has significant effect upon the pressure drop behavior in horizontal wells completed with multiple slotted liners. The apparent friction factor usually drops as the phasing decreases when the other parameters being held equal. The same thing is true in multiple perforation cases (See Figures 11 and 12). What we want to point out here is the effect of phasing is insignificant once the completion density or the influx over main flow rate ratio becomes too small. Under very small perforation density situations (the distance between two neighboring openings is greater than 8 times of the pipe diameter), the single perforation modeling from Yuan's study should be used to analyze the flow behavior in horizontal wells.

Experimental data are compared for the three perforation densities when the influx to average main flow rate ratios equal to 1/50 and 1/1000 (Figure 13 and Figure 14). Figure 13 shows that for influx to main flow rate ratio equal to 1/50, f_T is higher for the higher perforation density case. Figure 14 shows that for influx to main flow rate ratio equal 1/1000, f_T for the three perforation densities are almost the same at low Reynolds number region while f_T is slightly smaller when the density is 20 shots/ft. As discussed in Yuan's study, at very small influx/main flow rate ratios, f_T usually is lower for high perforation density case. One probable reason for this is that the third term on the right-hand side of Eq. 14 (influx contribution to total apparent friction factor) is the dominant term at high influx to main flow rate ratios, while the first term (wall friction factor) is the dominant term at low influx to main flow rate ratios.

5. Evaluation and Comparison

In this section, the new apparent friction factor correlation for the multiple perforation cases is evaluated together with the Asheim[1] *et al*. model and the Ouyang[16] *et al.* model against the experimental data obtained in this study.

Figures 15, 16 and 17 give comparisons among the three correlations, and the Blasius formula for smooth pipe and the experimental data of test section 6 for three influx over main flow rate ratios (the ratios are 1/50, 1/500 and 1/1000 respectively). From the comparisons, it's obvious that Ouyang *et al.* model almost always predicts the smallest friction factor under turbulent flow regime. In their study, they claimed that influx increases the friction factor for laminar flow and reduces the friction factor for turbulent flow. However in our study it was found that inflow could reduce the apparent friction factor only when the influx to main flow rate ratio is very small. And since no consideration was given to the perforation distribution in their modeling, the model can not differentiate between different perforation distributions. The Asheim *et al.* model in general gives closer prediction than the Ouyang *et al*. model especially when the influx from the perforated openings is small. However as we can see in Figure 15, the Asheim *et al.* model over-predicts when the influx over main flow rate ratio is high. This observation is consistent with what Yuan has observed when the Asheim *et al.* model was compared with the single perforation correlation.

Figure 18 shows the variations in prediction of pressure drop for different horizontal well hydraulics models using the horizontal well data provided in Ouyang *et al.*'s study. For simplicity, we used average fluid properties in the calculation of pressure drop. As we can see from the plot, Ouyang *et al.*'s model predicts the smallest pressure drop over the wellbore while the correlations we obtained in this study predict the largest pressure drop.

Reservoir Performance Modeling and Comprehensive Model

1. Pressure Response and Asymptotic Solution for a Continuous Point Source

We derived our solution for the slab reservoir with sealed top and bottom boundaries.

1.1 Continuous Point-Source in Laplace Domain

We can derive the pressure response $\overline{\Delta p}_{pt}(x, y, z, x_w, y_w, z_w, s)$ in a slab reservoir of thickness of h in Laplace domain for a continuous point source extracting fluid with a rate of Q at point of (x_w, y_w, z_w) by using image principle, where (*x, y, z*) is any location and s is the Laplace variable. This is illustrated in Fig. 19.

$$\overline{\Delta p} = \frac{141.2\overline{Q}(s)\,\mu}{kh}\{K_0(R_D\sqrt{s}) + 2\sum_{n=1}^{n=\infty} K_0(R_D\varepsilon_n) \cdot \cos(n\pi\frac{z_D}{h_D}) \cdot \cos(n\pi\frac{z_{wD}}{h_D})\} \quad (22a)$$

where,

$$\varepsilon_n = \sqrt{s + n^2\pi^2 / h_D^{\ 2}} \quad (22b)$$

$$R_D^2 = (x_D - x_{wD})^2 + (y_D - y_{wD})^2 \quad (22c)$$

$\overline{\Delta p}_{pt}$ is a Fourier Bessel series. The infinite series in the solution is due to the image system used to generate the effect of the sealed top and bottom boundaries.

1.2 Long-Time Asymptotic Solution for Pseudo-Radial Flow

We are not interested in the short-time solution but concerned about the long-time solution for our well completion optimization problem. In addition, we consider only pseudo-radial flow instead of seeking solution in boundary-dominated flow (pseudo-steady-state flow or steady-state flow) as shown in Fig. 20. The reason for this approach is as follows. Because the convergence of flow toward the wellbore openings will take place in the near vicinity of the well, the outer portions of the reservoir including the boundaries will not be affected by the existence of the openings. Therefore, if we derive the pseudo-skin expressions by comparing the transient pressure solutions of the open-hole completed and slotted-liner completed or perforated horizontal wells, then the same pseudo-skin factors can be incorporated into the bounded reservoir solutions for open-hole completed horizontal wells. This would represent the solution for a slotted-liner completed or perforated horizontal well in a bounded reservoir.

2. Pressure Response for a Perforated Horizontal Well

2.1 Pressure Response for Single Perforation

<u>(a). *Perforation Inclination $\psi_j \neq 0$ or π:*</u>

The geometry relationship between the j-th perforation and the i-th observation point is illustrated in Fig. 21. The real 3-D wellbore is considered. Assume the length of perforation is $L_{p,j}$, the inclination angle is ψ_j. The permeability anisotropy should be included to obtain the distorted dimensionless wellbore radius, perforation length, and inclination angle as follows

$$R_{wDj} = \frac{R_w}{l}\sqrt{\frac{k_r}{k_z}\cos^2\Psi_j + \sin^2\Psi_j} \quad (23a)$$

$$L_{PDj} = \frac{L_P}{l}\sqrt{\frac{k_r}{k_z}\cos^2\Psi_j + \sin^2\Psi_j} \quad (23b)$$

$$\Psi_j^{'} = \tan^{-1}[\sqrt{\frac{k_r}{k_z}} + \tan\Psi_j] \quad (23c)$$

where *l* is the characteristic length and we choose (*Lh/2*) as *l*. In order to use Eq. (22), we need to know R_{Dij}, the dimensionless horizontal distance between an arbitrary point on the j-th Perforation and the i-th observation point. For the triangle I'D'M', we use the law of cosine and get the distance

$$R_{Dji} = [r_D'^2 + r_{Dj,i}'^2 + 2 r_D' r_{Dj,i}' \cos\theta_{ji}]^{1/2} \quad (24a)$$

where,

$$r_{Dji} = [(x_{iD} - x_{jD})^2 + (R_{wDj} + \frac{L_{PDj}}{2})^2 \sin^2 \Psi_j']^{1/2} \quad (24b)$$

$$\theta_{ij} = \tan^{-1}[\frac{(x_{iD} - x_{jD})}{(R_{wDj} + \frac{L_{PDj}}{2}) \sin \Psi_j'}] \quad (24c)$$

To use Eq. (22), we also need to compute the dimensionless z coordinates for source point and for observation point. The z-coordinate for I-th observation point is

$$z_{iD} = z_{wD} + R_{W0D} \quad (25a)$$

where

$$R_{w0D} = \frac{R_w}{l}\sqrt{\frac{k_r}{k_z}} \quad (25b)$$

The z-coordinate for an arbitrary point on the j-th perforation is

$$z_D' = z_{wD} + \cot \Psi_j' \, r_D' + \cos \Psi_j' (R_{wDj} + \frac{L_{PDj}}{2}) \quad (25c)$$

In the following, we denote $\bar{\gamma}_{ji}$ as the follows

$$\bar{\gamma}_{ji} = K_0(R_{Dji}\sqrt{s}) + 2\sum_{n=1}^{n=\infty} K_0(R_{Dji}\varepsilon_n) \cdot \cos(n\pi \frac{z_{iD}}{h_D}) \cdot \cos(n\pi \frac{z_D'}{h_D}) \quad (26a)$$

The pressure response for single perforation can be obtained by integrating Eq. (22) along the perforation length

$$\overline{\Delta p}_{jD} = \frac{\bar{q}_{jD}(s)}{L_{PDj} \sin \Psi_j'} \int_{-.5L_{PDj}\sin\Psi_j'}^{-.5L_{PDj}\sin\Psi_j'} \bar{\gamma}_j \, dr_D' \quad (26b)$$

where $\tilde{q}_{jD}$ is the dimensionless flux through the j-th perforation

$$\bar{q}_{jD} = \frac{\bar{q}_j(L_{PDj} l)}{q} \quad (26c)$$

<u>(b). *Perforation Inclination $\psi_j = 0$ or π:*</u>

The geometric relationship between the j-th vertical perforation and the i-th is much simpler than the case of inclined perforation

$$\overline{\Delta p}_{jD} = \frac{\bar{q}_{jD}(s)}{L_{PDj}} \int_{-.5L_{PDj}}^{-.5L_{PDj}} \bar{\gamma}_j d\varepsilon_{zD} \quad (27a)$$

$$R_{Dji} = | x_{iD} - x_{iD} | \quad (27b)$$

$$z_{iD} = z_{wD} + R_{W0D} \quad (27c)$$

$$z_D' = z_{wD} + \frac{L_{PDj}}{2} + \varepsilon_{zD} + R_{w0D} \quad (27d)$$

2.2 Pressure Response for Multiple Perforations and the Asymptotic Solution

For *NP* number of perforations, we obtain the total pressure response $\overline{\Delta p}_D$ by using superposition principle. Fig. 22 shows the 3-D geometry for multiple perforations. The pressure response at any location as the sum of all the perforation sources is as follows

$$\overline{\Delta p}_D = \sum_{j=1}^{NP} \overline{\Delta p}_{jD} = \overline{\Delta p}_{Df} + \overline{\Delta p}_{D2} \quad (28a)$$

where

$$\overline{\Delta p}_{Df} = \sum_{j=1}^{NP} \frac{\bar{q}_{jD}(s)}{L_{PDj}\sin\Psi_j^{'}} \int_{-.5L_{PDj}\sin\Psi_j^{'}}^{-.5L_{PDj}\sin\Psi_j^{'}} K_0(R_{Dji}\sqrt{s})\,dr_D^{'} \quad (28b)$$

$$\overline{\Delta p}_{Df} = 2\sum_{j=1}^{NP} \{\frac{\bar{q}_{jD}(s)}{L_{PDj}\sin\Psi_j^{'}} \sum_{n=1}^{\infty}[\int_{-.5L_{PDj}\sin\Psi_j^{'}}^{-.5L_{PDj}\sin\Psi_j^{'}} \sum_{n=1}^{n=\infty} K_0(\frac{n\pi}{h_D}R_{Dji})\cos(n\pi\frac{z_D^{'}}{h_D})\,dr_D^{'} \quad (28c)$$
$$\cdot\cos(n\pi\frac{z_{iD}}{h_D})]\}$$

With long-time approximation we finally obtain the asymptotic solution by taking inverse Laplace transform

$$\Delta p_D = A + \Delta p_{D1} + \Delta p_{D2} \quad (29a)$$

where

$$\Delta p_{D1} = -\sum_{j=1}^{NP} \frac{q_{jD}}{L_{PDj}\sin\Psi_j^{'}} \int_{-.5L_{PDj}\sin\Psi_j^{'}}^{-.5L_{PDj}\sin\Psi_j^{'}} \ln(R_{Dji})\,dr_D^{'} \quad (29b)$$

$$\Delta p_{D2} = 2\sum_{j=1}^{NP} \{\frac{q_{jD}}{L_{PDj}\sin\Psi_j^{'}} \sum_{n=1}^{\infty}[\int_{-.5L_{PDj}\sin\Psi_j^{'}}^{-.5L_{PDj}\sin\Psi_j^{'}} \sum_{n=1}^{n=\infty} K_0(\frac{n\pi}{h_D}R_{Dji})\cos(n\pi\frac{z_D^{'}}{h_D})\,dr_D^{'} \quad (29c)$$
$$\cdot\cos(n\pi\frac{z_{iD}}{h_D})]\}$$

$$A = \frac{1}{2}[\ln(t_D) + 0.80709] \quad (29d)$$

For vertical perforations (ψ' = 0 or π), the term of sin(ψ') in Eq. (29) will be replaced by 1, and R_{DjI}, z_D', and z_{iD} are computed with Eqs. (27a)~(27c).

3. Pressure Response for Slotted-Liner Completed Horizontal Wells

Similarly to the case of perforating completion, we integrate the point-sink solution $\overline{\Delta p}_{D,pt}$ to obtain the solution for the m-th slot with length l_m and center at (x_m, y_m, z_m). Using superposition principle to incorporate the effect of all slots (*MS* is the total number of slots on the wellbore), we obtain the total pressure response in Laplace domain, which is obviously the function of slot geometry and distribution. Fig. 23 shows the multiple-slot geometry. For long-time asymptotic solution, we take the approximate expression for *K0(x)* and evaluate the integrands for each slot. Finally, we get the total pressure response on the wellbore (specifically at the top of the wellbore with $y_D = 0$, $z_D = z_{wD}+r_{wD}$) in real time domain by taking inverse Laplace transform. The pressure response is as follows

$$\Delta p_D = A + \Delta p_{D1} + \Delta p_{D2} \quad (30a)$$

$$\Delta p_{D1}(x_D) = -\sum_{m=1}^{MS} \frac{q_{mD}}{l_{mD}} \int_{x_{mD}-\frac{l_{mD}}{2}}^{x_{mD}+\frac{l_{mD}}{2}} \ln[\sqrt{(x_D - x_D^{'})^2 + y_{mD}^{\ 2}}]\,dx_D^{'} \quad (30b)$$

$$\Delta p_{D2}(x_D) = 2\cdot\sum_{m=1}^{MS}\{\frac{q_{mD}}{l_{mD}}\sum_{n=1}^{\infty}[\int_{x_{mD}-\frac{l_{mD}}{2}}^{x_{mD}+\frac{l_{mD}}{2}} K_0[n\pi L_D\cdot\sqrt{(x_D - x_D^{'})^2 + y_{mD}^{\ 2}}]\,dx_D^{'} \quad (30c)$$
$$\cdot\cos[n\pi\,(z_{wD} + R_{W0D})]\cdot\cos(n\pi\, z_{m,D})]\}$$

where q_{mD} is the dimensionless flux through the m-th slot

$$q_{mD} = \frac{q_m l_m}{q} \quad (30d)$$

4. Discrete Form of Pressure Response for Perforated Horizontal Wells

A perforated horizontal well has much more perforations than a vertical well (*e.g.*, there exist (1000 ft * 4 spf) = 4000 shots of perforations on a wellbore of 1000 ft long). We discretize the wellbore length into *2M* segments with uniform flux in each segment. The segment length is equal to (*Lh/2M*). Let $m_0(I)$ be the starting sequential number and $m_1(I)$ be the ending sequential number of the perforations in the *I*-th segment. For any location x_{DJ} on the wellbore, the pressure response is

$$\Delta p_D(x_{DJ}) = A - \sum_{I=1}^{2M} q_{mD}(I)\{\sum_{m=m_0(I)}^{m_1(I)} I_1\} + 2.\sum_{I=1}^{2M} q_{mD}(I)\{\sum_{m=m_0(I)}^{m_1(I)} I_2\} \quad (31a)$$

where,

$$I_1 = \frac{1}{L_{PD\,m}\sin\Psi'_m}\int_{-\frac{L_{PD\,m}\cdot\sin\Psi m}{2}}^{\frac{L_{PD\,m}\cdot\sin\Psi m}{2}} \ln[R_{DmJ}]\,dr'_D \quad (31b)$$

$$I_2 = \sum_{n=1}^{\infty}\frac{1}{L_{PD\,m}\sin\Psi'_m}\int_{-\frac{L_{PD\,m}\sin\Psi'_m}{2}}^{\frac{L_{PD\,m}\sin\Psi'_m}{2}} K_0[\frac{n\pi}{h_D}\cdot R_{DmJ}]\cdot\cos[\frac{n\pi}{h_D}z'_{Dm})]dr'_D\cdot\cos(\frac{n\pi}{h_D}z_{JD}) \quad (31c)$$

Note that $q_{mD}(I)$ is the dimensionless flux through the *m*-th perforation on the *I*-th segment (*I=1,2,…,2M*). We need to relate it with $q_{hD}(I)$, the flux in the *I*-th segment in order to solve the coupling equation that will be discussed later

$$q_{hD}(I) = \frac{q_h(I)L_h}{q} \quad (32a)$$

Assume that there is *MP* number of perforations in the *I*-th wellbore segment, and the total flow rate into the *I*-th segment, $\tilde{q}_h(I)$, is

$$q_h(I) = \frac{q_m(I)\cdot L_{PD\,m} l\cdot MP}{L_h/(2M)} \quad (32b)$$

Thus

$$q_{mD} = \frac{q_m(I)\cdot(L_{PD\,m}l)}{q} = \frac{q_m(I)\cdot(L_{PD\,m}l)}{L_h/(2M)\cdot q}\frac{MP}{MP}\frac{L_h}{2M} = \frac{q_h L_h}{q}\frac{1}{(MP\cdot 2M)} = \frac{q_{hD}}{2M\cdot MP}. \quad (32c)$$

In fact, $2M\cdot MP$ is the total number of perforations on the whole wellbore. Replace $q_{mD}(I)$ with $q_{hD}(I)$ and we get

$$\Delta p_D(x_{DJ}) = A + \sum_{I=1}^{2M} q_{hD}(I)\cdot(\sigma_I + F_I) \quad (33a)$$

where

$$\sigma_I = -\frac{1}{2M\cdot MP}\{\sum_{m=m_0(I)}^{m_1(I)} I_1\} \quad (33b)$$

$$F_I = \frac{2}{2M\cdot MP}\{\sum_{m=m_0(I)}^{m_1(I)} I_2\} \quad (33c)$$

Notice that I_2 includes the integration of modified Bessel function. The integrand is in the form of $K_0(u)$. We can use Chebyshev polynomials to approximate the modified Bessel function of K_0 with

Clenshaw's recurrence formula for the summation. Finally we numerically solve the integration with Chebyshev's coefficients. In addition, I_1 can be evaluated by deriving an accurate analytical expression.

5. Discrete Form of Pressure Response for Slotted-Liner Completed Horizontal Wells

Similarly to the perforated well, we divide the wellbore length into *2M* segments with each segment of length ($L_h/2M$). For any location x_{DJ} on the wellbore, the pressure response is

$$\Delta p_D^*(x_{DJ}) = A - \sum_{I=1}^{2M} \frac{q_{mD}(I)}{l_{mD}} \{ \sum_{m=m_0(I)}^{m_1(I)} I_1 \} + 2.\sum_{I=1}^{2M} \frac{q_{mD}(I)}{l_{mD}} \{ \sum_{m=m_0(I)}^{m_1(I)} \sum_{n=1}^{\infty} I_2 \} , \quad (34a)$$

where,

$$I_1 = \int_{x_{mD}-\frac{l_{mD}}{2}}^{x_{mD}+\frac{l_{mD}}{2}} \ln[\sqrt{(x_{DJ} - x_D')^2 + y_{mD}^{\;2}}]\, dx_D' \quad (34b)$$

$$I_2 = \int_{x_{mD}-\frac{l_{mD}}{2}}^{x_{mD}+\frac{l_{mD}}{2}} K_0[n\pi L_D \cdot \sqrt{(x_D - x_D')^2 + y_{mD}^{\;2}}]\, dx_D' \cdot \cos[n\pi (z_{wD} + r_{WD})] \cdot \cos(n\pi z_{mD}) . \quad (34c)$$

Notice that $q_{mD}(I) = q_m l_m / q$ is used in the above equations. Based on the same reason as discussed in perforated wells, we need to invert $q_{mD}(I)$ into $q_{hD}(I)$. Assume that there are NR rings of slots in each wellbore segment, and NS slots on each ring, as shown in Fig. 24. Thus the total number of slots in one segment is (NR*NS), and the total flow rate into the *I*-th segment is $q_m(I) \cdot l_m \cdot (NR \cdot NS)$. The flux on the *I*-th segment (the length is $L_h/2M$) is

$$\tilde{q}_h(I) = \frac{\tilde{q}_m(I) \cdot l_m \cdot (NR \cdot NS)}{L_h/(2M)} \quad (35a)$$

Thus

$$\begin{aligned} q_{mD} &= \frac{q_m(I) l_m}{q} \\ &= \frac{q_m(I) l_m}{q} \frac{(NR \cdot NS)}{L_h/(2M)} L_h \frac{L_h/(2M)}{L_h} \frac{1}{(NR \cdot NS)} . \\ &= \frac{q_h L_h}{q} \frac{L_h/(2M)}{L_h} \frac{1}{(NR \cdot NS)} \\ &= \frac{q_{hD}}{2M \cdot (NR \cdot NS)} \end{aligned} \quad (35b)$$

In fact, $2M \cdot (NR \cdot NS)$ is the total number of slots on the whole wellbore. Replace $q_{mD}(I)$ with $q_{hD}(I)$ and we get

$$\Delta p_D^*(x_{DJ}) = A + \sum_{I=1}^{2M} q_{hD}(I) \cdot (\sigma_I + F_I) \quad (36a)$$

where

$$\sigma_I = -\frac{1}{l_{mD} \cdot 2M \cdot NR \cdot NS} \{ \sum_{m=m_0(I)}^{m_1(I)} I_1 \} \quad (36b)$$

$$F_I = 2.\frac{1}{l_{mD} \cdot 2M \cdot NR \cdot NS} \{ \sum_{m=m_0(I)}^{m_1(I)} \sum_{n=1}^{\infty} I_2 \} \quad (36c)$$

Notice that I_2 includes the integration of modified Bessel function. The integrand is in the form of $K_0(\sqrt{x^2 + a^2})$. We can use Chebyshev polynomials to approximate the modified Bessel function of K_0 with Clenshaw's recurrence formula for the summation. Finally we numerically solve the integration with Chebyshev's coefficients. In addition, I_1 can be evaluated by deriving an analytical expression. However, such accurate calculations are only adopted for the long slots (for instance, the partially completed openhole can be treated as slotted liner wellbore with several long slots and long distances between slots). For short slots, we can simply take mean value of the integrand multiplying by the integration interval length to obtain approximate but fast results for the integration. Normally, it gives sufficiently accurate results.

6. Mechanical Skin

In the process of drilling and completing a horizontal well, the formation is usually damaged by mud filtration fluid or solid debris. Some dirt may also plug some completion openings (perforations or slots). Any of these factors which result in changes in the natural productivity may be categorized into the term of mechanical skin factor, *SF*. In this model, the skin along the wellbore may change from location to location. We use the following modification to consider of the contribution of skin.

$$p_D(x_{Dj}) = p_D(x_{Dj}) + q_{hD,J} \cdot SF(J), \quad J = 1, ..., 2M \tag{37}$$

We perform this simple addition of the skin effect is based on our following understanding. The skin only causes an additional pressure drop to the wellbore, but does not affect the reservoir pressure distribution. Furthermore, we incorporate the effect of skin only onto the segment where the skin exists. This is because that skin only produces additional pressure drop on its own segment.

7. Coupling Procedure and Numerical Solution Algorithm

The wellbore hydraulics equation can be expressed as

$$p_{wD}(t_D) - p_D(x_{D,} t_D) = \frac{f_t N_{\mathrm{Re},t}}{16} \frac{\pi}{C_{hD}} [2x_D - \int_0^{x_D}\int_0^{x_D'} \frac{D}{N_{\mathrm{Re}t} f_t} q_{hD} dx_D'' dx_D'] \tag{38a}$$

where C_{hD} and D are functions of wellbore Reynolds' number and friction factor

$$C_{hD} = \frac{7.395\times 10^{13} r_w^4}{kh L_h}, \tag{38b}$$

$$D = N_{\mathrm{Re}}^2 \frac{df_F}{dN_{\mathrm{Re}}} + 2N_{\mathrm{Re}} f_F . \tag{38c}$$

By dividing the wellbore into *M* segments we obtain the discrete form of the above equation

$$p_{wD}(t_D) - p_D(x_{DJ,} t_D) = \frac{f_t N_{\mathrm{Re},t}}{8} \frac{\pi x_{DJ}}{C_{hD}} - \frac{\pi}{16 \cdot C_{hD}} \{\sum_{I=1}^{J-1} (x_{DJ,} - \frac{2I-1}{2M}) \frac{1}{M} D_I\, q_{hD,I} + \frac{D_J q_{hD,J}}{8M^2}\} \tag{39}$$

We have (2M +1) unknowns (p_{wD} and $q_{hD,,I}$, I=1,…,2M). We obtain 2M equations by evaluating Eq. (13) at x_{DJ}, the center of each segment. An additional equation is that the sum of the dimensionless flux is 2M. Let $Q_I = p_{wD}$, $Q_I = q_{hD,,I-1}$ (*I=2,…,2M+1*). The (2M+1) equation can be written as

$$\sum_{I}^{2M+1} Q_I \cdot G(J,I) = B(J), \quad J = 1, ..., 2M+1 \tag{40a}$$

where G(I,J) is a function of flux distribution. Thus, this is a non-linear system. Let

$$F(Q) = G(Q)\cdot Q - B \qquad (40b)$$

where $G(Q)$ is a square matrix with dimension of (2M+1), and B and Q are (2M+1)×1 vectors.

We use Newton's iteration method by computing Jacobian matrix evaluated at $Q^{(k)}$ (the k-th iteration) and calculate incremental the vector $\delta Q^{(k)}$ by calling matrix solver of *LU* decomposition subroutine.

For computational implementation, we need to construct an algorithm to cope with the summation of the huge number of perforations or slots. The geometric distribution of the perforations is controlled by perforation length, perforation density, and phase angle. The geometric distribution of the slots should be controlled by inputting some limited parameters such as the length of slot (***ls***), the distance between two adjacent slot rings (***ee***), slot array phasing (***PHA***), slot number of each concentrated slot array (***NG***), distance of adjacent slots in one concentrated slot array (***de***), and the starting location of spacing slots ($\boldsymbol{x_0}$). These parameters are illustrated in Fig.?

8. Application of New Correlations for Apparent-Friction Factor

The regression correlations for the apparent friction factor, f_T, are obtained from the experimental data. According to the principle of modeling and similitude, the f_T correlation from small-scale model experiment can be directly used to the situation of large pipe diameter. This is because the regression equations are expressed in the dimensionless form of $f_T = function(N_{\mathrm{Re}}, d_{pipe} * \varphi, \frac{q_{\inf lux}}{\overline{Q}})$, where, N_{Re}, the Reynolds number, is related to dynamic similarity, $d_{pipe}*\varphi$, is related to geometry similarity, and $\frac{q_{\inf lux}}{\overline{Q}}$ is related to kinematic similarity. With same dimensionless parameters, the f_T given by the model will be equal to the corresponding f_T for the prototype. Thus, we can substitute the actual value of parameters into the model equations when applying them to the comprehensive coupling equations.

However, the empirical correlations from regression analysis are only valid over the range of parameters covered by the experiments. It will be risk to extrapolate beyond the range of parameters. For example, the multiple perforation pipe tests were performed with density of 5, 10 and 20 shots which may higher than the conventionally used perforation density (normally 1~8 shots per feet). Thus it is unwise to use the empirical relationship to the cases with density less than 5 shots/ft. Of course, if the distance between neighboring perforations is larger than 8*d_{pipe} (e.g., 5 *in.* casing with density of 0.2 spf, or one perforation controls 1/0.2 = 5 *ft.* = 60 *in.* > 8 d_{pipe} =40 *in.*), the single-perforation model should be used. For most of the cases (developing flow), the perforation distribution is denser than the single-perforation situation (developed flow), and sparser than the conditions of 5~20 spf. In such cases, we need to figure out an approach to use the correlations alternatively.

As we know that the inflow flux around the pipe affects the friction significantly, we may assume that the friction factor f_T be the same when the inflow flux (mass rate) keeps the same value for a unit length of pipe. Based on this idea, we infer that the f_T be the same if the opening areas on the pipe circumference are the same in unit length. At the same time, we need to consider the aspect ratio (d_{pipe}/L_{pipe}) which expresses the geometric similarity. Using subscript "m" for model, we need equal aspect ratio

$$\frac{d}{L} = \frac{d_m}{L_m} \tag{41a}$$

For example, d_m = 1 *in.*, d = 6 *in.*, L_m : L = 1 : 6.

The equivalent open-flow-area principle, for applying model correlations to actual situation where perforation density is out of the testing range, is expressed as follows

$$\frac{A_{p,m}\varphi_{p,m}}{L_m} = \frac{A_{p.}\varphi_p}{L} \tag{41b}$$

or

$$\varphi_{p,m} = \frac{A_{p.}}{A_{p,m}}\frac{d_m}{d}\varphi_p \tag{41c}$$

where,

$\varphi_{p,m}$ = the transferred perforation density which will be used in the model equation, *spf*;
φ_p = the real perforation density, *spf*;
A_p = the real single-perforation area, *in.*^{2};
$A_{p,m}$ = the single-perforation area used on the model pipe, *in.*^{2};
d = the real pipe diameter, *ft.;*
d_m = the model pipe diameter, *ft.*

For instance, d_m = 1 *in.*, d = 6 *in.*, $d_{p,m}$ = 1/8 *in.*, d_p = 3/4 *in.*, φ_p = 2 spf, then

$$\varphi_{p,m} = \frac{\frac{\pi}{4}(\frac{6}{8})^2}{\frac{\pi}{4}(\frac{1}{8})^2}\cdot\frac{1}{6}\cdot 2 = \frac{6^2\cdot 2}{6} = 12\,(spf)$$

Thus, we transfer φ_p = 2 spf to $\varphi_{p,m}$ = 12 spf which is in the range of the tested density.

Meanwhile, the second term on the R.H.S of f_T correlation (Eq. (15)), $C_n \cdot 2d\varphi \cdot q_{in} / \overline{Q}$, includes the product of $d\varphi$. For the same reason we should substitute $d_m\varphi_m$ into the calculation. For above example, we need to use $d_m\varphi_m$ = (12 spf * 1/12 ft) = 1. The real value of $d\varphi$ should be avoided because we are treating the formula apparently for the data out of the range.

For slotted liner, the density value seems in the normal range, and we can use the actual parameter values in the calculation of f_T.

To summarize, our model combines the reservoir model, the wellbore hydraulics, the non-uniform mechanical skin, and the restricted entry (slots or perforations) together to obtain a correct picture of the flow characteristics for the purpose of completion optimization. This is illustrated in Fig. 25.

Results and Discussion

1. Flux and Pressure drop Distribution Characteristics

We take the limiting case of slotted-liner completion by setting slots distance close to zero, and only one row of slots on the top of the wellbore. This should represent the openhole performance.

Fig. 26a gives the comparison for flow rate and flux between the openhole and the limiting case of slotted liner. Obviously, they match very well.

Fig. 26b indicates good match for DP and PI. The DP of the limited slotted liner is 47.5, and the PI is 632, while the openhole well has pressure drop of 48.7

and PI of 607.7. The small discrepancy is due to our neglecting the effect of no-flow wellbore surface in our model. The results show that such approximation is acceptable for the long-term behavior of completed horizontal wells.

2. The Effect of Slot Length and Slot Distance, and Slot Density

We take the limited case of Fig. 26b as the base case with slot length *ls* = 73 *in.,* and the distance between the adjacent slots *e* = 0.02 *in*.. We know it has DP of 47.5 psi, and the PI of 632 b/d/psi..

Now increasing the distance to *ee* = 36.5 in. by keeping *ls* = 73 in., we get larger pressure drop with DP = 51 psi and smaller productivity of PI = 582 b/d/psi.. If we further reduce the length to *ls* = 36.5 in. and keep ee = 36.5 in., we get even larger pressure drop of DP = 56.7 psi., and smaller productivity of PI = 528 b/d/psi..

In addition, if we reduce both length and distance by taking *ls = 2.5* in. and *ee* = 2.5 in., we will get less pressure drop with DP = 47.4 psi, and increase productivity to PI = 632.3 psi.. Although the densities are the same for the case of *ls = 2.5 in*. and *ee = 2.5 in.,* and for the case of *ls* = 36.5 in. and *ee* = 36.5 in., the different combinations of length and distance sizes give different results.

Thus, we find that both slot length and slot distance have significant effect on the pressure drop and productivity. Even for constant density, smaller slots give higher PI.

We also find that in case of low productivity, flux is more uniform and less skewed. In other words, low slot density results in higher pressure drawdown, and more uniform flux distribution. Fig. (27a) and Fig. (27b) compare high density case (*ls* = 2.5, *ee* =2.5, PI = 632) with low density case (*ls* = 0.5*, ee* = 5, PI = 364).

3. The Effect of Slot Phasing Angle and Slot Concentration

The slot phase angle (phasing) is defined as the angle between two adjacent slot array (slot concentration) in one slot ring, measured in degree. If the phasing is zero (one single slot array in one slot ring), we may call phasing of 360°. One slot ring may include slot arrays of number of 360/*PHA*, while each slot array has *NG* concentrated slots with the distance of ***de*** between the adjacent concentrated slots. For example, we may have phasing of 90° with 4 slot arrays (4=360/90), while each array includes 3 slots with distance of 0.5 in. between the neighboring slots within the array.

We have tested the effect of slot phasing on the PI and DP. We found phasing has little effect on DP and PI. For instance, given slot length of *ls* = 1.5 in. and distance between neighboring slot rings of *ee* = 10 in., we get (1) PI = 423 (DP=70.8) for PHA = 360°, (2) PI = 417.5 (DP = 71.9) for PHA = 120°, and (3) PI = 417.9 (DP = 71.8) for PHA=60°.

In addition, slot concentration has little effect. For example, given PHA = 360°, *ls* = 1.5 in, *ee* = 10 in., we get (1) PI = 423 (DP=70.8) for NG=1 (one slot in one array), (2) PI = 422 (DP = 71.1) for NG = 4 and de = 1 in.(4 slots in one array with distance of 1 in.).

The above results can be explained by analyzing the model equations. Eq. (33) expresses that the pressure response on the reservoir side, Δp_D, is function of $\sigma_I(I)$ and $F_I(I)$, I = 1, …, $2M$. $\sigma_I(I)$ and $F_I(I)$ are functions of I_1 and I_2 (defined in Eqs. (31b) and (31c)) respectively,

$$\sigma_I = -\frac{1}{l_{mD} \cdot 2M \cdot NR \cdot NS}\{\sum_{m=m_0(I)}^{m_1(I)} I_1(m)\}, \quad (33b)$$

$$F_I = 2.\frac{1}{l_{mD} \cdot 2M \cdot NR \cdot NS}\{\sum_{m=m_0(I)}^{m_1(I)} \sum_{n=1}^{\infty} I_2(m)\} \quad (33c)$$

I_1 and I_2 are related to slot geometry (length l_m, coordinates of x_m, y_m, and z_m). y_m, and z_m are small quantities (small wellbore diameter restrains y_m and z_m), while x_m varies on large scale for the long horizontal wellbore. The change of phasing will only affect y_m, and z_m. Thus the difference from the varying phasing will be very small. Nevertheless, we may say that phasing also changes the total number of slots in one segment. Then, why does it not change the system response? For example, let us say there are NS slots in one ring, and NR rings in one segment. Suppose we have NS = 1 for phasing of 360°, then we have NS = 4 for phasing of 90°. Thus, we have more I_1 and I_2 term in the summations of Eqs. (33b) and (33c). However, each of the extra terms is almost the same as the original terms for the slots in the same ring (y_m and z_m result in little difference for the slots in one ring). Although the NS increases in one ring, the summation divided by NS (Eq. (33b) and (33c)) gives almost the same results. This explains the negligible effect of slot concentration in one slot array. In other words, the long wellbore makes the phasing and concentration effects invisible. For very short wellbore length (vertical well), the phasing plays much more important rule.

4. The Effect of Perforating Parameters (Density, Phasing and Length)

The sensitivity analysis indicates that perforation density has more significant effect on the PI than that of phasing and perforation penetration length.

Fig. 28 shows the effect of perforation density on PI. From the figure we find that the perforation density has obvious effect on PI before density reaches 1 shot/ft. Beyond density of 1 shot/ft, the effect becomes small.

Perforation penetrating depth has smaller effect on horizontal well performance, as shown in Fig. 29.

Perforation phase angle has the least effect on productivity. The difference is very small among the PI values for different phasing. Under significant permeability anisotropy, the phase has slightly increasing effect. Phasing of 90° is the best one, and phase of 360 is the worst one. In the middle are 45°, 60°, and 180° in the order from high to low effects.

5. The Effect of Partial Completion

For slotted-liner or perforting completion along the long horizontal wellbore, we may have some blind-pipe segments which insulate the flow from the reservoir. This scenario also need to be simulated. The following are the results obtained from our program.

Assume a slotted-liner completed horizontal well. First, we space slots along the full length of the wellbore with *ls =2.5* in., and *ee* = 2.5 in.. We get PI = 632.2 (DP = 47.4). Then we divide the wellbore into seven segments, and space slots on four separated segments out of the seven (two end segments are open to reservoir flow). We get PI = 571 (DP = 52.5). Finally, we divide the wellbore into six segments and space slots on three separated segments out of the six. We get much smaller PI of 497 (DP = 60.3). Fig. 30 shows the flux distribution characteristics as we expected, *i.e*, the flux distribution inside each segment is also a skewed U-shaped curve. We reach conclusion that both the penetration ratio and the open segment locations affect the productivity.

Software Development with Window Graphical Interface

1. HORCOM --- a Borland C++ 4.0 GUI Software

The model computing algorithm has been written into a computer program. After preliminary testing and running, we started to develop a Window interface software that provides a user-friendly graphical environment. The software is developed on the Borland C++ Builder 4 which can easily use the Win32 GUI (graphical user interface) and also for console C++ application. We name the software **HORCOM** Version 1.0 that represents "HORizontal well COMpletion Optimization". HORCOM can be used independently without the software development environment.

First, create a directory in your computer hard disk to operate the software, and copy HORCOM into your the directory. Then you are ready to use it.

2. Main Menu and Speedbar

Double click the executable file HORCOM, the window graphic interface pops on the screen. A graphic icon with a conceptual picture of horizontal well and colored text of "Horizontal Well Completion Optimization" appears in front of the gray background. On the top of the screen are the design and analysis menus which can be chosen by mouse click. Below the top main window menu are some toolbars which help to open files, edit, and view multiple windows.

We illustrate main menu and toolbar with the examples of **File**, and **Edit** in the following.

The menu File consists of drop-down menu: New, Open, Close, Save, Save as, and Exit. The underscore letter means that you may use "Alt. plus letter" approach to activate the menu bar. For instance, you may press "Alt" then press "F", the File menu will be activated. Without releasing the key of "Alt", further press "O" will pop out the open dialogue frame. You may use mouse to directly point the menu and sub-menu. It is required that only text file can be operated with the File menu. Click "Exit" under File menu will end the application of HORCOM. Another alternate way is to click the toolbar to perform "Open" and "New" file operation.

Menu "Edit" is used to perform "Cut", "Copy" and "Paste". You can also click the toolbar for the same operation after you select you text object.

3. Data Files for Input and Output

The menu of "File" and "Edit" is mainly used to perform data file operation. We have four **input data files**: "Reservoir1.dat", "WellborFluid1.dat", "Slot1.dat" and "Perf1.dat".

We also have some **output data files** such as "Ohor1.dat", "Ohor2.dat", and "Ohor3.dat" for open-hole completion. For slot-liner completion output files, we add *SL after the filenames for openhole, such as* "Ohor1_SL.dat", "Ohor2_SL.dat", and "Ohor3_SL.dat". For perforation completion, we have files of "Ohor1_PERF.dat", "Ohor2_ PERF.dat", and "Ohor3_ PERF.dat".

The menu of "**Window**" is used to perform such operation on the multiple window as "Cascade", "Tile Horizontally", ", "Tile Vertically", and "Minimize All". You can also use the toolbar below the main menu.

4. Graphical Input Menu

We have four submenus under the main menu of "**INPUT**".

If you click Reservoir File under "Input", a sub-window will be created for inputing the data for reservoir characteristics. Use mouse to point to the text box or radial-group box and input the values by entering the keyboard. Clicking "OK" button will save your input to the file of "Reservoir1.dat" and exit this window.

Similarly, if you click "Perforating File" under the "Input" menu, you will be prompted to input related value by using edit box, radial-group box and form. The row number of the form will changed with your input value of "Number of Wellbore Section". You may use mouse to click and move the column and row of the form.

The same operation is need to submenu of "Slotted Liner File". If you are not sure about the meaning of the slot parameters, you may click "**Help**" to open the picture of slotted-line completion geometry.

Similar operation is need for "WellboreFluid File".

5. RUN

Under this menu, there are three submenus for "Openhole Well", "Slotted-Liner Completion" and "Perforated Completion". By clicking you choice, the software automatically accomplishes the computation for the three kinds of completion. You can watch some output information on the screen that indicate where the program is running and whether or not it has finished running.

6. Output

After running the program or if you have already created output files from previous run, you may perform the operation of the menu "**Output**".

The Output menu includes three submenus: "**Data**", "**Lable**" and "**Plot**". At the beginning, the Output menu is dimmed (unable). Once you click "Open" under the "File" menu, the Output menu is enabled. Then you can click "Data" and a form is presented for your to take a look at the data. You can even change some values of the data. You may click "Label" to change the default axis labels of "x" and "y" to the axis labels you desire to use. Then you may click the "Plot" submenu to plot the figure. You can press "Print Screen" button on the top right of you keyboard. Then you can paste your picture to other places (e.g. paste to Word or Powerpoint".

For checking the version of this product, you can click "**Help**" and click "**About",** the brief information of the production will be shown up with a TU logo.

Some Guideline for Horizontal Completion

1. Wellbore hydraulics plays an important role in the well performance. The new experimental correlations for friction factor from this study should be used for accurate prediction and design of horizontal well completion. When the control length by single slot/perforation is larger than eight times the pipe diameter, the single-opening model should be used. Otherwise, the multiple-opening models should be used.
2. For slotted-liner or perforating completed well, use phase angle of 90°. One reason is to obtain more uniform flux distribution and less flow convergence toward the wellbore. Another reason is to reduce the friction to the flow in the wellbore.
3. The slot-penetration ratio (slotted section length over the total section length) or perforation density has significant effect on the productivity of horizontal well. For slotted liner

completed well, the slot-penetration ratio should reach to 50%. For perforated well, the perforation density should be larger than 0.4 spf but less than 1.0 spf in order to obtain sufficiently large productivity and cost-effective operation.

4. Slot length has significant effect on the productivity. Under fixed slot-penetration ratio, choosing slot length as small as several inches might be a better practice than choosing longer slot length as large as several feet.
5. Perforation penetration depth does not affect well flow performance. For perforation completion, we can use small shaped-charge (e.g., charge weight less than less than 20g) to guarantee smallest damage to the casing mechanical strength and integrity, and also control permeability reduction in the perforation crushed-zone, without significant loss of productivity.

References

1. Asheim, H., Kolnes, J., and Oudeman, P.: "A Flow Resistance Correlation for Completed Wellbore," Journal of Petroleum Science and Engineering, 8 (1992), pp. 97-104

2. Kloster, J.: “Experimental Research on Flow Resistance in Perforated Pipe,” M.S. Thesis, Norwegian Institute of Techjnology, 1990

3. Yuan, H: “Investigation of Single Phase Liquid Flow Behavior in a Single Perforation Horizontal Well,” M.S. Thesis, The University of Tulsa, 1994.

4. Yuan, H: “Investigation of Single Phase Liquid Flow Behavior in Horizontal Wells,” Ph.D. Dissertation, The University of Tulsa, 1997.

5. Dikken, B.J.: “Pressure Drop in Horizontal Wells and Its Effect on Production Performance,” *JPT* (Nov. 1990) 1426-1433.

6. Ozkan, E., Sarica, C., Haciislamoglu, M., and Raghavan, R.: “Effect of Conductivity on Horizontal Well Pressure Behavior,” *SPE Advanced Technology Series*, Vol. 3, No. 1 (March 1995) 85-94.

7. Ozkan, E., Sarica, C., Haciislamoglu, M., and Raghavan, R.: “The Influence of Pressure Drop along the Wellbore on Horizontal Well Productivity,” paper SPE 25502 presented at the SPE Production Operation Symposium, Oklahoma City, OK, Mar. 21-23, 1993.

8. Yildiz, T., Ozkan, E.: “Transient Pressure Behavior of Selectively Completed Horizontal Wells,” paper SPE 28388 presented at the SPE 69th Annual Technical Conference and Exhibition held in New Orleans, LA, U.S.A., Sept. 25-28, 1994.

9. Ahmed, G., Horne, R.N., and Brigham, W.E.: “Theorical Development of Flow into a Well through Perforations,” DOE/BC/14126-25 (1990).

10. Spivak, D., and Horne, R.N.: “Unsteady-State Pressure Response due to Production with a Slotted Liner Completion,” paper SPE 10785 presented at the 1982 SPE California Regional Meeting, San Francisco, CA, March. 24-26, 1982.

11. Hazenberg, G., and Panu, U.S.: “Theoretical Analysis of Flow Rate into Perforated Drain Tubes,” *Water*

Resources Research, Vol.27, No.7, 1411-1418 (July 1991).

12. Landman, M.J., Goldthorpe, W.H.: "Optimization of Perforation Distribution for Horizontal Wells," paper SPE 23005 presented at the SPE Asia-Pacific Conference held in Perth, Western Australia, Nov. 4-7, 1991.

13. Yildiz, T., Ozkan, E.: "Pressure-Transient Analysis for Perforated Wells," paper SPE 49138 prepared for presentation at the 1998 SPE Annual Technical Conference and Exhibition held in New Orleans, LA, U.S.A., Sept. 27-30, 1998.

14. Ozkan, E., Yildiz, T., and Raghavan, R.: "Pressure-Transient Analysis of Perforated Slant and Horizontal Wells," paper SPE 56421 presented at the 1999 SPE Annual Technical Conference and Exhibition held in Houston, Texas, 3–6 October 1999.

15. Ouyang, L., Arbabi, S. and Aziz, K.: "General Wellbore Flow Model for Horizontal, Vertical, and Slanted Well Completions," SPE 36608, presented at 1996 SPE Annual Technical Conference and Exhibition, Denver, Colorado, Oct. 6-9, 1996.